The Open University of Sri Lanka

ECU 4302 System Dynamics / ECX 6242 Modern Control Systems
Monday 17th April 2006, 0930 - 1230 hrs.

Three hours

Up to five questions may be attempted, selecting at least **one** question from each of the sections A and B and C. However, full credit may be obtained for exceptionally good answers to only four questions. All questions carry equal marks.

Section A

1. Consider a system described by the state equations:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -2.5 & 0.5 \\ 0.5 & -2.5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

where **x** is the state vector and **u** is the input vector.

An observer of the system can monitor an observation y given by:

$$y = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Answer the following questions, giving reasons:

i What do you understand by the statement that a system is linear? Is the above system linear or non-linear?

ii Does it represent a continuous system or a discrete system?

iii Determine whether the system is completely controllable.

iv Is this system observable?

v Is the systems described above stable?

2. A non-linear system may be described by the generic vector state equations:

$$\underline{\dot{x}}(t) = \underline{f}(\underline{x}(t), \underline{u}(t))$$

An approximate linear representation of the behaviour of the system for small perturbations $\underline{\delta x}$ around an equilibrium point $\underline{x_0}$ is: given by:

$$\underline{\delta \dot{x}}(t) = J\big|_{\underline{x_0}} \underline{\delta x}(t)$$

where $J\big|_{\underline{x_0}}$ is the Jacobian matrix evaluated at $\underline{x_0}$.

Consider the non-linear system described by the system equations:

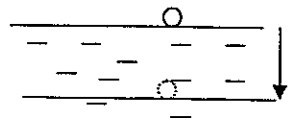
$$\dot{x}_1 = (x_1 + x_2)^2 + (x_1 + x_2)$$
$$\dot{x}_2 = (x_1 - x_2)^2 + (x_1 - x_2)$$

Show that it has an equilibrium point at the origin.

Write down the Jacobian matrix evaluated at the origin and hence the dynamic equations describing the approximate behaviour of the system around the origin.

Hence, determine whether the system is stable in the region around the origin.

3. A small sphere of mass m and volume v is gently placed on the surface of a deep lake as shown, so that the sphere, being heavier than the liquid at the surface, starts to move downwards, starting from rest. The density ρ of the liquid increases with depth x in accordance with the relation:



$$\rho = 1 + \alpha x$$

The sphere moving under gravity experiences a damping force proportional to the cube of its velocity. Write down the equations governing the motion of the sphere and determine its point of equilibrium.

If the sphere is subjected to an external vertical force u (subject to the constraint $|u_{max}| \leq 1$), formulate this as a time-optimal problem where u is to be chosen so as to move the sphere from the initial state to the final state in the minimal possible time.

4. The double integrator is frequently used as an example of a system for the study of various control techniques, because of its simplicity. In the continuous time domain, it may be represented by the state equations:

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = u$$

Express this in standard matrix form as $\dot{x} = Ax + Bu$

Evaluate

$$\Phi = e^{AT} = I + A\psi$$
$$\Gamma = \int_0^T e^{A\tau} d\tau B = \psi B$$

where

$$\psi = \int_0^T e^{AT} dt$$

and hence show that the discrete form $x(k+1) = \Phi x(k) + \Gamma u(k)$

of the system equations is $x(k+1) = \Phi x(k) + \Gamma u = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} x(k) + \begin{bmatrix} T^2/2 \\ T \end{bmatrix} u$

[T is the sampling interval]

If the output (observation) y is given by $y(k) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(k)$,

write down the pulse transfer function $G(z) = \dfrac{Y(z)}{U(z)}$ for T = 1

Section B

Read the introduction to the Advanced Nonlinear Control Group at the University of California at Los Angeles given at the end of this question paper before you attempt the questions in this section.

5. Explain what is meant by 'backstepping design methodology'

6. Discuss the difference between 'the traditional structure of concurrent on-line estimation and control' and the proposed mechanism of division into two phases, the active identification phase and the look-ahead control phase.

