

The Open University of Sri Lanka



ECX 5231 / ECU 3104 Network Theory

[ 8<sup>th</sup> May 2006, Time: 0930 hrs. – 1230 hrs]

Three hours

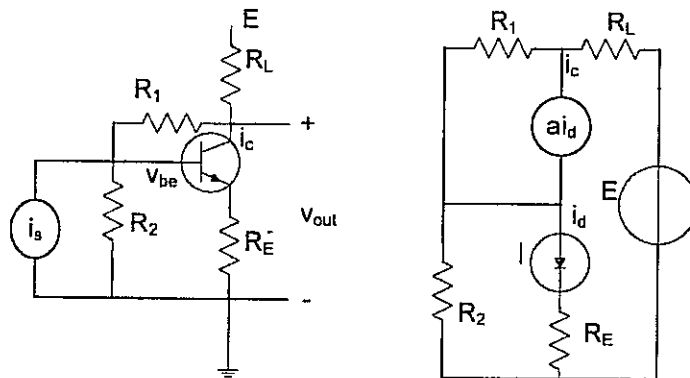
Instructions:

ECX 5231: Answer FIVE questions, selecting at least one question from each section.  
 ECU 3104: Answer any FIVE questions.

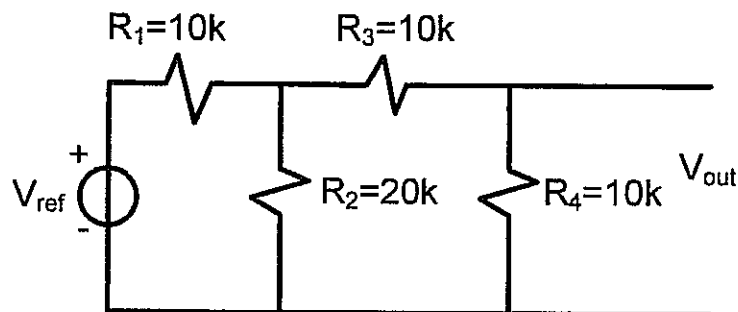
All questions carry equal marks.

Section A

- The figure shows a simple transistor amplifier and its Ebers-Moll model for the case when the input is zero. Replace the diode with its companion model so as to obtain the companion model of the complete network. Then, write the node equations for the iterative solution of the network to obtain the quiescent (with no input excitation) operating conditions for the circuit.



- The figure shows a R-2R ladder network used as a 2-bit digital to analogue converter: For the configuration shown, use Tellegen's Theorem to obtain expressions for the sensitivity of the output voltage to changes of each of the resistance values.



## Section B

3. Genetic algorithms have been used to design electrical circuits, in an attempt to automate the design process. The following is a description of a chromosome meant to represent the topology of a simple passive circuit having up to a maximum of eight nodes and eight elements.

The chromosome consists of a  $8 \times 8$  array, where each row describes one branch, the eight rows thus corresponding to (up to a maximum of) eight branches. The first two bits (of each row) describe the nature of the branch element as follows:

00	Short circuit
01	Resistance
10	Capacitance
11	Inductance

The next six bits refer to the two end nodes (three bits, eight values each). In the case of the circuit description giving rise to unconnected trees, the tree with the highest number of nodes only is considered, and the others ignored. Circuit elements connected to the same node are also ignored.

- a. Draw the circuit corresponding to the following (randomly generated) chromosome.

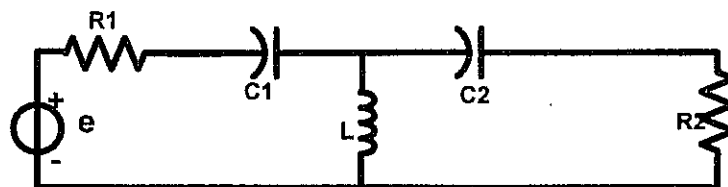
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1 0 0 0 1 0 1 0
0 0 0 1 0 0 0 0
1 1 0 1 1 0 0 0
0 1 0 1 1 0 0 1
1 0 0 1 0 0 1 1
0 1 1 0 0 0 0 1
1 1 0 1 1 1 0 1
1 0 1 0 1 0 1 1

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- b. Draw the circuit for chromosome that obtained by changing the elements (7,3) (mutation)
- c. Comment on the method of circuit representation used and suggest possible improvements to it.

4. Consider the circuit shown in the figure.



- a. How many state variables would be required to dynamically model this circuit? Give reasons for your answer.
- b. Choose a set of suitable state variables and directly write down the state equations describing the circuit, using nodal and / or mesh analysis.

