THE OPEN UNIVERSITY OF SRI LANKA
BACHELOR OF TECHNOLOGY – LEVEL 04
FINAL EXAMINATION – 2006
MPZ 4230 – ENGINEERING MATHEMATICS
DURATION : THREE (03) HOURS

Date : 24th March 2006 **Time: 14.00 - 17.00 hours**

Answer only Six (06) questions. State any assumptions, which you use.

01. Find curl $\underline{F}$ if

i. $\underline{F} = 3\underline{r}$

ii. $\underline{F} = f(r)\underline{n}$

iii. $\underline{F} = grad\ \phi$, where ϕ is a scalar field.

iv. $\underline{F} = (3y^2 + 2z)\underline{i} + (2xy + 8z)\underline{j} + (2z^2 + 5xz + y)\underline{k}$

here $\underline{r} = x\underline{i} + y\underline{j} + z\underline{k}$

$|\underline{r}|$ = length of $\underline{r}$ and

$\underline{n}$ = unit vector of $\underline{r}$.

02. i. Show that both functions $e^x \sin y$ and $\log\sqrt{x^2+y^2}$ satisfy Laplace's equation, $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

ii. Find the stationary points of the function $f(x,y) = x^3+y^3-3(x+y)$

iii. The heat generated in a resistance weld is given by $H = Ki^2Rt$, where K is a constant, $\boldsymbol{i}$ is the current between the electrodes and **t** is the time for which current flows. $\boldsymbol{H}$ must not vary by more than 5% if the weld is to remain good. It is possible to control **t** to within 0.5% and $\boldsymbol{R}$ to within 2.5%. Estimate the maximum possible variation in current if the weld is to retain its quality.

03. i. State Cauchy-Reimann equations for complex functions.
Find the value of k in order that (x^3-kxy-2y) may form the real part of a analytic function f(z). Show that (z^2-2iz+i) is one such function.

ii. Find the 6^{th} root of unity. Plot these points on the complex plane.

04. i. Find the Fourier series expansion of the following odd and even functions
$f(x) = (x-\pi)$ is $0<x<\pi$, period $=2\pi$
$= (n-x)$ is $n<x<2\pi$

ii. Obtain the general solution of the following differential equation using the power series method $y'' + 3y' + 4xy = 0$

05. i. Find a vector orthogonal to (1,-2,2) and (2,-1,2)

ii. Determine the dimension and a basis for the solution space of the following system of equations.

$$x_1 + x_2 - x_3 = 0$$
$$-2x_1 - x_2 + 2x_3 = 0$$
$$-x_1 + x_3 = 0$$

iii. Given the basis (1,0,1),(1,-2,1),(0,1,1) use the gram-schmidt process to obtain an orthonormal basis.

06 i. Find the eigen values of the matrix A given by,

$$A = \begin{pmatrix} 2 & -2 & 2 \\ -2 & -1 & 4 \\ 2 & 4 & -1 \end{pmatrix}$$

Hence obtain a matrix P such that $P^{-1}AP$ is a diagonal matrix.

ii. Using the result above, reduce the quadratic form
$Q(x) = 2x_1^2 - x_2^2 - x_3^2 - 4x_1x_2 + 8x_2x_3 + 4x_3x_1$
to a diagonal form $(\lambda_1 y_1^2 + \lambda_2 y_2^2 + \lambda_3 y_3^2)$ and state the equation of transformation.

07. The temperature θ (x,t) satisfies the equation, $\frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial x^2}, \theta < x < 1, t > 0$ with boundary and initial conditions,

$u(0,t) = u(1,t) = 0, \quad t > 0$

$u(x,0) = 2x, \quad 0 \le x \le \frac{1}{2}$

$\quad = 2(x-\frac{1}{2}), \quad \frac{1}{2} \le x \le 1$

Find numerical solution for θ (x,t) where,

$x = \frac{1}{4}, \frac{1}{2}, \frac{3}{4}$ and

$t = \frac{1}{64}, \frac{2}{64}$

{The explicit solution is given by, $u_{i,j+1} = u_{i,j} + r(u_{i-1,j} - 2u_{i,j} + u_{i+1,j})$ }

08. i. Use the Taylor series method to find y at, x = 0.1 for the differential equation,

$y'' + (x+1)y^2 = 0$ where

y(0)=1, y'(0)=1. use 3 decimal places.

ii. Use a predictor-corrector method to find y at x = 0.2 for the differential equation, $y' = (x - y^2)$. It is given that y (0)=0. Use 3 decimal places for your workings.

The 4th order Runge-Kutta algorithm in usual notation is,

$k_1 = hf(x_n, y_n)$

$k_2 = hf(x_n + \frac{h}{2}, y_n + \frac{k_1}{2})$

$k_3 = hf(x_n + \frac{h}{2}, y_n + \frac{k_2}{2})$

$k_4 = hf(x_n + h, y_n + k_3)$

$y_{n+1} = y_n + (k_1 + 2k_2 + 2k_3 + k_4)/6$

Where h = step size

For the Adam's – Moulton formula

Predictor : $y^p_{n+1} = y_n + \frac{h}{12}(23f_n - 16f_{n-1} + 5f_{n-2})$

Corrector : $y^c_{n+1} = y_n + \frac{h}{12}(5f_{n+1} + 8f_n - f_{n-1})$

09. i. State conditions under which the Poisson distribution is a good approximation to the Binomial distribution and conditions for the Normal distribution to be good approximation to the Binomial distribution.

ii. A machine produces components of which 1% are defective. A random sample of 80 components is taken. Find the probabilities of finding
(i) no defectives (ii) less than 3 defective components in the sample, using (a) the **Binomial distribution** and (b) the **Poisson approximation**.

During a week 20 random samples of 80 components are taken. Using a Normal approximation, calculate the probability that more than half of the 20- random samples will contain no defectives.

10. i. A supplier guarantees that each box of small electronic components will contain 250 components on the average. From a large consignment of such boxes a sample of 36 are checked and the average contents were found to be 248 with a standard deviation of 3 components. At the 0.1% significance level is there any reason to reject the supplier's guarantee?

Also compute the 99% confidence interval of the population mean of the electronic components.

ii. A manufacturer claims that the average tensile strength of thread A exceeds the average tensile strength of thread B by at least 12 Kilograms. To test his claim, 50 pieces of each type of thread are tested under similar conditions. Type A thread had an average tensile strength of 86.7 Kilograms with a standard deviation of 6.28 Kilograms, while type B thread had an average tensile strength of 77.8 Kilograms with a standard deviation of 5.61 Kilograms. Test the manufacturer's claim using a 0.05 level of significance.

11. The effect of the temperature of the deodorizing process on the colour of the finished product was determined experimentally. The data collected were as follows:

Temperature X	Colour Y
460	0.3
450	0.3
440	0.4
430	0.4
420	0.6
410	0.5
450	0.5
440	0.6
430	0.6
420	0.6
410	0.7
400	0.6
420	0.6
410	0.6
400	0.6

i. Plot these data on a graph paper, if it is required to predict colour from temperature.

ii. Calculate the linear regression equation of y on x and include it on your plot.

iii. Find an estimate of the error variance.

iv. Test whether the slope parameter is zero.

v. Find 99% confidence interval for the slope parameter.

vi. From your linear regression equation, find the colour for temperature at 425.

vii. Find the correlation coefficient between x and y. Comment your answer.