MPZ 4230 - Assignment No. 03 – Academic Year 2007

1. A function f(x) is defined on the interval $[-\pi, \pi]$ by

$$f(x) = \begin{cases} -\pi & : x \in [-\pi, 0] \\ x & : x \in [0, \pi] \end{cases}$$

Show that the Fourier series expansion for f is given by

$$f(x) = -\frac{\pi}{4} - \frac{2}{\pi}\sum_{n\ odd} \frac{\cos nx}{n^2} + 3\sum_{n\ odd} \frac{\sin nx}{n} - \sum_{n\ even} \frac{\sin nx}{n}$$

Hence deduce each of the following

i). $1 + \frac{1}{9} + \frac{1}{25} + \frac{1}{49} + \frac{1}{81} + \ldots\ldots = \frac{\pi^2}{8}$

ii). $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \ldots\ldots\ldots\ldots = \frac{\pi}{4}$

iii). $1 - \frac{1}{9} - \frac{1}{25} + \frac{1}{49} + \frac{1}{81} - \frac{1}{211} - \frac{1}{169} + - - - - = \frac{\pi^2\sqrt{2}}{16}$

What is the sum of the series

$$1 + \frac{1}{3} - \frac{1}{5} - \frac{1}{7} + \frac{1}{9} + \frac{1}{11} - \frac{1}{13} - \frac{1}{15} + - - - -$$

2. Find the Fourier series of the periodic function f(t) of period T.

$f(t) = -1$; $(-1 < t < 0)$

$2t$; $(0 < t < 1)$

Assume that

$$\int x \sin ax\, dx = \frac{1}{a^2}\sin ax - \frac{x}{a}\cos ax + c$$

& $T = 2$

3. Find the Fourier series for following

a). A string of 2L plucked at the right end and fixed at the left. The functional form of this configuration is $f(x) = \frac{x}{2L}$

b). (i). A symmetric triangle is pinned an x distance, which is $\left(\frac{1}{m}\right)^{th}$ of the distance L. The displacement a function x is, then

$$f_m(x) = \begin{cases} \frac{mx}{L} & \text{for } 0 \le x \le \frac{L}{m} \\ 1 - \frac{m}{(m-1)L}\left[x - \frac{L}{m}\right] & \text{for } \frac{L}{m} < x < 2L - \frac{L}{m} \\ \frac{m}{L}(x - 2L) & \text{for } 2L - \frac{L}{m} < x < 2L \end{cases}$$

ii). Then taking m = 2 gives the Fourier series as below $f(x) = \frac{8}{\pi^2} \sum_{n=1,3,5}^{\infty} \frac{(-1)^{(n-1)/2}}{n^2} \sin\left(\frac{n\pi x}{L}\right)$

4. Suppose a large lake form by damming a river hold initially 100 millions galloons of water, because a near by agricultural field was spread with a pesticide the water has been contaminated. The concentration of the pesticide has been measured and is equal to 35 parts per millions gallons (35×10^{-6} ppm). The river continuously flows in to the lake at a rate of 300 galloons per minute. The river only slide contaminated with pesticide an has a concentration of 5×10^{-6} ppm. The flow of water over the dam can be control and is set at 400 gallons per minutes.

   a). How long it will be before the water reaches and acceptable level of concentration is 5×10^{-6} ppm.

   b). The flowing rate of water is 400 galloons per minutes and the out following rate is 450 galloons per minute and all the other given data remains same. What will be the concentration of the tank after 450,000 minutes?

5). An R-L circle t has an emf is 3 sin 2t. A resistance of 10 Ω, Induction of 0.5 H and an initial current is 6A, find the current in the circuit at any time t.

6. a). Find the radius of convergence of the series

   i). $\sum \frac{nx^n}{2^{n+1}}$

   ii). $\sum \frac{(-1)^n n!\, x^n}{n^n}$

   b). Evaluate by means of Taylor series expansion the following problem of x = 0. 1, 0.2 to four significant figures

   $y'' - x(y')^2 + y^2 = 0$; $y(0) = 1$, $y'(0) = 0$

Model Answer 03 – MPZ 4230
Academic Year 2007

(01). $f(x) = \begin{cases} -\pi & : x \in [-\pi, 0] \\ x & : x \in [0, \pi] \end{cases}$

Let $f(x) = \frac{a_0}{2} + a_1 \cos x + a_2 \cos 2x + \ldots\ldots\ldots\ldots\ldots + a_n \cos nx + \ldots\ldots\ldots +$
$b_1 \sin x + b_2 \sin 2x + \ldots\ldots\ldots + b_n \sin nx + \ldots\ldots\ldots\ldots\ldots$ ------------ (1)

