MPZ 4230 – Engineering Mathematics II
Assignment No. 02 – Academic Year 2006

1. A company has a fleet of vehicles & is trying to predict the annual maintenance costs per vehicle. The following data have been supplied for a sample of vehicles.

Vehicle No.	Age in years (X)	Maintenance cost Rs.(x10) Y
1	2	60
2	8	132
3	6	100
4	8	120
5	10	150
6	4	84
7	4	90
8	2	68
9	6	104
10	10	140

(i). Draw a scatter diagram using above data.
(ii). Calculate the linear regression equation, to allow managers to predict the likely maintenance cost, knowing age of the vehicle.
(iii). Prepare the table of maintenance costs covering vehicles from 1-10 years of age based on your calculation in. part (ii).
(iv). Estimate the maintenance cost of 12 years old vehicle & comment on the validity of making such an estimate.
(Hint: Calculate conditional standard deviation ($s_{y/x}$)
Original variability in y values (s_y) and
Co-efficient of determination)
(v). Find 95% confidence interval for the true slop & interpret your answer.

2. (i). State conditions under which the passion distribution is a good approximation to the binomial distribution & conditions for the normal distribution to be good approximation to the Binomial distribution.
(ii). A machine produces 3% defectives. A random sample of 15 items is selected from the output of this machine. Calculate the probability that this sample contains
(i). Zero defectives
(ii). Less than 2 defectives using
(a). Binomial distribution
(b). Poisson distribution

3. A machine produces components of mean diameter 1.535 cm & standard deviation 0.005 cm. The diameters are assumed to be normally distributed. If all components with diameters out side the range 1.528 cm to 1.540 cm are rejected what proportion of components are rejected?

What is the probability the 4 components selected, at least there will be rejected?
If 28% of components have diameters less than a given value, what is the value?

4. $\frac{dy}{dx} = xy + 1$, y(0) = 1 , find y(0.1) & y(0.2). By using

(i). Euler Method

(ii). Taylor Series Method

(iii). Second order Runge-Kutta method

5. (i). State the classification conditions for a partial differential equation in to elliptic, parabolic & hyperbolic form.

(ii). The four sides of a square plate of side 12 cm made of homogeneous material are kept at constant temperatures 0^0C, 75^0 C & 100^0C as shown in the figure. Using a grid of mesh 4 cm find the temperature at the mesh points.

In the case of independence of time, the heat equation $\frac{\partial u}{\partial t} = c^2\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$ reduces to the Laplace equation $\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) = 0$

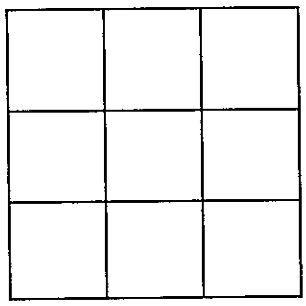


Figure -1

6. (i). Show that both functions $e^x \sin y$ & $\log\sqrt{x^2 + y^2}$ satisfy Laplace's equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

(ii). Find the stationary points of the function $f(x, y) = x^3 + y^3 - 3(x + y)$

