THE OPEN UNIVERSITY OF SRI LANKA
BACHELOR OF TECHNOLOGY – LEVEL 05
FINAL EXAMINATION – 2005
MPU 3305 - ENGINEERING MATHEMATICS II
DURATION : FOUR (04) HOURS

Date : 20th March 2006 **Time: 9.30-13.30 hours**

Answer only **seven** questions. State any assumptions which are required.

01. i. The output of a factory is 500 units a day and is provided by three machines A, B and C, which produce respectively 200, 175 and 125 units a day. Over a long period it is found that the percentage of defective output is 5% from machine A, 4% from machine B and 4% from machine C.

a) Find the probability that a unit chosen at random from the output of the factory is defective.

b) If a unit is chosen from the output of the factory and is found to be defective, find the probability that it came from machine A.

c) On three different days a unit is chosen at random from the output of the factory. Find the probability that exactly one of these units will be defective.

d) If one unit is chosen at random from the output of each machine, find the probability that exactly one unit will be defective.

02. i. A set of N readings X_1, X_2, ...X_n has mean m_1 and standard deviation S_1. Another set of M readings Y_1, Y_2, ...Y_n has mean m_2 and standard deviation S_2. Given that the mean of the combined set of (N+M) readings has mean P with standard deviation Q, show that,

a) $(N+M)P = Nm_1+Mm_2$

b) $(N+M)(Q^2+P^2) = N[S_1^2+m_1^2]+M[S_2^2+m_2^2]$

The mean of 20 measurements is 1.50 with standard deviation 0.95. A further measurements of 2.50 is then included. Find the mean and the standard deviation of the set of 21 measurements.

ii. A and B are two events such that, $P(A) = 0.8$, $P(B)=0.4$ and $P(A \cup B)=0.9$. Find

a) $P(A \cap B)$ b) $P(B/A)$

c) $P(\overline{B}/A)$ and d) $P(A/\overline{B})$

Also test whether event A and B are mutually exclusive or independent.

03. i. X is a continuous Random variable. Show that the function,

$$f(x) = \lambda e^{-\lambda x} \quad x \geq 0$$
$$= 0 \quad x < 0$$

is a probability distribution function. Hence find,

a) $F(X)$ b) $E(X)$

c) $E(X^2)$ d) $V(X)$

($F(X)$ is the CDF of X)

X is a random variable such that $X \sim N(\mu, \sigma^2)$ in usual notation, Show that $Z \sim N(0,1)$ where $Z = (x - \mu)/\sigma$.

ii. The finished inside diameter of a piston ring is normally distributed with a mean of 10cm and a variance of 0.0025 cm^2.

Calculate

a) Proportion of rings which will have inside diameter less than 10.075cm.
b) Probability that a certain piston ring will have an inside diameter between 10.050 cm and 10.060 cm.

04. The probability that a particular automobile part is defective is known to be 0.001. 3000 parts are required in the assembly of a car.

a) Use the binomial probability distribution to calculate the probability that there are no defectives. Now calculate the approximate probability using the Poisson distribution.

b) Calculate the probability of exactly 5 defectives. Try using both the binomial distribution and the Poisson approximation to obtain your answer.

05. i. A supplier guarantees that each box of small electronic components will contain 250 components on the average. From a large consignment of such boxes a sample of 36 are checked and the average contents were found to be 248 with a standard deviation of 3 components. At the 0.1% significance level is there any reason to reject the supplier's guarantee?

Also computer the 99% confidence interval of the population mean of the electronic components.

ii. A large automobile manufacturing company is trying to decide whether to purchase brand A or brand B tyres for its new models. To help arrive at a decision an experiment is conducted using 12 of each brand. The tyres are run until they wear out. The results are,

brand A : $x_1 = 37{,}900$ km, $S_1 = 5100$ km
brand B : $x_2 = 39{,}800$ km, $S_2 = 5900$ km

Test the hypothesis at the 0.05 level of significance that there is no difference in the two brands of tyres. Assume the population to be approximately normally distributed. x and S are the mean and standard deviation is usual notation. You should work from first principles.

06. It is thought that the number of cans damaged in a boxcar shipment of cans is a function of the speed of the boxcar at impact. Thirteen boxcars selected at random were used to examine whether this appeared to be true. The data collected were as follows:

Speed of Car at Impact	No. of Cans Damaged
X	Y
4	27
3	54
5	86
8	136
4	65
3	109
3	28
4	75
3	53
5	33
7	168
3	47
8	52

a) Plot these data on a graph paper, if it is required to predict Y from X.

b) Calculate the linear regression equation of Y on X. Draw this on your plot.

c) Find the corelation coefficient between X and Y. Comment your answer.

d) Find an estimate of the error variance.

e) Test whether the slope parameter is zero.

f) Find 95% confidence interval for the slope parameter.

07. A firm produces three items A, B and C and requires two types of resources man hours and row material. The following L.P problems has been formulated to determine the optimum production schedule that maximizes the total profit.

Maximize $Z = 3y_1 + y_2 + 5y_3$

Subject to $6y_1 + 3y_2 + 5y_3 \leq 45$

$3y_1 + 4y_2 + 5y_3 \leq 30$

$y_1 \geq 0$, $y_2 \leq 0$,

Where y_1, y_2, y_3 are the number of items A, B and C.

i. Find the optimal solution using simplex method.

ii. Formulate the dual of the above problem and use the solution obtained in part (i) to find its optimal solution.

08. i. Find by the use of the 2nd order Tayler series method, solution of the differential equation,

$$\frac{dy}{dx} = x^2 + y^2, \; y(1) = 0 \text{ at}$$

x=1.1,1.2

ii. Obtain the same solution for the above differential equation using the Runge-Kutta (RK4) method. Briefly comment why the two solutions are nearly equal.

(Equations for RK4 method are given by,

$$Y_{n+1} = Y_n + \frac{k_1 + 2k_2 + 2k_3 + k_4}{6} \qquad K_1 = hf(x_n, y_n)$$

$$k_2 = hf(x_n + \frac{h}{2}, y_n + \frac{k_1}{2})$$

$$k_3 = hf(x_n + \frac{h}{2}, y_n + \frac{k_2}{2})$$

$$k_4 = hf(x_n + h, y_n + k_3)$$

Take h=0.1 and state any assumption you use.

09. i. Find the Fourier sine and cosine transform of (a) e^{-x}(b) xe^{-x}.

ii. Write down the Laplace transforms of

a) t

b) te^{-1}

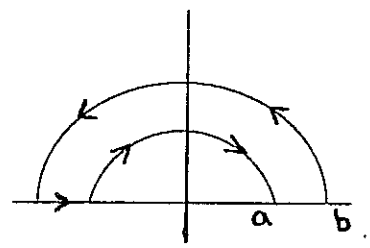
and use the Convolution Theorem to find the Inverse Laplace transform of $\frac{1}{S^2(S+1)^2}$.

Hence find the solution of the differential equation

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = 1$$

Given that $\frac{dy}{dt} = 1$ and y = 0 when t=0.

10. i. Evaluate the integral



$$\int_L \frac{\bar{z}}{z} dz$$

Where L is the contour shown in the figure, by

a) Residue theorem.

b) Any other method.

11. In the mapping $W = \frac{az+b}{cz+d}$, a,b,c,d are complex constants.
Find the values of those constants so that upper half of the z-plane is mapped on to the inside of the unit circle.

12. Use a numerical method to find the dominant eigen values and the corresponding eigen vector of the Matrix "A" where

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & -2 & 1 \\ 3 & 1 & 3 \end{bmatrix}$$

Describe briefly. the method you have used.