MPZ 4230 – Engineering Mathematics II
Assignment No. 04 – Academic Year 2006

1.(a). Show that $\underline{V} = R^2$ is not a vector space over R with respect to operation call
$(a, b) + (c, d) = (a + c, b + d)$ $\quad$ $k(a, b) = (k^2a, k^2b)$

(b). Suppose $\underline{U}, \underline{V}, \underline{W}$ are sub space of vector space $\underline{V}$. Prove that $(\underline{U} \cap \underline{V}) + (\underline{U} \cap \underline{W})$ is a subset of $\underline{U} \cap (\underline{V} + \underline{W})$ $\quad$ [Hint $\underline{U} + \underline{W} = \{\underline{u} + \underline{w};\ \underline{u} \in \underline{U};\ \underline{w} \in \underline{W}\}$]

2.(a). Show that W is a subspace of $V \in R^3$; W consist of those vectors each whose sum of the component is 0

(b). Find k, so that $w = (1, k, 5)$ is a combination of $\underline{u}$ and $\underline{v}$ $\quad$ $\underline{u} = (1,-3,2)$ $\quad$ $\underline{v} = (2,-1, 1)$

(c). Determine whether (1, -3, 7), (2, 0, -6), (3, -1, -1), (2, 4, 5) are lineally dependent.

(d). Show that dim $R^n = n$

3. (a). There is a linear map $T : R^2 \rightarrow R$ for which $T(1, 1) = 3$ and $T(0, 1) = -2$. Find the formula

(b). Consider $F : R^3 \rightarrow R^2$ define by $F(x, y, z) = (|x|, 0)$ show that F is not linear.

(c). Consider the basis $\{\underline{v}_1, \underline{v}_2\}$ for R^2, where $v_1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$, $v_2 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$. Show that the linear transformation

$T : R^2 \rightarrow R^3$ define by $T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \dfrac{2x-y}{3} \\ \dfrac{2x-y}{3} \end{bmatrix}$. $\quad$ Find ker T and Rank T.

4. (a). Consider the vector space $\underline{v}_3(R)$ with the Euclidean product. Apply the Gram process to transfer the basis $\underline{u}_1 = (1,1,1)$ $\underline{u}_2 = (1,1,0)$, $\underline{u}_3 = (1,0,0)$

(b). Using v_1, v_2, v_3 verify the theorem [Theorem: If $\underline{u}$ and $\underline{v}$ are orthogonal vectors in an inner product space, then $\|\underline{u} + \underline{v}\|^2 = \|\underline{u}\|^2 + \|\underline{v}\|^2$]

5. (a). Find the eigon value and the corresponding eigon vector for $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$

A is diagonalizable. If so find a matrix P which diagonalize A & determine $P^{-1}AP$ by direct computation.

(b). Find the eigon value and the corresponding eigon vector of the matrix $A = \begin{bmatrix} 2 & \sqrt{2} \\ \sqrt{2} & 1 \end{bmatrix}$

(c). Show that $\lambda_1, \lambda_2, \lambda_3 \ldots\ldots \lambda_n$ are eigon values of matrix A
Then inverse of A (1f it is exist, i.e. A^{-1}) has eigon values $\lambda_1^{-1}, \lambda_2^{-1}, \ldots\ldots \lambda_n^{-1}$ [Hint: $A^{-1}A = I$]

6. (a). Show that $\underline{W}$ is not a subspace of $\underline{V} = R^3$, where $\underline{W}$ consist of those vector whose length does not exceed

(b). Write the polynomial $\underline{V} = t^2 + 4t - 3$ over R as a linear combination of polynomial. $e_1 = t^2 - 2t + 5$
$e_2 = 2t^2 - 3t$, $e_3 = t + 3$

(c). Show that 2 vectors are dependent if and only if one of them is a multiple of the other.

