033

THE OPEN UNIVERSITY OF SRI LANKA
BACHELOR OF TECHNOLOGY (level 05)
ECU 3205
FIELD THEORY
FINAL EXAMINATION 2005

DATE : 10th May 2006 **TIME : 09.30 – 12.30 hours**

Answer FIVE questions

1.

(a) Verify whether the divergence theorem is true for the field

$$\mathbf{A} = \mathbf{a_x}\, x^3 + \mathbf{a_y}\, y^3 + \mathbf{a_z}\, z^3$$

when the closed surface is the sphere $x^2 + y^2 + z^2 = a^2$.

(b) The scalar potential of a certain field in spherical coordinates is given by

$V = 50\ K/r$; where K is a constant.

Use the relationship $\mathbf{F} = -\text{grad}V$ to find the vector field associated with this field.

Check whether F is
(i) solenoidal
(ii) irrotational

2. A cylindrical capacitor consists of an inner conductor of radius r_1 and an outer conductor whose inner radius is r_2. The space between the conductors is filled with a dielectric of permittivity ε_r and the length of the capacitor is L. If the outer conductor is grounded and the inner conductor is maintained at a potential V_0. Find the following:

(a) the electric field intensity $\mathbf{E}(r_1)$ at the surface of the inner conductor in terms of r_1, r_2 and V_0.

(b) r_1 when r_2= 5 mm such that $\mathbf{E}(r_1)$ is minimized

(c) the capacitance per unit length under the conditions of part(b), if $\varepsilon_r = 2$ and V_0=150 V.

3. The field components of a plane wave in a non-conducting medium vary only in the x direction. For the condition of no losses where $\gamma = j\omega\ (\varepsilon\mu)^{1/2}$ and using $\eta = (\mu/\varepsilon)^{1/2}$ show that $E_x = H_x = 0$; $E_y = \eta\ H_z$ and $E_z = -\ \eta\ H_y$.

4.

(a) The electric field E could be given in terms of the magnetic vector potential A and the scalar electric potential V by

$$\mathbf{E} = -\partial \mathbf{A}/\partial t - \text{grad}V$$

By substituting into one of the Maxwell's equations deduce an expression for the magnetic field **H** in free space in terms of potential functions.

(b) If **A** and V are related by div**A** + $\mu_0\varepsilon_0\ \partial V/\partial t = 0$, derive equations for **A** and V in terms of the sources of the fields.

5. Using the complex form of Maxwell's equations curl **E** = - $j\omega$**B**, curl **H** = **J** + $j\omega$**D**, Div **D** = ρ = 0, Div **B** = 0.

(a) Derive for free space (ρ = **J** = 0) the complex form of the wave equation for **E**.
(b) Derive the conservation of charge equation div**J** = - $j\omega\ \rho$ for the case **J** = σ **E**

6. For a uniform plane wave having a frequency of 350 MHz and an electric field of peak value 5 V/m traveling in the positive z direction in a lossless medium for which $\varepsilon_r = 3$, $\mu = \mu_0$ find the following:

(a) the wave length
(b) the intrinsic impedance
(c) the phase velocity
(d) the group velocity
(e) the time average power density

7. A current of density $\mathbf{J} = \mathbf{a}_z J_0$, flows along an infinitely long solid cylindrical conductor of radius 'b' oriented along the z axis.

(a) calculate the magnetic flux density, inside and outside the conductor
(b) find the Poyinting vector on the surface of the conductor
(c) verify Poynting theorem

8. The far field of an antenna could be expressed as given below in spherical coordinates:

$$\mathbf{E}(r, \theta) = \mathbf{a}_\theta \, [\, (k_1/r) \sin\theta] \, e^{-j\beta r}$$
$$\mathbf{H}(r, \theta) = \mathbf{a}_\varphi \, [(k_2/r) \sin\theta] \, e^{-j\beta r}$$

Where k_1 and k_2 are constants and other symbols carry their usual meaning.