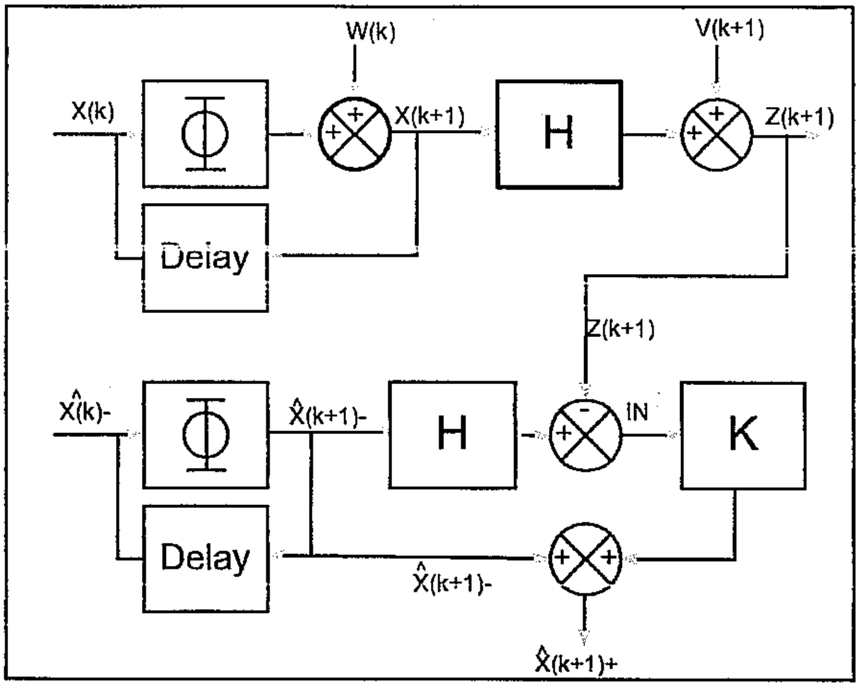
7. The introduction to the ANCG at UCLA also discusses Intelligent Transport Systems and Automated Highway Systems. It categorises research and development in this area into three main areas.

   Complete the table below, and then very briefly comment on their relationship to the operation of railway transport systems.

System	Source of information	Type of information	Performance capability	Relative cost and time for commercialisation
Autonomous systems				
Cooperative systems				
Automated highway systems				

Section C

8. The figure shows a schematic diagram of a discrete Kalman filter for state estimation, where x represents the state, Φ represents the state transition matrix, H represents the observation matrix and K represents the Kalman gain matrix. The system and observation noise sequences are w and v respectively. The hat represents estimates, and the - and + signs denotes estimates before and after an observation.



The first step in one cycle of updating the state estimate, described in words, is:

i Predict the estimate of $\hat{x}(k+1)_-$ from $\hat{x}(k)_+$, using the system model Φ. It is not possible to allow for the system noise w(k) in this.

Write down the rest of the steps **in words**, describing each step as accurately as possible, without using mathematical expressions.

9. Consider the system described by

$$x(k+1) = \begin{bmatrix} 0.5 & 1 \\ 0 & 0.3 \end{bmatrix} x(k) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(k)$$

Determine the eigen values of the system, and transform the system equations to diagonal form. Hence determine whether the system is completely state controllable (reachable). Verify your results by determining the rank of the controllability matrix.

Show that a linear state feedback controller $u(k) = -L\,x(k)$ can be used to shift one of the poles to an arbitrary location, but not the other.

Text for Section B

Introduction

The Advanced Nonlinear Control Group at UCLA focuses on nonlinear control design research, from purely theoretical topics to very applied projects. The objective of this research is to bring nonlinear control theory and practice closer together, by:

- developing theoretical design tools that address real-world problems,
- customizing these tools to fit specific applications, and
- performing experiments with real systems to demonstrate the improvements resulting from the use of these new theoretical tools.

Adaptive nonlinear control theory

Our theoretical research focuses on *constructive* design methods for nonlinear control. Our approach is based on the conjecture that no general theory could ever encompass **all** nonlinear systems and still be useful: as the class of systems one considers becomes more and more general, the control objectives that can be achieved become less and less ambitious. Therefore, we consider classes of nonlinear systems which are general enough to be of practical interest (in other words, there are lots of real-world nonlinear systems that control engineers have to deal with in their applications and that can be cast into our framework), yet specific enough to allow the construction of explicit expressions for our controllers. This is the best way to achieve our main goal, which is to bring nonlinear control theory and practice closer together; if a new method solves an important problem but is so abstract in its formulation that the actual implementation of a controller is rendered impossible, then its usefulness to a practicing engineer is severely limited.

One of the cornerstones of our research is the **backstepping** design methodology, which makes the nonlinear control design problem simple and systematic by:

- breaking down complex nonlinear systems into smaller subsystems,
- designing *partial Lyapunov functions* and *virtual controllers* for these subsystems, and
- integrating these individual controllers into an actual controller, by "stepping back" through the system and re-assembling it from its component subsystems.

This design methodology, with additional enhancements that deal with uncertainty, such as adaptation and robustification, has been used to solve several long-standing problems in nonlinear control; the common characteristic of the resulting solutions is the fact that they are **global**, because they are constructed with truly nonlinear design tools. In contrast, the often-used approach of designing controllers for nonlinear systems via tools that are "adapted" from the linear literature, produces at best local solutions whose region of validity can be estimated only through simulation.

The backstepping design methodology has been developed in journal and conference papers, several of which have received Best Paper awards. Currently, the most comprehensive reference on backstepping is the book **Nonlinear and Adaptive Control Design** by Krstic, Kanellakopoulos, and Kokotovic.