Please send your answers according to the Activity Diary Due Date to the following address
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Model Answer 04 – MPZ 4230
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(1) (i) Axiom [A8] $a, b \in c$, $\underline{u} \in \underline{v}$

Then $u(a+b) = au + bu$

Check the Axiom [A8] for

Let $a = 1$, $b = 2$, $\underline{u} = (3,4)$

L.H.S $\underline{u}(a+b) = (1+2)(3,4)$

$= 3(3, 4)$

$= (27,37)$ $[\because k(a,b) = k^2a,\ k^2b]$

R.H.S. $au + bu = 1(3,4) + 2(3,4)$

$= (3, 4) + (12, 16)$ $[\because k(a,b) = k^2a,\ k^2b]$

$= [15, 20]$ $[\because (a,b) + (c,b) = \{(a+c), (b+d)\}]$

L.H.S $\neq$ R.H.S

Axiom [A8] is contradict

This is not a vector space

(ii) Let u be arbitrary element $[(\underline{U} \cap \underline{V}) + (U \cap \underline{W})]$

$\underline{u} \in [(U \cap \underline{V}) + (\underline{U} \cap \underline{W})]$

Then $\underline{u} = \underline{u}_1 + \underline{u}_2$ $[Where\ \underline{u}_1 \in \underline{U} \cap \underline{V},\ \underline{u}_2 \in \underline{u}U \cap \underline{W}]$

Then $\underline{u}_1 \in U \cap \underline{V} \Rightarrow \underline{u}_1 \in U$ ---------- (1) and

$\underline{u}_1 \in \underline{V}$ --------- (2)

$\underline{u}_2 \in \underline{U} \cap \underline{W} \Rightarrow \underline{u}_2 \in \underline{V}$ ---------- (3)

$\underline{u}_2 \in \underline{W}$ --------- (4)

By (1) & (3)

$\underline{u} = \underline{u}_1 + \underline{u}_2 \in \underline{U}$ -------------- (5)

By (2) & (4)

$\underline{u}_1 \in V$ and $\underline{u}_2 \in \underline{W}$

$\Rightarrow \underline{u} = \underline{u}_1 + \underline{u}_2 \in (\underline{V} + \underline{W})$ ------- (6)

(5) & (6)

$\underline{u} \in \underline{U}$ and $\underline{u} \in \underline{V} + \underline{W}$

$\underline{u} \in [\underline{U} \cap (\underline{V} + \underline{W})]$

$\Rightarrow [(\underline{U} \cap \underline{V}) + (\underline{U} \cap \underline{W})] \in [\underline{U} \cap (\underline{V} + \underline{W})]$

(2) (a) $\underline{W} = \{x, y, z;\ x + y + z = 0,\ x, y, z \in R\}$

We want to prove

$\underline{u}, \underline{w} \in \underline{W} \Rightarrow \alpha\underline{u} + \beta\underline{w} \in W$ for every $\alpha, \beta \in K$

Let $\underline{u}, \underline{w} \in \underline{W}$

$$\underline{u} = (a_1, b_1, c_1)\ ,\ \underline{w} = (a_2, b_2, c_2)\ \left[\begin{array}{r} \text{where } a_1 + b_1 + c_1 = 0 \\ a_2 + b_2 + c_2 = 0 \end{array}\right]$$

$$\begin{aligned} \alpha u + \beta w &= \alpha(a_1, b_1, c_1) + \beta(a_2, b_2, c_2) \\ &= (\alpha a_1, \alpha b_1, \alpha c_1) + (\beta a_2 + \beta b_2 + \beta c_2) \\ &= [(\alpha a_1 + \beta a_2), (\alpha b_1 + \beta b_2), (\alpha c_1 + \beta c_2)] \\ &= (a_3, b_3, c_3) \quad \left[\begin{array}{r} \text{where } a_3 = \alpha a_1 + \beta a_2 \\ b_2 = \alpha b_1 + \beta b_2 \\ c_2 = \alpha c_1 + \beta c_2 \end{array}\right] \end{aligned}$$

Then find $(a_3 + b_3 + c_3) = \alpha a_1 + \beta a_2 + \alpha b_1 + \beta b_2 + \alpha c_1 + \beta c_2$

$$\begin{aligned} &= \alpha(a_1 + b_1 + c_1) + \beta(a_2 + b_2 + c_2) \\ &= \alpha.0 + \alpha.0 \\ &= 0 \end{aligned}$$