(a) write an expression for the instantaneous Poyinting vector
(b) find the total average power radiated by the antenna

VECTOR RELATIONS

UNIT VECTORS

$\mathbf{a}_x, \mathbf{a}_y, \mathbf{a}_z$ — rectangular
$\mathbf{a}_\rho, \mathbf{a}_\phi, \mathbf{a}_z$ — cylindrical
$\mathbf{a}_r, \mathbf{a}_\theta, \mathbf{a}_\phi$ — spherical

COORDINATE TRANSFORMATIONS

$$x = \rho\cos\phi = r\sin\theta\cos\phi$$
$$y = \rho\sin\phi = r\sin\theta\sin\phi$$
$$z = r\cos\theta$$
$$\rho = \sqrt{x^2 + y^2} = r\sin\theta$$
$$\phi = \tan^{-1} y/x$$
$$r = \sqrt{x^2 + y^2 + z^2} = \sqrt{\rho^2 + z^2}$$
$$\theta = \tan^{-1}(\sqrt{x^2 + y^2}/z) = \tan^{-1}(\rho/z)$$

COORDINATE COMPONENT TRANSFORMATIONS

$$A_x = A_\rho\cos\phi - A_\phi\sin\phi = A_r\sin\theta\cos\phi + A_\theta\cos\theta\cos\phi - A_\phi\sin\phi$$
$$A_y = A_\rho\sin\phi + A_\phi\cos\phi = A_r\sin\theta\sin\phi + A_\theta\cos\theta\sin\phi + A_\phi\cos\phi$$
$$A_z = A_r\cos\theta - A_\theta\sin\theta$$
$$A_\rho = A_x\cos\phi + A_y\sin\phi = A_r\sin\theta + A_\theta\cos\theta$$
$$A_\phi = -A_x\sin\phi + A_y\cos\phi$$
$$A_r = A_x\sin\theta\cos\phi + A_y\sin\theta\sin\phi + A_z\cos\theta = A_\rho\sin\theta + A_z\cos\theta$$
$$A_\theta = A_x\cos\theta\cos\phi + A_y\cos\theta\sin\phi - A_z\sin\theta = A_\rho\cos\theta - A_z\sin\theta$$

DIFFERENTIAL ELEMENTS OF VECTOR LENGTH

$$d\mathbf{l} = \begin{cases} \mathbf{a}_x\,dx + \mathbf{a}_y\,dy + \mathbf{a}_z\,dz \\ \mathbf{a}_\rho\,d\rho + \mathbf{a}_\phi\rho\,d\phi + \mathbf{a}_z\,dz \\ \mathbf{a}_r\,dr + \mathbf{a}_\theta r\,d\theta + \mathbf{a}_\phi r\sin\theta\,d\phi \end{cases}$$

DIFFERENTIAL ELEMENTS OF VECTOR AREA

$$d\mathbf{s} = \begin{cases} \mathbf{a}_x\,dy\,dz + \mathbf{a}_y\,dx\,dz + \mathbf{a}_z\,dx\,dy \\ \mathbf{a}_\rho\rho\,d\phi\,dz + \mathbf{a}_\phi\,d\rho\,dz + \mathbf{a}_z\,\rho\,d\rho\,d\phi \\ \mathbf{a}_r r^2\sin\theta\,d\theta\,d\phi + \mathbf{a}_\theta r\sin\theta\,dr\,d\phi + \mathbf{a}_\phi r\,dr\,d\theta \end{cases}$$

DIFFERENTIAL ELEMENTS OF VOLUME

$$dv = \begin{cases} dx\,dy\,dz \\ \rho\,d\rho\,d\phi\,dz \\ r^2\sin\theta\,dr\,d\theta\,d\phi \end{cases}$$

VECTOR OPERATIONS—RECTANGULAR COORDINATES

$$\nabla\alpha = \mathbf{a}_x\frac{\partial\alpha}{\partial x} + \mathbf{a}_y\frac{\partial\alpha}{\partial y} + \mathbf{a}_z\frac{\partial\alpha}{\partial z}$$
$$\nabla\cdot\mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$
$$\nabla\times\mathbf{A} = \mathbf{a}_x\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \mathbf{a}_y\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \mathbf{a}_z\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$
$$\nabla^2\alpha = \frac{\partial^2\alpha}{\partial x^2} + \frac{\partial^2\alpha}{\partial y^2} + \frac{\partial^2\alpha}{\partial z^2} = \nabla\cdot\nabla\alpha$$
$$\nabla^2\mathbf{A} = \mathbf{a}_x\nabla^2 A_x + \mathbf{a}_y\nabla^2 A_y + \mathbf{a}_z\nabla^2 A_z = \nabla(\nabla\cdot\mathbf{A}) - \nabla\times(\nabla\times\mathbf{A})$$

