THE OPEN UNIVERSITY OF SRI LANKA

BACHELOR OF TECHNOLOGY – LEVEL 05

FINAL EXAMINATION – 2006

MPU 3304 - ENGINEERING MATHEMATICS I

DURATION : FOUR (04) HOURS

063

Date : 05th May 2006 Time:14.00 – 18.00 hours

Instructions:

- **Answer only Seven (07) questions.**
- **State any assumption you required**
- **Do not spend more than 30 minutes for one problem.**
- **Show all your workings.**
- **All symbols are in standard notation.**

01. OABC is a tetrahedran. The planes OAC, OBC are mutually perpendicular. BC is perpendicular to OC and CA is perpendicular to OA. Prove that BA is perpendicular to OA.
If OB = r and $C\hat{O}A = C\hat{O}B = 45^0$. Determine the lengths BC, CA, OA.

02. i. $\underline{a} = \underline{a}$ (t) is a vector function of a scalar parameter t.
Define $\frac{d\underline{a}}{dt}$

ii. The position vector of a particle varies with time according to the equation,

$$\underline{r}(t) = -(t^4 + 3t)\,\underline{i} + 8t^2\,\underline{j} + 4\underline{k}$$

Find,

a) The particle's velocity and acceleration vectors at t = 0.
b) The particle's velocity and acceleration vectors at t = 1.
c) The particles speed and its direction of motion at t = 1.

03. i. Given that vectors $\underline{A} = 2\underline{i} - \underline{j} + 2\underline{k}$ and $\underline{B} = \underline{i} + 2\underline{j} - 2\underline{k}$,
find vectors $\underline{C}$ and $\underline{D}$ that satisfy the following conditions,

a) $\underline{C}$ is parallel to B
b) $\underline{D}$ is perpendicular to B
c) $\underline{A} = \underline{C} + \underline{D}$

ii. Use $A = \cos\alpha\,\underline{i} + \sin\alpha\,\underline{j}$, $B = \cos\beta\,\underline{i} + \sin\beta\,\underline{j}$ to prove that
$\cos(\alpha + \beta) = \cos\alpha . \cos\beta - \sin\alpha . \sin\beta$.

04. i. Calculate the line integral of the vector function
$f(x,y,z) = x\underline{i} + y\underline{j} + (xz - y)\underline{k}$ from (0,0,0) to (1,2,4) along a line segment.

ii. Find $\int_C (x^2 + y^2)\, ds$ when C is the curve defined.
by $x = a\,(\text{Cos}t + t\ \text{Sin}\ t)$
and $y = a\,(\text{Sin}t - t\ \text{Cos}t)$
and $0 \leq t \leq 2\pi$.

05. Using the vector identity $\underline{\nabla}.(\underline{A} \times \underline{B}) = \underline{B}.\ \underline{\nabla} \times \underline{A} - \underline{A}.\ \underline{\nabla} \times \underline{B}$ or otherwise show that, Div ($\underline{F}$ x grad f) = grad f. curl $\underline{F}$ where $\underline{F}$ is a vector field, while f is a scalar field. Hence prove that

$$\iiint_V \text{Grad f. curl } \underline{F}\, dv = \iint_S (\underline{F} \times \text{grad f}).n\ \ ds$$

Where v is for volume, s is for the surface enclosing v.

06. i. Find the domain in which $f(z) = \frac{1}{z^2}$
is analytic. f(z) is a function of the complex variable z.

Using the above result, evaluate $\int_{-i}^{i} \frac{dz}{z^2}$

ii. Evaluate $(\cos\theta + i\sin\theta)^4$ by using De Moivors theorem. Hence find a formula for $\sin 4\theta$ in terms of $\cos\theta$ and $\sin\theta$.

07. i. A matrix is given as,

$$R = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & -2 \\ 0 & -2 & 3 \end{bmatrix},$$

a) Find eigen values and corresponding eigen vectors of R.
b) An orthogonal matrix P, diagonalizes R such that $P^{-1} RP = DM$ where DM is a diagonal matrix. Determine the matrices P and DM.

ii. A quadratic function is given in standard notation by the equation,
$Q(x,y,z) = (3x^2+3y^2+3z^2-4yz) = XRX^t$, where R is the matrix given in part (i) and X = (x,y,z).

Reduce Q to the form of sum of squares.

08. i. Liquid fills a spherical vessel of radius $\boldsymbol{a}$ to a depth h_0. At time $t = 0$ fluid is allowed to drain out of an orifice at the lowest point of the vessel at a volume rate $k\sqrt{h}$ where k is a constant. Derive the differential equation for the variation of depth h with time and find an expression for the time taken to empty the vessel.

ii. Solve the following differential equations.

a) $$\frac{dy}{dx} + y\cot x = x^2$$

b) $$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = x^4e^x$$

09. i. A circular coil of n turns of area A whose inductance is L henrys and resistance R ohms is rotated with angular velocity ω about a diameter perpendicular to a field of strength H Am^{-1}. The current **i** induced in the coil is given by

$$L\frac{di}{dt} + Ri = n\omega HA\cos\omega t$$

Find the current at time t, assuming that initially it is zero.

ii. Find the complete solution to the following differential equation.

$$\frac{d^2m}{dx^2} + \frac{dm}{dx} - 2m = 5\sin 3x$$

<u>Answer any two parts for the question No. 10</u>.

10. i. Derive Newton's formula to solve an equation f(x)=0, for a real unknown x. Show that the equation.

$\mu e^m = 1$, where $m = 2\mu\pi$

Will have one real solution for the unknown μ which stands for the coefficient of friction. By doing a single iteration of the Newton-Raphson show how to solve the above equation to 3 decimal places.

ii. A solid body is symmetrical about a vertical axis. The radius(r) of the body at distances (d) from the base is given as follows:-

d(cm)	0	1	2	3	4	5	6
r(cm)	18	17	16	15	12	6	4

Find a good estimate for the volume of the solid body using a numerical method.

iii. Values of the function $y = \log_e(\cos x)$ are tabulated as follows:-

x:	0.0	0.1	0.2	0.3	0.4
y:	0.0	-0.0050	-0.0201	-0.0457	-0.0822

Use this table to find $\log_e 0.125$, by the use of the forward difference interpolation method.

Use the safe table to construct a table of approximate values for tan x, at x=0.5,1.5,2.5,3.5. From the table for tan x, so constructed. Show how to estimate x, which satisfies tan x = 0.3.

11. Electrical currents in four parts of a circuit satisfy the equations,

$$\begin{pmatrix} 2 & 1 & 2 \\ 2 & 3 & 4 \\ 2 & 2 & 7 \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \\ i_3 \end{pmatrix} = \begin{pmatrix} 6 \\ 8 \\ 11 \end{pmatrix}$$

Find three currents i_1, i_2 and i_3 using Gauss- Seidal, Jacobi or any other numerical method (use 4 steps and work to 3 decimal place)

12. The strain tensor at a point of a body is given by

$$\epsilon_{ij} = \begin{pmatrix} 1 & 0 & 3 \\ 0 & -2 & 0 \\ 3 & 0 & 1 \end{pmatrix} \times 10^{-2}$$

i. Calculate the principal strain and the principal directions.

ii. Write the equation of co-ordinate transformation.