$\therefore \alpha\underline{u} + \beta\underline{w} \in \underline{W}\quad [\because\ a_3 + b_3 + c_3 = 0]$

$\therefore \underline{W}$ is a subspace $V \in R^3$

(b) $(1, k, 5) = a_1(1, -3,, 2) + a_2(2. -1, 1)$

$(1, k, 5) = (a_1 + 2a_2,\ -3a_1 - a_2,\ 2a_1 + a_2)$

$a_1 + 2a_2 = 1$ ------- (1)

$-3a_1 - a_2 = k$ ------- (2)

$2a_1 + a_2 = 5$ -------- (3)

(1) & (2) $\quad -3a_2 = 3$

$$a_2 = -1$$

$$a_1 = 3$$

Then by (3)

$$k = -8$$

(c) $\quad a_1(1,-3,7) + a_2(2,0,-6) + a_3(3,-1,-1) + a_4(2,4,-5) = 0$

$a_1 + 2a_2 + 3a_3 + 2a_4 = 0$ ---------- (1)

$-3a_1 \quad + \; a_3 + 4a_4 = 0$ ---------- (2)

$7a_1 - 6a_2 - a_3 - 5a_4 = 0$ ---------- (3)

take one of a = t, then find other "a"

a_1, a_2, a_3, a_4 have infinite solution

One of a is multiple of other.

$\therefore$ Given vectors are linearly dependent

(d) $\quad$ take $\underline{u} \in R^n$

$\underline{u}_1 = [1,0,... \quad 0] \quad \underline{u}_1 \in R^n$

$\underline{u}_2 = [0,1,.... \quad ,0] \quad \underline{u}_2 \in R^n$

$\underline{u}_3 = [...$

....

$\underline{u}_n[0,0,0,.... \quad ,1] \quad \underline{u}_n \in R^n$

$a_1\underline{u}_1 + a_2\underline{u}_2 + ...a_n\underline{u}_n = 0$

$\Rightarrow a_1 = a_2 = ... = a_n = 0$

$\therefore \underline{u}_1, \underline{u}_2.....\underline{u}_n$ are lineally independent

Basis $R^n = \{\underline{u}_1, \underline{u}_2, ...\underline{u}_n\}$

$\therefore \dim R^n = n$

(3) (a) If T is a linear transformation

Then $T(\alpha\underline{u}+\beta\underline{v})=\alpha T(u)+\beta T(v)$ --------- (A)

$T(\alpha\underline{u}+\beta\underline{v})=T\{\alpha(1,1)+\beta(0,1)\}$

$= T(\alpha,\alpha+\beta)$ ----------- (1)

$\alpha T(u)+\beta T(v)=\alpha T(1,1)+\beta T(0,1)$

$= \alpha.3+\beta.(-2)$

$= 3\alpha-2\beta$ ----------- (2)

(1) & (2) substituting (A)

$T(\alpha,\alpha+\beta)=3\alpha-2\beta$ --------- (3)

Take $T(x,y)=T(\alpha,\alpha+\beta)$

Then $x=\alpha\Rightarrow\alpha=x$

$y=\alpha+\beta\Rightarrow\beta=y-\alpha=y-x$

Then (3) $T(x,y)=3x-2(y-x)$

$= 5x-2y$

(b) $F:R^3\to R^2$

Let $\underline{u}=(x_1,y_1,z_1)$

$\underline{v}=(x_2,y_2,z_2)$ [where $u,v\in R^3$]

If F is linear transformation

$F(\alpha\underline{u}+\beta\underline{v})=\alpha F(\underline{u})+\beta F(v)$ ----------(A)

$F(\alpha\underline{u}+\beta\underline{v})=F\{\alpha(x_1,y_1,z_1)+\beta(x_2,y_2,z_2)\}$

$= F(\alpha x_1+\beta x_2,\alpha y_1+\beta y_2,\alpha z_1+\beta z_2)$

$= (|\alpha x_1+\beta x_2|,0)$ ------------ (1)

$\alpha F(\underline{u})+\beta F(\underline{v})=\alpha(|x_1|,0)+\beta(|x_2|,0)$

$= (\alpha|x_1|+\beta|x_2|,0)$ ----------- (2)

By (1) & (2)

$F(\alpha\underline{u}+\beta\underline{v})\neq\alpha F(\underline{u})+\beta F(\underline{v})$ $[\because|\alpha x_1+\beta x_2|\leq\alpha|x_1|+\beta|x_2|]$

By (1) & (2), F is not satisfy the linear transformation conditions.