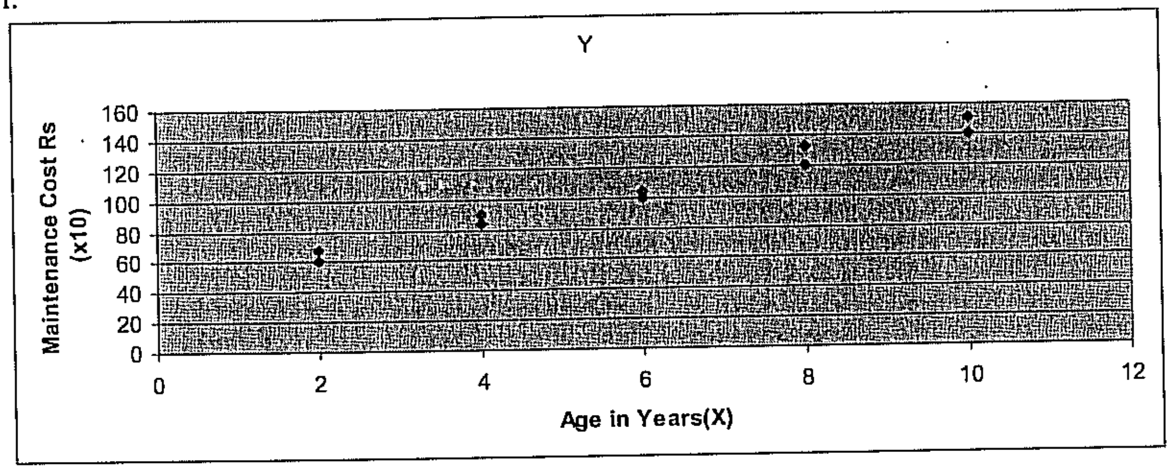
(iii). The heat generated in a resistance weld is given by $H = k i^2 Rt$, where k is a constant, i is the current between the electrodes and t is the time for which current flows. H must not vary by more than 5% of the weld is to remain good. It is possible to control t to within 0.5% & R to within 2.5%. Estimate the maximum possible variation in current if the weld is to retain its quality.

Please send your answers according to the Activity Diary due date to the following address.

Course Coordinator – MPZ 4230
Dept. of Mathematics & Philosophy of Engineering
Faculty of Engineering Technology
The Open University of Sri Lanka
Nawala.

MPZ 4230 –Engineering Mathematics II
Model Answer – 02 Academic Year 2006

1.
i.



ii.

	X	Y(x10)	XY(x10)	X^2	Y^2(x100)
	2	60	120	4	3600
	8	132	1056	64	17424
	6	100	600	36	10000
	8	120	960	64	14400
	10	150	1500	100	22500
	4	84	336	16	7056
	4	90	360	16	8100
	2	68	136	4	4624
	6	104	624	36	10816
	10	140	1400	100	19600
$\sum$	60	1048	7092	440	118120

Regression Equation is Y= a + bX , n= 10
Where

$$b = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - (\sum x)^2} \qquad a = \frac{\sum y - b\sum x}{n}$$

$$= \frac{10 \times 7092 - 60 \times 1048}{10 \times 440 - 3600} \qquad = \frac{1048 - 10.5 \times 60}{10}$$

$$= 10.5 \qquad = 44.5$$

Then $Y = 44.5 + 10.5\,X$

iii

Age in Years(X)	Maintenance Cost(Y) Rs(x10)
1	54.55
2	64.60
3	74.65
4	84.70
5	94.75
6	104.80
7	114.85
8	124.90
9	134.95
10	145.00

iv. $$S_{y/x} = \sqrt{\frac{(\sum y^2 - a\sum y - b\sum xy)}{n-2}}$$

$$= \sqrt{\frac{118120 - 46636 - 71274.6}{8}}$$

$$= 5.116151$$

$$S_y = \sqrt{\frac{\sum y^2 - (\sum y)^2 / n}{n-1}}$$

$$= \sqrt{\frac{18120 - 1048^2/10}{9}}$$

$$= 30.34908$$

$$r = \frac{n\sum xy - \sum x \sum y}{\sqrt{[n\sum x^2 - (\sum x)^2][n\sum y^2 - (\sum y)^2]}}$$

$$= \frac{70920 - 62880}{\sqrt{(4400 - 3600)(1181200 - 1098404)}}$$

$$= \frac{8040}{8143.513}$$

$$= 0.987289$$

$$r^2 = 0.974739$$

The correlation coefficient r, is 0.987289 which just barely places the correlation into the strong category.
The coefficient of determination r^2, is 0.974739 which means that 97.4739 % of the variation in y can be explained by relationship between x and y. The other 2.5261 % of the total variation in y remains unexplained.

v. $$S_b = \frac{S_{y/x}}{\sqrt{\sum x^2 - (\sum x)^2/n}}$$

$$= \frac{5.116151}{\sqrt{440 - 360}}$$

$$= 0.572003$$

confidence interval $= b \pm ts_b$

$= 10.05 \pm 2.306 \times 0.572003$

confidence interval is [8.9730961 – 11.36904]

2.

i.

