070

The Open University of Sri Lanka
Department of Electrical and Computer Engineering
Final Examination 2005/2006
ECX5233 – Radio and Line Communication

Time: 0930 – 1230 hrs. Date: 2006-04-20

Answer any FIVE questions

1.

(a) *Fourier Transform F(ω)* of a signal *f(t)* is defined as

$$F(\omega)=\int_{-\infty}^{+\infty} f(t).e^{-j\omega t}\,dt$$

Using the above definition prove that the *Fourier Transform F'(ω)* of $f(t-t_0)$ is given by

$$F'(\omega)=\int_{-\infty}^{+\infty} f(t-t_0).e^{-j\omega t}\,dt=F(\omega).e^{-j\omega t_0} \text{ - Time shifting}$$

[Note: similarly the *Frequency shifting* says that the Fourier Transform of

$f(t).e^{j\omega_0 T}$ is $F(\omega-\omega_0)$. where $\omega_0 = 2\pi/T$]

(b) *Dirac Impulse δ(t)* has following properties:

$$\delta(t)=0, \quad if\ t\neq 0$$
$$\int_{-\infty}^{+\infty}\delta(t)\,dt=1$$

Show that the Fourier Transform of *δ(t)* is 1

(c) Some properties related to *the Fourier Transform F(ω)* of a signal *f(t)* is given below:

(i) *Fourier Transform of* $f(at)=\frac{1}{a}F\left(\frac{\omega}{a}\right)$ - *Scaling Property*

(ii) *Fourier Transform of* $\frac{d[f(t)]}{dt}=j\omega F(\omega)$ - *Time Differentiation*

(iii) *Fourier Transform of* $F(t) = 2\pi f(-\omega)$ – *Symmetry or Duality*

Prove any of the above properties (i), (ii) or (iii).

(d)

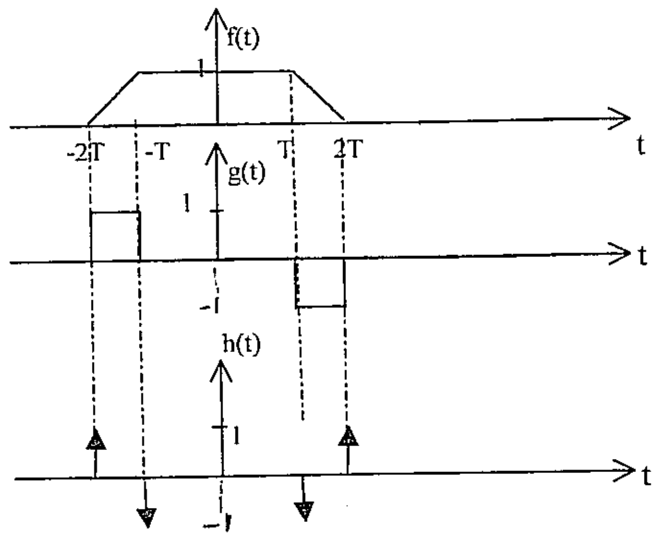


Fig.1

Three waveforms f(t), g(t) and h(t) are shown in Fig.1.

(i) Find the relationship between f(t), g(t) and h(t).[Hint: Differentiate f(t) and g(t)].

(ii) Using the results of (d) (i) and (c) (ii) calculate and sketch the Fourier Transforms of f(t), g(t) and h(t).

2.

(a) The *Convolution* of two waveforms g(t) and h(t) is defined as follows:

$$s(t) = g(t) * h(t) = \int_{-\infty}^{+\infty} g(\tau).h(t-\tau)d\tau$$

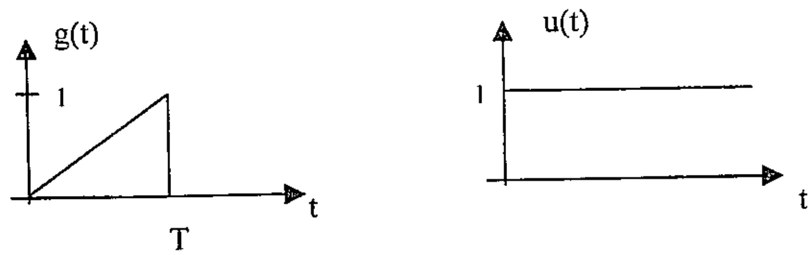


Fig.2 (a)

A sawtooth waveform g(t) and a step-waveform u(t) is shown in Fig.2 (a).