VECTOR OPERATIONS—CYLINDRICAL COORDINATES

$$\nabla\alpha = \mathbf{a}_\rho\frac{\partial\alpha}{\partial\rho} + \mathbf{a}_\phi\frac{1}{\rho}\frac{\partial\alpha}{\partial\phi} + \mathbf{a}_z\frac{\partial\alpha}{\partial z}$$
$$\nabla\cdot\mathbf{A} = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho A_\rho) + \frac{1}{\rho}\frac{\partial A_\phi}{\partial\phi} + \frac{\partial A_z}{\partial z}$$
$$\nabla\times\mathbf{A} = \mathbf{a}_\rho\left(\frac{1}{\rho}\frac{\partial A_z}{\partial\phi} - \frac{\partial A_\phi}{\partial z}\right) + \mathbf{a}_\phi\left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial\rho}\right) + \mathbf{a}_z\frac{1}{\rho}\left(\frac{\partial}{\partial\rho}(\rho A_\phi) - \frac{\partial A_\rho}{\partial\phi}\right)$$
$$\nabla^2\alpha = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial\alpha}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2\alpha}{\partial\phi^2} + \frac{\partial^2\alpha}{\partial z^2}$$
$$\nabla^2\mathbf{A} = \mathbf{a}_\rho\left(\nabla^2 A_\rho - \frac{2}{\rho^2}\frac{\partial A_\phi}{\partial\phi} - \frac{A_\rho}{\rho^2}\right) + \mathbf{a}_\phi\left(\nabla^2 A_\phi + \frac{2}{\rho^2}\frac{\partial A_\rho}{\partial\phi} - \frac{A_\phi}{\rho^2}\right) + \mathbf{a}_z\nabla^2 A_z$$

Vector Operations - Spherical coordinates

$$\nabla \alpha = a_r \cdot \frac{\partial \alpha}{\partial r} + a_\theta \frac{1}{r} \frac{\partial \alpha}{\partial \theta} + a_\phi \cdot \frac{1}{r \sin\theta} \cdot \frac{\partial \alpha}{\partial \phi}$$

$$\nabla \cdot A = \frac{1}{r^2} \frac{\partial}{\partial r}\left(r^2 A_r\right) + \frac{1}{r\sin\theta} \frac{\partial}{\partial \theta}\left(A_\theta \cdot \sin\theta\right) + \frac{1}{r\sin\theta} \frac{\partial A_\phi}{\partial \phi}$$

$$\nabla \times A = \frac{1}{r^2 \sin\theta} \begin{vmatrix} a_r & r \cdot a_\theta & r\sin\theta \cdot a_\phi \\ \partial/\partial r & \partial/\partial\theta & \partial/\partial\phi \\ A_r & r \cdot A_\theta & r\sin\theta \cdot A_\phi \end{vmatrix}$$

$$\nabla^2 \alpha = \frac{1}{r^2} \frac{\partial}{\partial r}\left(r^2 \frac{\partial \alpha}{\partial r}\right) + \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial \theta}\left(\sin\theta \cdot \frac{\partial \alpha}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta} \frac{\partial^2 \alpha}{\partial \phi^2}$$

$$\nabla \times \nabla \times A = \nabla(\nabla \cdot A) - \nabla^2 A$$

$$\frac{d}{dx}(\sin ax) = a \cos x$$

$$\frac{d}{dx}(\cos ax) = -a \sin x$$

$$\int \sin ax \cdot dx = -(\cos ax)/a$$

$$\int \cos ax \cdot dx = \sin ax / a$$

$$\int_V \nabla\cdot\bar{A}\, dv = \oint_S \bar{A}\cdot ds \qquad \int_S (\nabla\times\bar{A})\cdot ds = \oint_C \bar{A}\cdot dl$$

$$\bar{D} = \varepsilon\bar{E},\ \bar{B} = \mu\bar{H},\ \bar{J} = \sigma\bar{E}$$

$$\nabla\times(\nabla V) \equiv 0\ ;\ \nabla\cdot(\nabla\times\bar{A}) \equiv 0\ ;\ \bar{F} = -\nabla V + \nabla\times\bar{A}$$

$$\nabla\times\bar{E} = -\partial\bar{B}/\partial t \qquad \oint_C \bar{E}\cdot dl = -d\phi/dt \qquad E_{1t} = E_{2t}$$

$$\nabla\times\bar{H} = \bar{J} + \partial\bar{D}/\partial t \qquad \oint_C \bar{H}\cdot dl = I + \int_S \partial\bar{D}/\partial t\, ds \qquad B_{1n} = B_{2n}$$

