THE OPEN UNIVERSITY OF SRI LANKA
Department of Civil Engineering
Bachelor of Technology - Level 5

052

CEX 5231 - MECHANICS OF FLUIDS
CEU 3204 – FLUID MECHANICS

FINAL EXAMINATION - 2005/2006

Time Allowed : **Three Hours** *Index No.*

Date : 13th March, 2006 Time : 0930 - 1230

ANSWER **ALL THREE** QUESTIONS IN PART A AND **ANY TWO** QUESTIONS IN PART B.
ALL QUESTIONS CARRY EQUAL MARKS.

PART A

Answer **all three** questions in this section.

1) The Navier-Stokes equations can be written in the form

$$\frac{\partial \underline{q}}{\partial t}+(\underline{q}.\nabla)\underline{q}=-\frac{1}{\rho}\underline{\nabla}(p^*)+\frac{\mu}{\rho}\nabla^2\underline{q} \quad (1) \quad \text{and} \quad \nabla.\underline{q}=0 \quad (2)$$

where $\underline{q}=u\underline{i}+v\underline{j}+w\underline{k}$ is the velocity vector and $p^*=p+\rho gz$ is the piezometric pressure with z measured in the $\underline{k}$ direction.

a) List the assumptions that have been made in the derivation of equations (1) and (2).

b) Explain the physical significance of the terms in the left hand side of equation (1).

A fluid (of density ρ and dynamic viscosity μ) flows between two parallel plates as shown in Figure 1. The distance between the plates is h. The upper plate moves with respect to the lower plate at a steady velocity V.

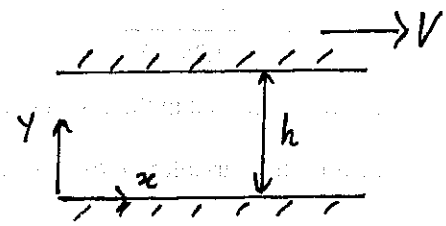

FIGURE 1

The differential equation for the velocity between these two plates can be derived from equations (1) and (2) as $\mu\frac{d^2u}{dy^2}=\frac{dp^*}{dx}$.

c) List the assumptions made in deriving the simplified equation given above.

d) Show that the discharge, **Q**, per unit width between the two plates is given by $Q=\frac{Vh}{2}-\frac{h^3}{12\mu}\frac{dp^*}{dx}$

e) Show that the shear stress, τ_w, *on the moving plate* is given by $\tau_w=-\mu\frac{V}{h}-\frac{h}{2}\frac{dp^*}{dx}$ where x is measured in the direction of motion of the moving plate.

2) A cylindrical piston moves downward with a *steady* velocity, **V** in a cylinder filled with an incompressible fluid (of density ρ and dynamic viscosity μ) under the action of a force **F**, as shown in Figure 2. The diameter of the cylinder is **D**, while the gap between the cylinder and the piston is **d**. The length of the piston is **L** while the cylinder is filled with the fluid to a height **H** as shown in the figure. The piston is made of a material of density ρ_s.

a) Obtain the non-dimensional form of the relationship between the force, **F** and the other relevant variables.

b) Discuss any relationship between the Reynolds number and one or more of the non-dimensional variables in your answer to section a).

c) Indicate all the forces on the piston on a neat diagram.

For flow between parallel plates where one plate is moving relative to the other the discharge, Q, per unit width, and the shear stress, τ_w, on the moving plate are given by the following expressions.

$Q=\frac{Vh}{2}-\frac{h^3}{12\mu}\frac{dp^*}{dx}$, $\tau_w=-\mu\frac{V}{h}-\frac{h}{2}\frac{dp^*}{dx}$ where V is the velocity of the moving plate and $\frac{dp^*}{dx}$ is the gradient of the piezometric pressure and x is measured in the direction of motion of the plate.

d) Explain how you would use these two expressions to obtain a relationship between the force on the piston and the steady velocity of the piston. List all the assumptions that have to be made.