(i) Draw the functions of u(τ), u(-τ) and u(t-τ).

(ii) Draw the two functions g(τ) and u(t-τ) on the same diagram.

Indicate the area represented by $\int_{-\infty}^{+\infty} g(\tau).u(t-\tau)d\tau$ on your diagram.

(iii) Evaluate s(t) = $g(t) * u(t)$ using the result 2 (a) (ii).

(b) (i) A rectangular pulse p(t) can be given by the following expression:

$p(t) = 1$ for $|t| \leq T/2$

$= 0$ otherwise

Express p(t) in terms of u(t) (refer Fig.2 (a))

Hence or otherwise calculate g(t)*p(t) and sketch it.

(ii) The rectangular pulse q(t) shown in Fig.2(b) is sent through a channel whose impulse response is g(t). Calculate the output waveform y(t) of the channel and sketch it.

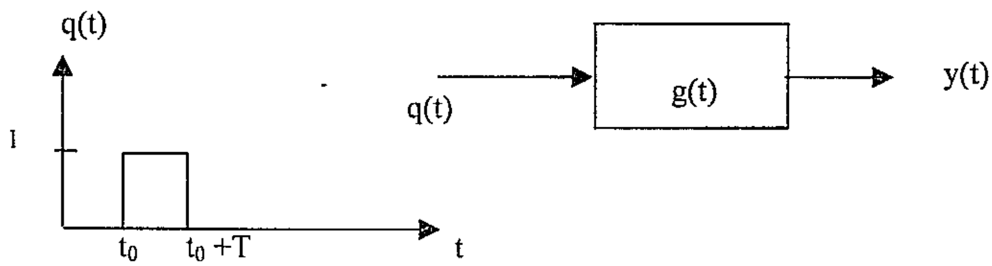


Fig.2(b)

(c) (i) Find the *Fourier Transform* of p(t) and hence the *Fourier Transform* of q(t).

(ii) Using the *Fourier Transform* of q(t) or otherwise calculate the Fourier Transform of g(t) (Hint: Use the *Time differentiation* property of a Fourier Transform).

(iii) Calculate and sketch the *Frequency spectrum* of y(t)

3.

(a) The complex number notation for $\cos\theta$ and $\sin\theta$ is given below:

$$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}; \ \sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

An information signal m(t) is amplitude modulated using a carrier signal $\cos\omega_c t$. The output y(t) of the modulator is given

$$y(t) = (1 + m(t)) \cos\omega_c t$$

If $M(\omega)$ is the Fourier Transform of m(t), find the Fourier Transform $Y(\omega)$ of the output signal y(t) from first principles. [Hint: Use the complex number notation of $\cos\theta$]
Hence sketch the frequency spectrum of the modulated signal.

(b) A certain filter has following amplitude- and phase characteristics:

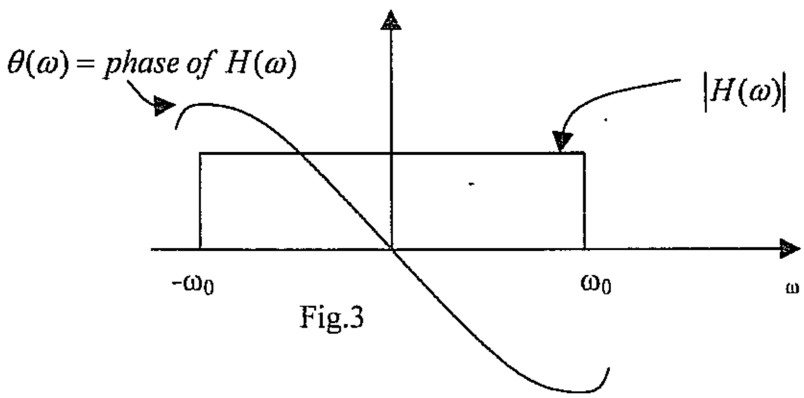


Fig.3

If $|H(\omega)| = 1$ and $\theta(\omega) = -\omega t_0 - k \sin(\omega T)$ where $k << 1$