$$\nabla\cdot\bar{D} = \rho \qquad \oint_S \bar{D}\cdot ds = Q \qquad \bar{a}_{n2}\times(\bar{H}_1 - \bar{H}_2) = \bar{J}_s$$

$$\nabla\cdot\bar{B} = 0 \qquad \oint_S \bar{B}\cdot ds = 0 \qquad \bar{a}_{n2}\cdot(\bar{D}_1 - \bar{D}_2) = \rho_s$$

$$\nabla\cdot\bar{J} + \partial\rho/\partial t = 0$$

$$\bar{B} = \nabla\times\bar{A} \qquad \nabla\cdot\bar{A} + \mu\varepsilon\,\partial V/\partial t = 0 \qquad \nabla^2\bar{A} - \mu\varepsilon\frac{\partial^2\bar{A}}{\partial t^2} = -\mu\bar{J}$$

$$\bar{E} = -\nabla V - \frac{\partial\bar{A}}{\partial t} \qquad \nabla^2 V - \mu\varepsilon\,\partial^2 V/\partial t^2 = -\rho/\varepsilon$$

$$V(R,t) = \frac{1}{4\pi\varepsilon}\int_{V'} \frac{\rho(t - R/u)}{R}\, dv' \qquad V(R) = \frac{1}{4\pi\varepsilon}\int_{V'} \frac{\rho e^{-jkR}}{R}\, dv'$$

$$\bar{A}(R,t) = \mu/4\pi \int_{V'} \frac{\bar{J}(t - R/u)}{R}\, dv' \qquad \bar{A}(R) = \mu/4\pi \int_{V'} \frac{\bar{J}\cdot e^{-jkR}}{R}\, dv'$$

$$\nabla\times\bar{E} = -j\omega\mu\bar{H} \qquad \nabla^2 V + k^2 V = -\rho/\varepsilon$$

$$\nabla\times\bar{H} = \bar{J} + j\omega\varepsilon\bar{E} \qquad \nabla^2\bar{A} + k^2\bar{A} = -\mu\bar{J} \qquad \beta = k = \omega\sqrt{\mu\varepsilon} = \frac{\omega}{u}$$

$$\nabla\cdot\bar{E} = \rho/\varepsilon \qquad \nabla\cdot\bar{A} + j\omega\mu\varepsilon V = 0 \qquad c = \frac{1}{\sqrt{\mu_0\varepsilon_0}} \qquad \eta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$\nabla\cdot\bar{H} = 0$$

$$\gamma = \alpha + j\beta\ ;\ \delta = 1/\alpha \qquad \alpha = \sqrt{\pi f\mu\sigma} \qquad k = 2\pi/\lambda$$

$$\frac{\sin\theta_t}{\sin\theta_i} = n_1/n_2\ ;\ n = c/u_p \qquad Z_0 = \left(\frac{R + j\omega L}{G + j\omega C}\right)^{1/2} \qquad v = f\lambda = \omega/\beta$$

$$\bar{P} = \bar{E}\times\bar{H} \qquad \gamma = [(R + j\omega L)(G + j\omega C)]^{1/2} \qquad \Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

$$P_{av.} = \frac{1}{2}\,\mathrm{Re}\,(\bar{E}\times\bar{H}^*) \qquad \tau = 1 + \Gamma$$

$$\text{total radiated power} = \frac{P_{av.}\times 4\pi r^2}{D} \qquad H = \frac{1}{\eta}E \qquad S = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

$$R_r = 80\pi^2\left(\frac{dl}{\lambda}\right)^2 \qquad \frac{\sin\theta_t}{\sin\theta_i} = \sqrt{\frac{\varepsilon_{r1}}{\varepsilon_{r2}}} \qquad \mu_0 = 4\pi\times 10^{-7}\ H/m$$

$$\Gamma_\perp = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}\ ;\ \Gamma_\parallel = \frac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i} \qquad \varepsilon_0 = 8.854\times 10^{-12}\ F/m$$

$$\tau_\perp = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}\ ;\ \tau_\parallel = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}$$

$$\sin\theta_{B\perp} = 1/(1 + \mu_1/\mu_2)^{1/2}\ ;\ \sin\theta_{B\parallel} = 1/(1 + \varepsilon_1/\varepsilon_2)^{1/2}$$