5. The system matrix describing a connected network is shown below:

$$\begin{bmatrix} 5 & -2 & 0 & 0 & -1 & 0 & 0 \\ -2 & 5 & -2 & 0 & 0 & 0 & 0 \\ 0 & -2 & 11 & -2 & 0 & 0 & 0 \\ 0 & 0 & -2 & 5 & -2 & 0 & 0 \\ -1 & 0 & 0 & -2 & 11 & -2 & -2 \\ 0 & 0 & 0 & 0 & -2 & 5 & 0 \\ 0 & 0 & 0 & 0 & -2 & 0 & 5 \end{bmatrix}$$

Less than half its elements are non-zero. Draw up a table indicating the storage of the elements of the first two rows of these values in memory using any reasonable method of sparsity programming. Add comments to allow a reader to interpret your method of storage. Explain how you would accommodate 'fill-ins' during processing.

Re-order the given matrix to conserve sparsity during processing (such as LU factorisation). You do not need to enter the actual values, but you may indicate the presence of a non-zero element by an ( \* ) in a matrix represented in non-sparse form.

Are there better methods of re-ordering than the one you used? When would such methods be useful?

### Section C

You know that active filters with only capacitors and resistors (and no inductors) may be constructed using devices such as gyrators and Negative Impedance Converters. It is also possible to dispense with the use of resistors and use only switched capacitor circuits as described in the following account from

<http://www.swarthmore.edu/NatSci/echeev1/Ref/FilterBkgrnd/SwitchedCap.html>

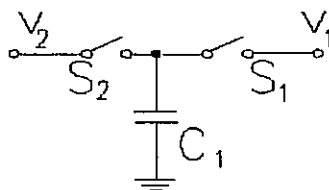
Read it carefully before you attempt questions 7, 8 or 9.

#### Switched Capacitor Circuits

*In the last decade or so many active filters with resistors and capacitors have been replaced with a special kind of filter called a switched capacitor filter. The switched capacitor filter allows for very sophisticated, accurate, and tuneable analogue circuits to be manufactured without using resistors. This is useful for several reasons. Chief among these is that resistors are hard to build on integrated circuits (they take up a lot of room), and the circuits can be made to depend on ratios of capacitor values (which can be set accurately), and not absolute values (which vary between manufacturing runs).*

#### The Switched Capacitor Resistor

*To understand how switched capacitor circuits work, consider the circuit shown with a capacitor connected to two switches and two different voltages.*



If  $S_2$  closes with  $S_1$  open, then  $S_1$  closes with switch  $S_2$  open, a charge  $\Delta q$  is transferred from  $v_2$  to  $v_1$  with

$$\Delta q = C_1(v_2 - v_1)$$

If this switching process is repeated  $N$  times in a time  $\Delta t$ , the amount of charge transferred per unit time is given by

$$\frac{\Delta q}{\Delta t} = C_1(v_2 - v_1) \frac{N}{\Delta t}$$

Recognizing that the left hand side represents charge per unit time, or current, and the number of cycles per unit time is the switching frequency (or clock frequency,  $f_{CLK}$ ) we can rewrite the equation as

$$i = C_1(v_2 - v_1)f_{CLK}$$

Rearranging we get

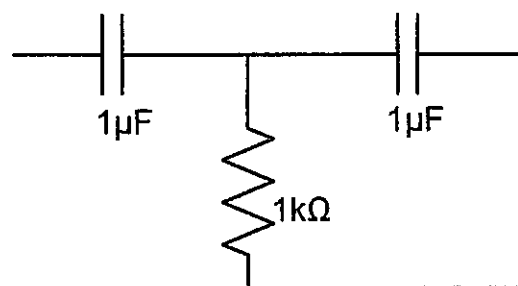
$$\frac{(v_2 - v_1)}{i} = \frac{1}{C_1 f_{CLK}} = R$$

which states that the switched capacitor is equivalent to a resistor. The value of this resistor decreases with increasing switching frequency or increasing capacitance, as either will increase the amount of charge transferred from  $v_2$  to  $v_1$  in a given time.

6. When is it advantageous to use switched capacitors instead of resistors in the manufacture of electronic circuits? What are its advantages in these circumstances?
7. Consider the relationship  $\frac{1}{C_1 f_{clk}} = \bar{R}$ , derived in the simplified analysis given in the text.

What choices are available to the circuit designer for the realisation of a particular resistor?

It is proposed to use this technique to construct the simple RC circuit shown in the figure.



What should be the switching frequency  $f_{clk}$  if the switched capacitor is to have the same value as the others?

8. The simple analysis presented in the text assumes instantaneous charge transfer upon switching, as represented by the equation

$$\Delta q = C_1(v_2 - v_1)$$

What factor(s) are likely to constrain this in a practical circuit? [Hint: Sketch a simple charge transfer curve from a capacitor, and you will see the answer!]