(i) write down an expression for $H(\omega)$.
(ii) find the filter output z (t) if the input signal to the filter is any band limited signal x(t)

[Hint: $e^{-jk\sin\omega T} = 1 - jk\sin\omega T$ if $k << 1$].

(iii) find the filter output z(t) if the modulator output y(t) mentioned in 3 (a) is applied as the input signal to the filter.

4.

(a) A sampling process uses a series of Dirac impulses as the sampling signal s(t).

$$s(t) = \sum_{n=-\infty}^{+\infty} \delta(t - nT_s)$$

Express s(t) as a complex Fourier series and find the Fourier coefficients.

[Express s (t) as $\sum_{n=-\infty}^{+\infty} c_n e^{jn\omega_0 t}$ and find the value of c_n for all n]

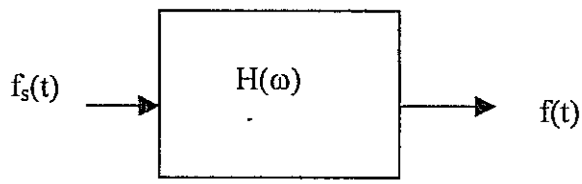
(b) Now the signal f(t) is sampled using s(t). Sampled signal $f_s(t)$ is given by

$f_s(t) = f(t).\ s(t).$

(i) Sketch $f_s(t)$.

(ii) Find the Fourier Transform of $f_s(t)$ in terms of F(ω). Sketch $F_s(\omega)$.
Show that F(ω) cannot be separated from $F_s(\omega)$ if $B > 2/T_0$.

(iii) Sketch the Frequency response H(ω) of a suitable filter which can reconstruct the original signal f(t) from the sampled signal if $B \leq 2/T_0$.



5.

(a) *Auto-correlation* function *R(t)* of *an Ergodic* random Process x (t) can be expressed as

$$R(\tau) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{+T} x(t)\, x(t+\tau)\, d\tau$$

(i) What is an Ergodic process?

(ii) If x(t) was not Ergodic, then how would you define the Auto-correlation function for the two random variables $x(t_1)$ and $x(t_2)$? Assume that the joined probability density function of two random variables $x_1 = x(t_1)$ and $x_2 = x(t_2)$ is $p_{x_1 x_2}(x_1, x_2)$.

(b) Consider the *Ergodic* periodic time signal shown in Fig.5 (a)

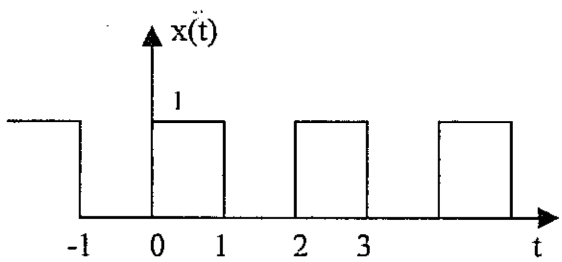


Fig.5 (a)

Calculate the Autocorrelation function $R(\tau)$

(i) Calculate the power of the signal.
(ii) Calculate the d.c component of the signal.
(iii) Calculate the variance σ_x^2.

(c) Power spectral density function (PSD) of a stationary ergodic process x(t) is shown in Fig.5 (b)

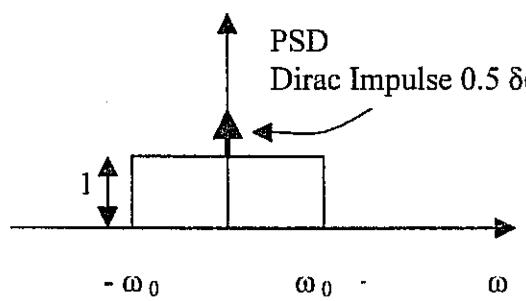


Fig.5(b)

If $\omega_0 = 2\pi / 100\ \text{Hz}^{-1}$, calculate the Autocorrelation function of x(t).