More recently, we have established the first a priori verifiable parameter convergence conditions for adaptive nonlinear systems. Given a specific nonlinear system and a specific reference signal, our procedures allow the user to determine a priori whether or not the signal is persistently exciting (PE), and, hence, whether or not the parameter estimates will converge. We show that the presence of nonlinearities usually reduces the sufficient richness (SR) requirements on the reference signals, and hence enhances parameter convergence. This is the first result on the relationship between persistent excitation and sufficient richness for adaptive nonlinear control systems.

Our most recent theoretical result is the first global stabilization and tracking result for **discrete-time** nonlinear systems. Instead of the traditional structure of concurrent on-line estimation and control, we adopt a two-phase control strategy: First, in the **active identification** phase, we use the control input to drive the system state to points in the state space which provide us with information about the unknown parameters. Our algorithm guarantees that the duration of this phase is finite and that at its end we will be able to compute future values of the system states. Then, in the **look-ahead control** phase, we use this prediction capability to treat the system as completely known and to drive it to its desired state in finite time.

We have also used backstepping to design nonlinear controllers which use nonlinearity in order to improve performance tradeoffs between competing requirements and objectives. In particular, we recently designed a new class of nonlinear controllers for active suspensions.

Vehicle automation

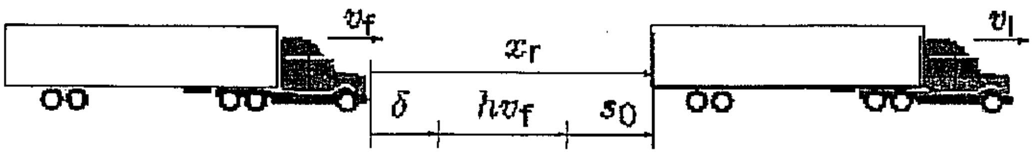
Intelligent Transportation Systems (ITS) technology is progressing at an ever-increasing rate, with exciting developments in all fronts, from driver information and assistance systems to Automated Highway Systems (AHS). This presentation discusses potential scenarios with varying levels of automation for commercial vehicles, and introduces a new sensor technology called IRIS (Intelligent Ranging with Infrared Sensors) and new nonlinear control algorithms which can be combined with existing sensors and actuators to produce economically feasible automation solutions.

Our ITS research was featured in the cover story (The Cutting Edge) of the Business section of the **Los Angeles Times** on January 12, 1998. Here is what the LA Times had to say.

One of the main points of contention in the ITS community is the level of vehicle-to-vehicle and vehicle-to-roadway cooperation that can be assumed in current research. In that respect, systems currently in various stages of research and development can be classified into three categories:

- **Autonomous systems** depend only on information obtained by the sensors located on the vehicle itself, usually relative distance and velocity to stationary objects and moving vehicles. They are therefore implementable in the immediate future, and in fact have started to appear as commercial products (collision warning, adaptive cruise control).
- **Cooperative systems** add information transmitted by neighboring vehicles, usually acceleration and steering inputs. Hence, they can perform more demanding tasks than autonomous systems, such as coordinated driving in a group, but their time to commercialization is likely to be longer.
- **Automated highway systems** add information obtained from the roadway infrastructure, such as messages regarding traffic conditions and road geometry, and lateral information from magnetic nails or reflective guardrails installed on the highway. Such systems can perform even more demanding tasks, like fully automated driving in a platoon, but must face many more obstacles (standardization, liability issues, public acceptance) on their way to implementation.

Commercial vehicles, in particular, will reap significant benefits from all stages of automation. Collision warning systems increase safety and reduce accidents, while adaptive cruise control enhances driver comfort and reduces fuel consumption and emissions. Fleet operators can further reduce their costs using cooperative scenarios like the *electronic towbar*, in which one manually driven vehicle is followed by two or three driverless automated vehicles. Finally, in automated highway systems, fully automated vehicles will be able to carry freight and passengers with significantly enhanced safety, increased fuel efficiency, and much more predictable travel times.



Two critical automation components in any of these scenarios, especially for commercial vehicles, are the sensors and the actuators. Existing sensor technologies can only provide the reliability required for these applications by increasing the cost to levels beyond those acceptable to the potential customers. Actuators, on the other hand, have built-in saturations and delays which significantly complicate the control task. These issues are addressed by our current research, in which we are developing:

□ very inexpensive and highly accurate **ranging sensors** that can be used as stand-alone sensors for adaptive cruise control and vehicle following, and also in combination with radar or vision for improved collision warning, lane tracking, as well as driving in platoons, and

□ **novel nonlinear control algorithms** which can deal with the severe actuator saturations and delays present in commercial vehicles to yield good performance in both autonomous and cooperative scenarios.

The reason for focusing on these projects is that we are interested in results that will be useful in all stages of automation and will potentially impact product development ranging from today's collision warning and adaptive cruise control to the future's fully automated AHS vehicles.