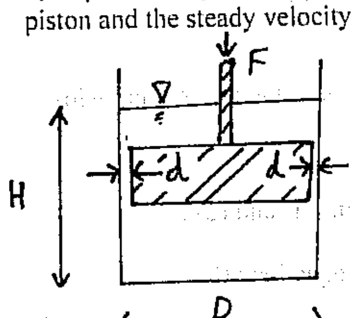

FIGURE 2

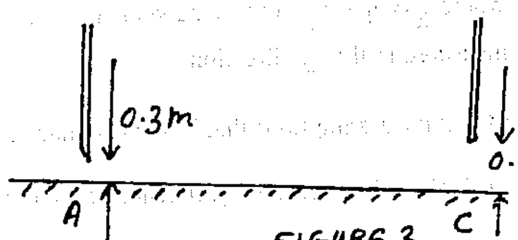

FIGURE 3

3) An equation used in the computation of flow profiles in open channel flows is $\frac{dy}{dx}=\frac{S_0-S_f}{1-F_r^2}$.

a) Identify the variables **dy/dx**, S_0, S_f and F_r in this equation using a neat diagram.

b) List the important assumptions that have been made to derive this equation.

A long, uniform open channel of rectangular cross-section has a width of 3 m, a slope of 0.001 and a Manning's coefficient of 0.015. The channel carries a discharge of 6 m^3/s. Two sluice gates are located on the channel at A and C as shown in Figure 3. The opening of the sluice gate at A is 0.3 m while the opening of the gate at C is 0.6 m. A hydraulic jump is observed at B, between A and C, and the depth just upstream of the jump is found to be 0.52 m.

c) Estimate the distance AB. Explain your answer.

d) Classify the surface profiles between A and C (from M1, M2, M3, S1, S2, S3, C1, C2, C3, etc.). Explain your answer.

Note : The equation $\frac{y_2}{y_1}=\frac{1}{2}\left(\sqrt{1+8F_{r1}^2}-1\right)$ may be used.

PART B

Answer any two of the five questions in this section.

4) Figure 4 shows a pipe system AB and CD that moves water from a large reservoir, X, to a tank Y, by means of a pump, P. The lengths of AB and CD are 5 m and 10 m, respectively. The bottom of the tank Y, is 10 m above the level of the reservoir, X. The cross-sectional area of the tank Y is 10 m^2. The tank Y discharges water to the atmosphere through the pipe EF, which has a length of 10 m. The point F is 5 m below the bottom of the tank Y. All pipes are of diameter 2.5 cm and have a friction factor of 0.01. At time t = 0, the tank Y is empty. The pump is started at time t = 0. The pump is tested at its operating speed and the results are given in Table 4.

Discharge (l/s)	0	15	25	30	35
Head (m)	20	15	10	5	0.5

Table 4

a) Sketch the change in the water level of tank Y with time.

b) Calculate the discharge in the pipeline when the tank Y is empty. Explain your answer.

c) Calculate the final water level in the tank.

d) Use your answers to sections b) and c) above to estimate the time at which the final water level in the tank is reached.

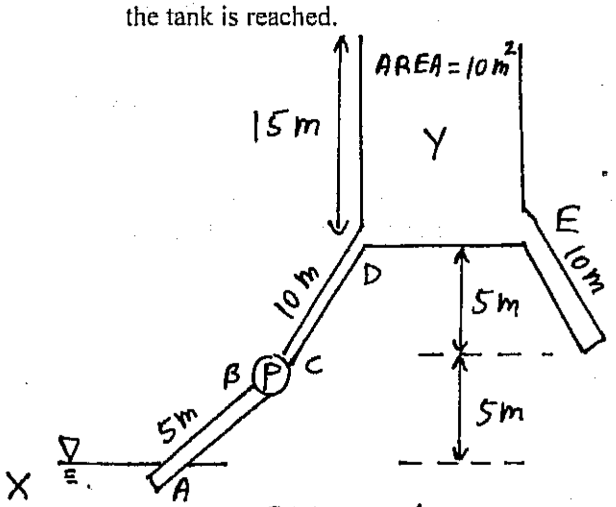

FIGURE 4

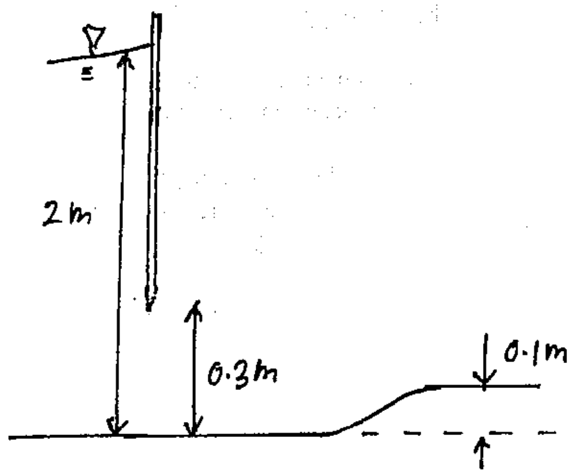

FIGURE 5

5) A horizontal open channel has a rectangular cross-section and a width of 2 m. A sluice gate is located on the channel as shown in Figure 5. There is a smooth step – of height 0.1 m - in the channel just downstream of the sluice gate. The sluice gate has an opening of 0.3 m while the depth of water upstream of the gate is 2 m as shown in the figure. The density of water is 1000 kg/m^3.