Poisson distribution Vs Binomial distribution

When a Binomial situation where the sample size is large & the proportion of success in the population is small the Poisson distribution is a good approximation.

Normal distribution Vs Binomial distribution

Normal distribution to be a good approximation to the Binomial distribution leads to reasonable large value of sample size(n)

ii. Let X denotes the number of defectives in sample size 15

i. Probability that sample contains zero defectives $p(x = 0)$

a) By using Binomial distribution

$$P(X = 0) = {}^{15}C_0 (0.03)^0 \ (0.97)^{15}$$
$$= 1 \times 0.633251$$
$$= 0.633251$$

b) By using Poisson distribution

$$\lambda = np$$
$$= 15 \times 0.03$$
$$= 0.45$$

$$P(X = 0) = \frac{e^{-0.45} (0.45)^0}{0!}$$
$$= e^{-0.45}$$
$$= 0.63762$$

ii Probability that sample size contain less than 2 defectives,

$$P(X<2) = P(X=0) + P(X=1)$$

a) By using Binomial distribution

$$P(X < 2) = {}^{15}C_0 (0.03)^0 \ (0.97)^{15} + {}^{15}C_1 (0.03)^1 \ (0.97)^{14}$$
$$= 1 \times 0.633251 + 15 \times 0.652836$$
$$= 0.927028$$

c) By using Poisson distribution

$$P(X < 2) = \frac{e^{-0.45} (0.45)^0}{0!} \quad \frac{e^{-0.45} (0.45)^1}{1!}$$
$$= e^{-0.45}(1 + 0.45)$$
$$= 0.924561$$

3.

Let D denotes the diameter of a component selected at random
$D \sim N(1.535, 0.005^2)$

Probability that a component select at random has diameter between 1.528 Cm & 1.540 Cm is $p(1.528 < D < 1.540)$

Let $Z = \dfrac{D - 1.535}{0.005}$

$$= p\left\{\frac{1.528-1.535}{0.005} < Z < \frac{1.540-1.535}{0.005}\right\}$$

$$= p\left\{-1.4 < Z < 1\right\}$$

$$= (0.9192 - 0.5) + (0.8413 - 0.5)$$

$$= 0.4192 + 0.3413$$

$$= 0.7605$$

Probability that a component selected at random will rejected = 1 -0.7605 = 0.2395

23.95% of the component will be rejected

ii Let p=0.2395
probability that at least 3 will be rejected $= {}^4c_3\, p^3 q + {}^4c_4\, p^4 q^0$
$= 4(0.2395)^3(0.7605) + (0.2395)^4$
$= 0.045081$

iii. Suppose 28% of component have diameters less than x,
$P(D < x) = 0.28$

$$Z = \frac{D - 1.535}{0.005}$$

$$P\left(Z < \frac{x - 1.535}{0.005}\right) = 0.28$$

$$\frac{x - 1.535}{0.005} = -d$$

$$\frac{x - 1.535}{0.005} = -0.58$$

$$x = -0.58 \times 0.005 + 1.535$$
$$= 1.5321 \text{ Cm}$$

4.

