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THE OPEN UNIVERSITY OF SRI LANKA
Faculty of Engineering Technology
Department of Electrical & Computer Engineering

Bachelor of Software Engineering Honours

Final Examination (2016/2017)
ECZ3161 – Mathematics for Computing

Date: 26th November 2017 (Sunday) **Time: 0930 hrs – 1230 hrs**

Instructions:

- *Answer **five** questions only.*
- *Show intermediate steps clearly.*
- *Programmable calculators are not allowed.*
- *The number of questions of the paper is eight (08).*
- *The number of pages of the paper is four (04).*

Q1

(a) Let A, B , C and D be Boolean variables. Prove the following identities.

(i) $(\overline{\bar{A}.B + B.\bar{C}}).\overline{A.\bar{B}} = \bar{A}.\bar{B} + A.B.C$

(ii) $AB + A(B + C) + B(B + C) = B + A.C$

(iii) $(A\bar{B}(C + BD) + \bar{A}\bar{B})C = \bar{B}C$ [9 marks]

(b) Let p, q and r be propositions. By constructing truth tables, show that the following propositions are equivalent.

(i) $p \leftrightarrow q \equiv (p \to q) \land (q \to p)$

(ii) $(p \lor q) \to r \equiv (p \to r) \land (q \to r)$ [6 marks]

(c) By using the Karnaugh maps method, minimize the following.
$\bar{P}.\bar{Q}.\bar{R} + \bar{P}.Q.\bar{R} + P.Q.\bar{R}$ [5 marks]

Q2

(a) If $A = \begin{bmatrix} 6 & -2 \\ -4 & 1 \end{bmatrix}$, then prove that $A^2 = 7A + 2I$, where I is the 2x2 identity matrix.
[3 marks]

(b)

(i) Let $\boldsymbol{A} = \begin{bmatrix} 2 & -2 \\ 2 & -2 \end{bmatrix}$. Is the matrix **A** nilpotent ? Justify your answer.
[3 marks]

(ii) If $A = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 6 \\ 1 & 4 \\ 5 & 2 \end{bmatrix}$ then find AB. [3 marks]

(iii) If $A = \begin{bmatrix} 0 & 3 & 4 \\ -3 & 0 & 7 \\ -4 & -7 & 0 \end{bmatrix}$, prove that $A^T + A = \bar{0}$, where $\bar{0}$ represents 3x3 zero matrix.

[3 marks]

(c) Using the method of Gaussian elimination, solve the following system of linear equations.

$$\begin{aligned} 4x + 8y - 4z &= 4 \\ 3x + 8y + 5z &= -11 \\ -2x + y + 12z &= -17 \end{aligned}$$

[8 marks]

Q3

(a) Find the equation of the tangent to the curve of $y = 2x^3 + 3x + 7$ at the point x =2.

[6 marks]

(b) Find $\dfrac{dy}{dx}$ for the following.

(i) $y = 5x^2e^{3x}$

(ii) $y = \dfrac{sin3x}{4+5cos2x}$

[8 marks]

(c) If $y_n = tan^n x$, then prove that $\dfrac{dy_n}{dx} = n(y_{n-1} + y_{n+1})$.

[6 marks]

Q4

(a) Find the following indefinite integral.

$$\int \frac{1}{x^3-1} dx$$

[6 marks]

(b) Using integration by parts, find the following indefinite integral.

$$\int sinx\ ln(cosx)\ dx$$

[8 marks]

(c) Evaluate the following definite integral.

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} sin^3 x cosx dx$$

[6 marks]

Q5

(a) Sketch the graph of the following function for $x \in [0, 2\pi]$.
Write-down the amplitude and period.

$$y = 2cos\left(x - \frac{\pi}{2}\right) + 1$$ [6 marks]

(b)

(i) Solve the equation $5sinx - 2cos^2x - 1 = 0$ for $0 \leq x < 360^0$.
[4 marks]

(ii) The angles of depression of the top and the bottom of a 12 m high tree from the top of the building are 45 degree and 60 degree respectively. Calculate the height of the building.
[4 marks]

(c) Prove the following identities.

(i) $tan^4x + tan^2x = sec^4x - sec^2x$

(ii) $(1 - cos^2x)(1 + cos^2x) = 2sin^2x - sin^4x$

(iii) $\frac{1+tan^2x}{1-tan^2x} = sec2x$ [6 marks]

Q6

(a) Using the first principles, find first derivatives of the following functions.

(i) $y = sinx + cosx$

(ii) $y = \frac{1}{1+x}$ [6 marks]

(b) Evaluate the following limits.

(i) $\lim_{x\to 4} \frac{2-\sqrt{x}}{4-x}$

(ii) $\lim_{x\to 0} \frac{x}{sin7x}$ [6 marks]

(c)

(i) If $A = \begin{bmatrix} 0 & 6 & 7 \\ -6 & 0 & 8 \\ 7 & -8 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix}$
Find AC, BC and (A+B)C.
Also, verify that (A+B)C = AC+BC.
[4 marks]

(ii) If $A = \begin{bmatrix} -2 \\ 4 \\ 5 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 3 & -6 \end{bmatrix}$, verify that $(AB)^T = B^T A^T$
[4 marks]

Q7

(a)

(i) Calculate the relative error of 0.94 ± 0.2. [2 marks]

(ii) The density of lead is $13.6\ gcm^{-3}$, but the measured and calculated value in lab was $12.9\ gcm^{-3}$. What is the percentage error? [3 marks]

(b) Construct Newton's forward difference table for the following data.

x	1	2	3	4	5
$f(x)$	1	4	9	16	25

Hence calculate an approximate value for $f(1.7)$ from Newton's forward interpolation method.

[10 marks]

(c) Let $f(x) = x^3 - 2x - 5$, where $x \in R$

(i) Find $f'(x)$. [1 Marks]

(ii) Show that $f(x)$ has a real root that lies between 2 and 3. [1 Marks]

(iii) Using Newton-Raphson method find the above real root to correct to five decimal places. [3 Marks]

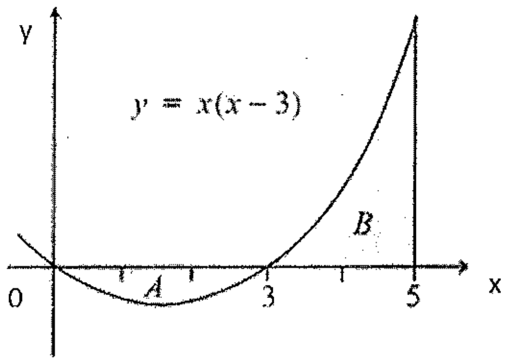
Q8

(a) Evaluate the following indefinite integral.

(i) $\int cos^2 x\ dx$

(ii) $\int xe^{x^2} dx$ [6 marks]

(b) The equation of the curve in the following figure is $y = x(x-3)$. Find the colored area (A+B).



[6 marks]

(c)

(i) Solve the equation $2sin^2 x = sinx$ for $0^0 \leq x \leq 360^0$. [4 marks]

(ii) Prove the following identity.

$$\tan\left(x + \frac{\pi}{4}\right)\tan\left(x - \frac{\pi}{4}\right) = -1$$ [4 marks]

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