6.

(a) A digital signal having *four* amplitude levels is transmitted over a noisy channel. What modification(s) should be done to the signal detection technique in order to cope with noise?

(b) A signal (q(t)) having 4 amplitude levels -4, -2 +2 and +4 is transmitted over a noisy communication channel. During the transmission, amplitude of the signal changes due to channel noise. The probability density function p_n of channel noise n(t) is shown in the Fig.6. Different signals of q(t) leaves the transmitter with different probabilities as given below:

$P(-4) = 1/6$; $P(+4) = 1/6$; $P(-2) = 1/3$; $P(+2) = P_0$

Noise signal n(t) is statistically independent from the data signal q(t)

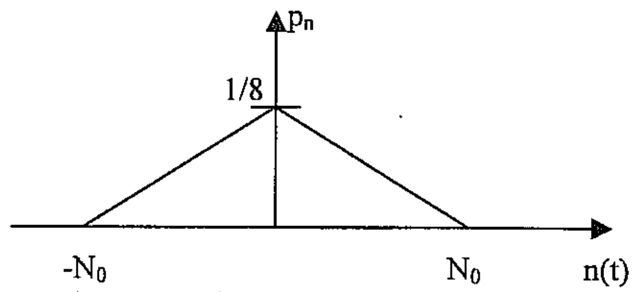


Fig.6

To minimize the error due to additive noise, following criteria is used at the receiver to modify the threshold level of different signal components:
If the received signal is s(t) and the corrected signal level is a(t) then

$$a(t) = \begin{cases} +4 & \text{if } \quad s(t) > +3 \\ +2 & \text{if } +3 \geq s(t) > +1 \\ -2 & \text{if } +1 \geq s(t) > -3 \\ -4 & \text{if } -3 \geq s(t) \end{cases}$$

(i) Calculate the values of P_0 and N_0.
(ii) Calculate the noise power.
(iii) Calculate the average power of s(t).
(iv) Calculate the probability that q = +4 will be correctly detected.
(v) Calculate the probability that q = +4 will be erroneously detected.
[Hint: use the result of 6 b (iv)]

7.
(a) It is necessary to match a load whose normalized admittance is $y_L = 0.2 - j0.8$, to a transmission line using the *double stub matching method.* The first short circuit stub whose length is l_1 is connected to the load itself. The second stub is connected at a distance $\lambda/8$ from the first stub. The length of the second stub is l_2. (When working out this question you must always stick to only one set of l_1 and l_2. Marks will be deducted if more than one set of values are specified. You must stick to one selected set of values and show your design based on that. Always use the Smith chart as an admittance chart).

(i) Locate $y_L = 0.2 - j0.8$ on the Smith Chart and denote it by A.
(ii) Find the new admittance y_l at the load when the first stub is added. Locate this point (B) on the Smith chart.
(iii) Find the value of l_1 in terms of λ. Indicate the path from A to B clearly on the Smith chart.

(iv) Indicate clearly the path from B to the second stub point (C) before the addition of the second stub. Locate C and calculate the corresponding admittance y_2 .

(v) Now the second stub is added. Find new admittance y_3 and locate that point (D) on your Smith chart. Indicate clearly the path from C to D on the Smith chart. Find the value of l_2 in terms of λ.

(vi) If the generator frequency is 100 MHz, calculate l_l and l_2 in centimeters.

(b) How does the magnitude of the *reflection coefficient* of a lossless transmission line changes along the line?

8.

Explain the following :

(a) Discrete Fourier Transform (DFT) of a signal and its practical significance.
(b) Eye diagram of a random binary pulse sequence.
(c) Frequency and the phase characteristics of the Impulse response of an ideal low pass filter.
(d) Calculation of the probability density function (PDF) of $z = x + y$, when the PDF's of the random variables x and y are known.
(e) Niquist's first Criterion for zero ISI