i By using Euler Method

Let h= 0.1, $x_0 = 0$, $y_0 = 1$,

Equation $y_{k+1} = y_k + hf((x_k, y_k)$

Computation can be tabulated as follows

k	x_k	$y_{k+1} \approx y_k + 0.1(1+x_k y_k)$	y_{k+1}
0	0.0	$y_1 \approx y_0 + 0.1(1+x_0 y_0)$	1+0.1(1) =1.1
1	0.1	$y_2 \approx y_1 + 0.1(1+x_1 y_1)$	1.1 + 0.1(1+0.1x1.1) = 1.211
2	0.2	$y_3 \approx y_2 + 0.1(1+x_2 y_2)$	1.211+0.1(1+ 0.2x1.211)=1.33522

$y(0.1) = 1.2110$
$y(0.2) = 1.33522$

ii. By using Taylor series method

Let $h = 0.1$

$$\frac{dy}{dx} = f(x,y) = 1 + xy \qquad \left.\frac{dy}{dx}\right|_{(0,1)} = 1$$

$$\frac{d^2y}{dx^2} = x\frac{dy}{dx} + y \qquad \left.\frac{d^2y}{dx^2}\right|_{(0,1)} = 1$$

$$\frac{d^3y}{dx^3} = x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} \qquad \left.\frac{d^3y}{dx^3}\right|_{(0,1)} = 2$$

$$\frac{d^4y}{dx^4} = x\frac{d^3y}{dx^3} + 3\frac{d^2y}{dx^2} \qquad \left.\frac{d^4y}{dx^4}\right|_{(0,1)} = 3$$

$$y(x_k+h) = y(x_k) + h\,y'(x_k)$$

$$y(0+0.1) = y(0) + 0.1\,y'(0) + \frac{0.1^2}{2!} y^2(0) + \frac{0.1^3}{3!} y^3(0)$$

$$= 1 + 0.1\text{x}1 + \frac{0.1^2}{2}\text{x } 1 + \frac{0.1^3}{6}\text{x } 2 + \frac{0.1^4}{24}\text{x } 3$$

$Y(0.1) = 1.1053$

To find next point , set the derivatives be constructed as follows

k	$(dy/dx)^k$	$y^k(x)$
1	1 + xy	1 + 0.1x1.1053 =1.11053
2	xy' + y	0.1x1.11053 + 1.1053 =1.216399
3	xy'' + 2y'	0.1x1.221583 + 2x1.11053 =2.3427
4	xy''' + 3y''	0.1x2.343218 + 3x1.221583 =3.883466

$$Y(0.2) = y(0.1) + 0.1\ y'(0.1) + \frac{0.1^2 y^2(0.1)}{2!} + \frac{0.1^3 y^3(0.1)}{3!} + \frac{0.1^4 y^4(0.1)}{4!}$$

$$= 1.1053 + 0.1 \times 1.11053 + \frac{0.1^2}{2} \times 1.216399 + \frac{0.1^3}{6} \times 2.3427 + \frac{0.1^4}{24} \times 3.883466$$

$$= 1.222887$$

iii. Using second order Runge Kutta Method

$y' = 1 + xy$, $y(0) = 1$ $x(0)=0$, $h=0.1$

$f(x_n,y_n) = 1 + x_n y_n$

$$y_1 = y_0 + \frac{h}{2}\left(f(x_0,y_0) + f(x_0+h, y_0+hf(x_0,y_0)) \right)$$

$$= 1 + \frac{0.1}{2}\left\{ 1+(1+0.1(1+0.1(1+0))) \right\}$$

$$= 1 + \frac{0.1}{2}\left\{ 1 + 1.11 \right\}$$

$$= 1.1055$$

$$y_2 = y_1 + \frac{h}{2}\left(f(x_1,y_1) + f(x_1+h, y_1 + hf(x_1,y_1)) \right)$$

$f(x_1,y_1) = 1 + 0.1 \times 1.1055 = 1.11055$

$x_1 + h = 0.2$

$y_1 + hf(x_1,y_1) = 1.1055 + 0.1 \times 1.11055 = 1.216555$

$$y_2 = 1.1055 + \frac{0.1}{2}\left\{ 1.11055 + (1 + 0.2 \times 1.216555) \right\}$$

$$= 1.223193$$

5.

i.