$\therefore$ F is not linear transformation

(4) (a) Step 1

$$\underline{v}_1 = \frac{\underline{u}_1}{\|\underline{u}_1\|} = \frac{1,1,1}{\sqrt{3}} = \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$$

Step 2

$$\underline{v}_2 = \frac{\underline{u}_2 - \text{proj}\ w_1 \underline{u}_2}{\|\underline{u}_2 - \text{proj}\ w_1 \underline{u}_2\|}$$

$$\underline{u}_2\text{-proj}\ \ w_1 u_2 = (1,1,0) - \frac{2}{\sqrt{3}}\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$$

$$= \left(\frac{1}{3}, \frac{1}{3}, -\frac{2}{3}\right)$$

$$\|\underline{u}_2 - \text{proj}\ \ w_1 \underline{u}_2\| = \frac{\sqrt{6}}{3}$$

$$\underline{v}_2 = \frac{3}{\sqrt{6}}\left(\frac{1}{3}, \frac{1}{3}, -\frac{2}{3}\right) = \left(\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, -\frac{2}{\sqrt{6}}\right)$$

Step 3

$$\underline{v}_3 = \frac{\underline{u}_3 - \text{proj}\ w_1 \underline{u}_3}{\|u_3 - \text{proj}\ w_1, u_3\|}$$

$$\underline{u}_3 - \textit{proj}\ w_1 \underline{u}_3 = (1,0,0) - (\underline{u}_3, \underline{v}_1)\underline{v}_1 - (\underline{u}_3 . \underline{u}_2)\underline{v}_2$$

$$= (1,0,0) - \frac{1}{\sqrt{3}}\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right) - \frac{1}{\sqrt{6}}\left(\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, -\frac{2}{\sqrt{6}}\right)$$

$$= \left[\frac{1}{2}, -\frac{1}{2}, 0\right]$$

$$\underline{v}_3 = \frac{\left(\frac{1}{2}, -\frac{1}{2}, 0\right)}{\frac{1}{\sqrt{2}}} = \left[\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0\right]$$

(b) $\underline{v}_1 = \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right), \underline{v}_2 = \left(\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, -\frac{2}{\sqrt{6}}\right), \quad \underline{v}_3 = \left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0\right)$

$\|\underline{v}_1\| = 1$ ------ (1) $\|\underline{v}_2\| = 1$ --- (2) $\|\underline{v}_3\| = 1$ --- (3)

$$\|\underline{v}_1 + \underline{v}_2\| = \left[\left(\frac{1}{\sqrt{3}} + \frac{1}{\sqrt{6}}\right)^2 + \left(\frac{1}{\sqrt{3}} + \frac{1}{\sqrt{6}}\right)^2 + \left(\frac{1}{\sqrt{3}} - \frac{2}{\sqrt{6}}\right)^2\right]^{1/2}$$

$$= \left[\frac{1}{3} + \frac{2}{\sqrt{18}} + \frac{1}{6} + \frac{1}{3} + \frac{2}{\sqrt{18}} + \frac{1}{6} + \frac{1}{3} - \frac{4}{\sqrt{18}} + \frac{4}{6}\right]$$

$$= \sqrt{2} \text{ ------- } (4)$$

$\therefore \|\underline{v}_1 + \underline{v}_2\|^2 = \|\underline{v}_1\|^2 + \|\underline{v}_2\|^2$

$\therefore$ Theorem (3) satisfy

Also $\|\underline{v}_1 + \underline{v}_3\| = \|\underline{v}_2 + \underline{v}_3\| = \sqrt{2}$

Then $\underline{v}_1, \underline{v}_2, \underline{v}_3$ orthogonal vectors using can verify the theorem 3.