a) Calculate the discharge in the channel. State all your assumptions.

b) Calculate the force on the sluice gate. State all your assumptions.

c) Calculate the depth of flow over the step in the channel.

d) What would happen if the height of the step was increased to 0.8 m? Explain your answer.

6) a) Can the Bernoulli's equation be used in an ideal fluid flow? Explain your answer.

An ideal fluid flow is created by placing a source of strength **m** in a uniform flow field of velocity **U**. The source is placed a distance **m/2U** from a long, straight wall, as shown in Figure 6. The wall is parallel to the direction of the uniform flow.

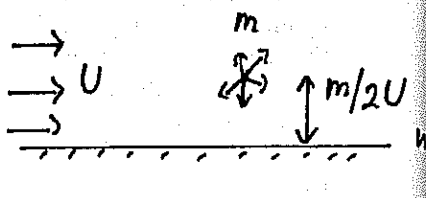

FIGURE 6

b) Obtain the complex potential of this flow.

c) Sketch the streamlines of this flow.

d) Determine any stagnation points.

e) Obtain an expression for the velocity distribution along the wall.

f) By considering the velocity and pressure far away from the source derive an expression for the pressure distribution along the wall.

7) a) Explain – using neat figures of velocity profiles - what you understand by the term "fully-developed boundary layer"?

b) Explain - using neat figures of velocity profiles – the difference between laminar and turbulent boundary layers on a flat plate. What are the differences in shear stress and boundary layer thickness for these two types of boundary layer for the same external flow?

c) Explain - using neat figures of velocity profiles – what is meant by "boundary layer separation" and how it may occur on a flat plate.

The velocity distribution **u(y)/U** in a laminar boundary layer flow over a flat plate is assumed to be given by a quadratic expression in η , where $\eta = y/\delta$ and **y** is the vertical distance from the plate. Here δ is the boundary layer thickness and **U** is the external velocity.

d) Using appropriate boundary conditions determine the coefficients of the quadratic expression.

The displacement thickness, δ^* and the momentum thickness θ^* are given by

$$\delta^* = \int_0^\delta \left(1 - \frac{u(y)}{U}\right) dy \quad \text{and} \quad \theta^* = \int_0^\delta \left(1 - \frac{u(y)}{U}\right)\frac{u(y)}{U} dy$$

e) Explain the physical significance of the displacement thickness and the momentum thickness.

f) Calculate the displacement thickness, the momentum thickness and the shear stress on the plate using the velocity distribution derived in section d).

8) The Meyer-Peter-Muller formula for bed load transport is given by

$$\frac{q_s \rho^{1/2}}{\left(\rho(s-1)gd\right)^{3/2}} = 8\left(\frac{\rho u_*^2}{\rho(s-1)gd} - 0.047\right)$$

where q_s is the rate of sediment transport, ρ the density of water, s the specific gravity of the sediment, d the grain diameter and u_* the shear velocity at the bed ($u_* = \sqrt{\frac{\tau_0}{\rho}}$).

a) Explain the difference between "bed load" and "suspended load".

b) What are the units of the rate of sediment transport q_s ?

c) Discuss the variables in this equation and make conclusions regarding the type of sediment transport and the type of flow conditions that this equation is meant for.

A long straight river is 20 m wide and has a slope of 0.0001. The river bottom consists of fine sand of diameter 0.1 mm and specific gravity 2.65 . The flow depth in the river is 3 m .

b) Calculate the discharge in the river if the flow is uniform. Assume that Manning's n for a flat sand bed is given by $n = 0.039d^{1/6}$ where d is the grain diameter in metres.

d) Calculate the rate of sediment transport in the river as the volume of sediment per unit time.

A short weir is constructed on the river.

e) Identify the flow profile upstream of the weir and predict the change in the sediment transport capacity of the river upstream of the weir. Recall how the friction slope is calculated in gradually varied flow to estimate the change in the bottom shear stress.