Consider the second order linear partial differential equation

$$a\frac{\partial^2 u}{\partial x^2} + b\frac{\partial^2 u}{\partial y \partial x} + c\frac{\partial^2 u}{\partial y^2} = 0$$

Where a,b,c are function of x & y

If

$b^2 - 4ac < 0$	elliptic equation
$b^2 - 4ac = 0$	parabolic equation
$b^2 - 4ac > 0$	hyperbolic equation

ii

$$\frac{\partial^2 u}{\partial x^2} = \frac{u_{i-1,j} - 2u_{i,j} - u_{i+1,j}}{(\Delta x)^2}$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{u_{i,j-1} - 2u_{i,j} - u_{i,j+1}}{(\Delta y)^2}$$

In this problem $\Delta x = \Delta y = 4$

From Laplace Equation,

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

$$u_{i-1,j} - 4u_{i,j} - u_{i+1,j} + u_{i,j-1} - u_{i,j+1} = 0$$

u_1 point,

$$100 - 4u_1 + u_2 + 100 + u_3 = 0$$

$$4u_1 - u_2 - u_3 = 200 \quad \text{-----------------} \quad 1$$

u_2 point,

$$u_1 + u_4 + 100 + 75 - 4u_2 = 0$$

$$-u_1 + 4u_2 - u_4 = 175 \quad \text{-----------------} \quad 2$$

u_3 point,

$$u_1 + u_4 + 0 + 100 - 4u_4 = 0$$

$$-u_1 + 4u_3 - u_4 = 100 \quad \text{-----------------} \quad 3$$

u_4 point,

$$u_2 + u_3 + 0 + 75 + 4u_4 = 0$$

$$-u_2 - u_3 + 4u_4 = 75 \quad \text{-----------------} \quad 4$$

By 1,2,3,4

$$\begin{bmatrix} 4 & -1 & -1 & 0 \\ -1 & 4 & 0 & -1 \\ -1 & 0 & 4 & -1 \\ 0 & -1 & -1 & 4 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} 200 \\ 175 \\ 100 \\ 75 \end{bmatrix}$$

$$\left[\begin{array}{cccc|c} 4 & -1 & -1 & 0 & 200 \\ -1 & 4 & 0 & -1 & 175 \\ -1 & 0 & 4 & -1 & 100 \\ 0 & -1 & -1 & 4 & 75 \end{array}\right]$$

$R_1 + 4R_2 \rightarrow R_2$
$R_1 + 4R_3 \rightarrow R_3$

$$\left[\begin{array}{cccc|c} 4 & -1 & -1 & 0 & 200 \\ 0 & 15 & -1 & -4 & 900 \\ 0 & -1 & 15 & -4 & 600 \\ 0 & -1 & -1 & 4 & 75 \end{array}\right]$$

$R_2 + 15R_3 \rightarrow R_3$
$R_1 + 15R_4 \rightarrow R_4$

$$\left[\begin{array}{cccc|c} 4 & -1 & -1 & 0 & 200 \\ 0 & 15 & -1 & -4 & 900 \\ 0 & 0 & 224 & -64 & 9900 \\ 0 & 0 & -16 & 56 & 2025 \end{array}\right]$$

$R_3 + 14R_4 \rightarrow R_4$

$$\left[\begin{array}{cccc|c} 4 & -1 & -1 & 0 & 200 \\ 0 & 15 & -1 & -4 & 900 \\ 0 & 0 & 224 & -64 & 9900 \\ 0 & 0 & 0 & 720 & 38250 \end{array}\right]$$

$720\, u_4 = 38250$ $\quad u_4 = 53.125$

$$u_3 = \frac{9900 + (53.125 \times 64)}{224} \qquad u_3 = 59.375$$

$$u_2 = \frac{900 + (53.125 \times 4) + 59.375}{15} \qquad u_2 = 78.125$$

$$u_1 = \frac{200 + 78.125 + 59.375}{4} \qquad u_1 = 84.375$$

$u_1 = 84.375$ $\quad u_2 = 78.125$ $\quad u_3 = 59.375$ $\quad u_4 = 53.125$

6.