(5) (i) $A - \lambda I = \begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 8-\lambda & -6 & 2 \\ -6 & 7-\lambda & -4 \\ 2 & -4 & 3-\lambda \end{pmatrix}$

$$8 - \lambda[(7-\lambda)(3-\lambda) - 16] - 6[-8 + 6(3-\lambda)] + 2[24 - 2(7-\lambda)]$$

$$(8-\lambda)[21 - 10\lambda + \lambda^2 - 16] - 6[-8 + 18 - 6\lambda] + 2[24 - 14 + 2\lambda]$$

$$40 - 8\lambda + 8\lambda^2 - 5\lambda + 10\lambda^2 - \lambda^3 - 60 + 36\lambda + 20 + 4\lambda$$

$$-\lambda^3 + 18\lambda^2 - 45\lambda = 0$$

Characteristic equation of A is $-\lambda^3 + 18\lambda^2 - 45\lambda = 0$

$$\lambda(\lambda^2 - 18\lambda + 45) = 0$$

$$\lambda(\lambda - 3)(\lambda - 15) = 0$$

$\therefore$ eigen values are $\lambda = 0, 3, 15$

Let $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$ be a vector corresponding $\lambda = 0$

When $\lambda = 0$ $AX = \lambda X$

$$\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \end{pmatrix} = 0\begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

$$\left.\begin{aligned} 8a-6b+2c=0 \\ -6a+7b-4c=0 \\ 2a-4b+3c=0 \end{aligned}\right\} \Rightarrow \begin{aligned} y=2x \\ z=2x \end{aligned}$$

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} a \\ 2a \\ 2a \end{pmatrix} = a\begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$$

Eigen vector of A corresponding $\lambda = 0; \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$

When $\lambda = 3$ AX=λX

$$\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \end{pmatrix} = 3\begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

$$\left.\begin{aligned} 8a-6b+2c=3a \\ -6a+7b-4c=3b \\ 2a-4b+3c=3c \end{aligned}\right\} \begin{aligned} b=\frac{1}{2}a \\ c=-a \end{aligned}$$

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} a \\ \frac{1}{2}a \\ -a \end{pmatrix} = \frac{1}{2}a\begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$$

Eigen vector of A corresponding to $\lambda = 3$; $\begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$

When $\lambda = 15$ $AX = \lambda X$

$$\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \end{pmatrix} = 15\begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

$$\left.\begin{aligned} 8a-6b+2c&=15a \\ -6a+7b-4c&=15b \\ 2a-4b+3c&=15c \end{aligned}\right\} \begin{aligned} b&=-a \\ c&=\frac{1}{2}a \end{aligned}$$

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix}=\begin{pmatrix} a \\ -a \\ \frac{1}{2}a \end{pmatrix}=2a\begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}$$

Eigen vector of A corresponding to $\lambda=15$; $\begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}$

Then eigen vectors $\begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix},\begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix},\begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}$ are lineally independent vectors

$\therefore$ A is diagonalizable

$$\text{Let } P=\begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{pmatrix}$$

$$|P|=-27$$

$$P^{-1}=\frac{1}{|P|}\begin{pmatrix} -3 & -6 & -6 \\ -6 & -3 & 6 \\ -6 & 6 & -3 \end{pmatrix}$$

$$=\frac{1}{-27}\begin{pmatrix} -3 & -6 & -6 \\ -6 & -3 & 6 \\ -6 & 6 & -3 \end{pmatrix}$$

$$P^{-}AP=\frac{1}{9}\begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{pmatrix}\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}\begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{pmatrix}$$

$$=\frac{1}{9}\begin{pmatrix} 0 & 0 & 0 \\ 6 & 3 & -6 \\ 30 & -30 & 15 \end{pmatrix}\begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{pmatrix}=\begin{pmatrix} 0 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 15 \end{pmatrix}$$