i.

a) $u = e^x \sin y$

$\frac{\partial u}{\partial x} = e^x \sin y$

$\frac{\partial u}{\partial y} = e^x \cos y$

$\frac{\partial^2 u}{\partial x^2} = e^x \sin y$

$\frac{\partial^2 u}{\partial y^2} = -e^x \sin y$

then $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

$e^x \sin y$ satisfy the Laplace equation

b). $u = \log\sqrt{x^2 + y^2}$

$\frac{\partial u}{\partial x} = \frac{1}{\sqrt{x^2+y^2}} \times \frac{2x}{2\sqrt{x^2+y^2}}$

$= \frac{x}{x^2+y^2}$

$\frac{\partial u}{\partial y} = \frac{1}{\sqrt{x^2+y^2}} \times \frac{2y}{2\sqrt{x^2+y^2}}$

$= \frac{y}{x^2+y^2}$

$\frac{\partial^2 u}{\partial x^2} = \frac{(x^2+y^2) - x.2x}{(x^2+y^2)^2}$

$\frac{\partial^2 u}{\partial x^2} = \frac{y^2 - x^2}{(x^2+y^2)^2}$

$\frac{\partial^2 u}{\partial y^2} = \frac{(x^2+y^2) - y.2y}{(x^2+y^2)^2}$

$\frac{\partial^2 u}{\partial y^2} = \frac{x^2 - y^2}{(x^2+y^2)^2}$

then $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

$\log\sqrt{x^2+y^2}$ satisfy the Laplace equation

ii.

$f(x,y) = x^3 + y^3 - 3(x+y)$

$\frac{\partial f}{\partial x} = 3x^2 - 3$

$\frac{\partial f}{\partial y} = 3y^2 - 3$

At Stationary points $\frac{\partial f}{\partial x} = 0$, $\frac{\partial f}{\partial y} = 0$

$3x^2 - 3 = 0$
$3(x^2 - 1) = 0$
$x^2 - 1 = 0$
$x = \pm 1$

$3y^2 - 3 = 0$
$3(y^2 - 1) = 0$
$y^2 - 1 = 0$
$y = \pm 1$

Stationary points are (1,-1) , (1,1) ,(-1,-1) , (-1,1) iii.

$H = ki^2Rt$

$$\frac{\partial H}{\partial i} = 2KiRt \text{ ------------------1}$$

$$\frac{\partial H}{\partial t} = Ki^2R \text{ ------------------2}$$

$$\frac{\partial H}{\partial R} = Ki^2t \text{ ------------------3}$$

From Taylor approximation, The maximum possible variation in Heat generation,

$$\delta H = \frac{\partial H}{\partial i}\delta i + \frac{\partial H}{\partial t}\delta t + \frac{\partial H}{\partial R}\delta R$$

Substituting 1,2,3, equations

$$\delta H = 2KiRt\ \delta i + Ki^2R\ \delta t + Ki^2t\ \delta R$$

devide by H

$$\frac{\delta H}{H} = 2\frac{\delta i}{i} + \frac{\delta t}{t} + \frac{\delta R}{R}$$

$$\frac{\delta H}{H} = 2\frac{\delta i}{i} + \frac{\delta t}{t} + \frac{\delta R}{R} \le \frac{5}{100}$$

$$2\frac{\delta i}{i} + \frac{0.5}{100} + \frac{2.5}{100} \le \frac{5}{100}$$

$$2\frac{\delta i}{i} \le \frac{5}{100} - \frac{0.5}{100} - \frac{2.5}{100}$$

$$\frac{\delta i}{i} \le \frac{1}{100}$$

maximum possible variation in current = 1%