5. ii $A - \lambda I = \begin{pmatrix} 2 & \sqrt{2} \\ \sqrt{2} & 1 \end{pmatrix} - \lambda\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$$\begin{pmatrix} 2-\lambda & \sqrt{2} \\ \sqrt{2} & 1-\lambda \end{pmatrix} = 0$$

$$(2-\lambda)(1-\lambda) - 2 = 0$$

$$\lambda(\lambda - 3) = 0$$

$\lambda = 0$ or $\lambda = 3$

The eigen values are $\lambda = 0,3$

Let $X = \begin{pmatrix} a \\ b \end{pmatrix}$ be a vector corresponding $\lambda = 0$

$$AX = \lambda X$$

When $\lambda = 0$

$$\begin{pmatrix} 2 & \sqrt{2} \\ \sqrt{2} & 1 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = 0\begin{pmatrix} a \\ b \end{pmatrix}$$

$$2a + \sqrt{2}b = a$$

$$a + \sqrt{2}b = 0$$

Let $a = t$

$$\therefore b = -\sqrt{2}t$$

$\therefore X = \begin{pmatrix} a \\ b \end{pmatrix} = t\begin{pmatrix} 1 \\ -\sqrt{2} \end{pmatrix}$ eigen vector of A

When $\lambda = 3$

$$\begin{pmatrix} 2 & \sqrt{2} \\ \sqrt{2} & 1 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = 3\begin{pmatrix} a \\ b \end{pmatrix}$$

$$-a + \sqrt{2}b = 0$$

$$\sqrt{2}a - 2b = 0$$

Let $a = t$

$$\therefore b = -\sqrt{2}t$$

Then eigen vector of A is

$$X = \begin{pmatrix} a \\ b \end{pmatrix} = t\begin{pmatrix} 1 \\ \sqrt{2} \end{pmatrix}$$

(c) $AX = \lambda X$

$A^{-}(AX) = A^{-1}\lambda X$

$IX = A^{-1}\lambda X$

$X = \lambda(A^{-1}X)$

$\frac{1}{\lambda}X = A^{-}X$

$\lambda^{-1}X = A^{-1}X$

λ^{-1} is a eigen value of A^{-1}

$\lambda_1, \lambda_2, ... \lambda_n$ are the eigen values of A

Then $\lambda_1^{-1}, \lambda_2^{-1}, ... \lambda_n^{-1}$ eigen values of A^{-1}

(6) (a) Let $w = \{(x, y, z); x, y, z \in R \ \& \ x^2 + y^2 + z^2 < 1\}$

Let $u_1 = \left(0, \frac{1}{2}, 0\right)$ & $u_2 = \left(\frac{1}{2}, \frac{1}{2}, 0\right)$

$u_1 + u_2 = \left(0, \frac{1}{2}, 0\right) + \left(\frac{1}{2}, \frac{1}{2}, 0\right)$

$= \left(\frac{1}{2}, 1, 0\right) \notin v \quad \left[\because \left(\frac{1}{2}\right)^2 + 1^2 + 0^2 > 1\right]$

$\therefore$ w is not a sub space of $v = R^2$

(b) $v = t^2 + 4t - 3$

$e_1 = t^2 - 2t + 5$ $\quad e_2 = 2t^2 - 3t$ $\quad e_3 = t + 3$

t^2; $\quad a + 2b = 1$ ---------- (1)

t: $\quad -2a - 3b + c = 4$ -------- (2)

$5a + 3c = -3$ -------------- (3)

(3)– (2) x 3

$11a + 9b = -15$ -------- (4)

(5) x 11 – (4)

$13b = 26$

$b = 2$

$a = -3$

$$c = 4$$

$$v = -3e_1 + 2e_2 + 4e_3$$

(c) Let v_1 and v_2 are dependent

$$a_1 v_1 + a_2 v_2 = 0 \quad ; \quad a_1, a_2 \neq 0$$

$$a_1 v_1 = -a_2 v_2$$

$$v_1 = -\frac{a_2}{a_1} v_2 \quad (\therefore a_1 \neq 0)$$

$$v_1 = k v_2 \qquad \left(k = -\frac{a}{a_1}\right)$$

Conversely v_1 is multiple of v_2

$$v_1 = k v_2; \ k \text{ is scalar}$$

$$v_1 - k v_2 = 0$$

$$1.v_1 - k v_2 = 0$$

v_1, v_2 are linearly dependent $[\therefore 1 \neq 0]$