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x).dx$$

$$a_0 = \frac{1}{\pi}\left[\int_{-\pi}^{0}(-\pi)dx + \int_{0}^{\pi} x.dx\right]$$

$$= \frac{1}{\pi}\left[-\pi(x)_{-\pi}^{0} + \left(\frac{x^2}{2}\right)_0^{\pi}\right]$$

$$= \frac{1}{\pi}\left[-\pi^2 + \frac{\pi^2}{2}\right]$$

$$a_0 = \frac{-\pi}{2}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x).\cos\ nx.dx$$

$$a_n = \frac{1}{\pi}\left[\int_{-\pi}^{0}(-\pi)\cos nx.dx + \int_{0}^{\pi} x.\cos nx.dx\right]$$

$$a_n = \frac{1}{\pi}\left[-\pi\left(\frac{\cos}{n}nx\right)_{-\pi}^{0} + \left(\frac{x\ \sin nx}{n}\right)_0^{\pi} + \left(\frac{\cos nx}{n^2}\right)_0^{\pi}\right]$$

$$a_n = \frac{1}{\pi}\left[0 + \frac{\cos nx}{n^2} - \frac{1}{n^2}\right]$$

$$a_n = \frac{1}{n^2\pi}(\cos n\pi - 1)$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\ dx$$

$$b_n = \frac{1}{\pi}\left[\int_{-\pi}^{0}(-\pi)\sin\ nx\ .\ dx + \int_{0}^{\pi} x \sin nx\ .\ dx\right]$$

$$= \frac{1}{\pi}\left[\pi\left(\frac{\cos nx}{n}\right)_{-\pi}^{0} + \left(-x\frac{\cos nx}{n} + \frac{\sin nx}{n^2}\right)_0^{\pi}\right]$$

$$= \frac{1}{\pi}\left[\frac{\pi}{n}(1-\cos nx) - \pi\frac{\cos nx}{n}\right]$$

$$b_n = \frac{1}{n}\ (1 - 2\cos nx)$$

From (1);

$$f(x) = \frac{-\pi}{4}\ \frac{-2}{\pi}\left[\cos x + \frac{\cos 3x}{3^2} + \frac{\cos 5x}{5^2} + ...\right] + 3\sin x - \frac{\sin 2x}{2} + \frac{3\sin 3x}{3} - \frac{\sin 4x}{4} + ...$$

$$\therefore\ f(x) = -\frac{\pi}{4} - \frac{2}{\pi}\sum_{n\ odd}\frac{\cos nx}{n^2} + 3\sum_{n\ odd}\frac{\sin nx}{n} - \sum_{n\ even}\frac{\sin nx}{n}$$

$$f(x) = \frac{-\pi}{4}\ \frac{-2}{x}\left[\cos x + \frac{\cos 3x}{3^2} + \frac{\cos 5x}{5^2} + \frac{\sin 3x}{3}.....\right] + 3[\sin\ x + ...] - \left[\frac{\sin 2x}{2} + \frac{\sin 4x}{4} + ...\right]$$

--- (2)

When $x = 0$;

$$f(0) = -\frac{\pi}{4} - \frac{2}{\pi}\left[1 + \frac{1}{3^2} + \frac{1}{5^2} - ....\right] \quad \text{---------------- (3)}$$

But f(x) is discontinuous at x = 0

But $f(0-0) = -\pi$ & $f(0+0) = 0$

$$\therefore\ f(0) = \frac{1}{2}\ [f(0-0) + f(0+0)] = -\frac{\pi}{2}$$

From (3); $-\frac{\pi}{2} = -\frac{\pi}{4} - \frac{2}{\pi}\left[\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + ...\right]$

$$\therefore\ \frac{\pi^2}{8} = 1 + \frac{1}{9} + \frac{1}{25} + ...$$

(ii). $f(x) = -\frac{\pi}{4} - \frac{2}{\pi}\left[\cos x + \frac{\cos 3x}{3^2} + \frac{\cos 5x}{5^2} + ...\right] + 3\sin x - \frac{\sin 2x}{2} + \frac{3\sin 3x}{3}$

When $x = \frac{\pi}{2}$;

$$f\left(\frac{\pi}{2}\right) = \frac{1}{2}\left[f\left(\frac{\pi}{2}-0\right) + f\left(\frac{\pi}{2}-0\right)\right] = \frac{\pi}{2}$$

$$\frac{\pi}{2} = -\frac{\pi}{4} + 3\left[1 - \frac{1}{3} + \frac{1}{5} - \text{-----}\right]$$

$$\frac{3\pi}{4} = 3\left[1 - \frac{1}{3} + \frac{1}{5} - \text{------}\right]$$

$$\frac{\pi}{4} = \left[1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + ...\right]$$

(iii). $f(x) = -\frac{\pi}{4} - \frac{2}{\pi}\left[\cos x + \frac{\cos 3x}{3^2} + \frac{\cos 5x}{5^2} + ---\right] + 3\sin x - \frac{\sin 2x}{2} + \frac{3\sin 3x}{3} + ------$

When $x = \frac{\pi}{4}$

$$f\left(\frac{\pi}{4}\right) = \frac{\pi}{4} = -\frac{\pi}{4} - \frac{2}{\pi}\left[\frac{1}{\sqrt{2}} - \frac{1}{9\sqrt{2}} - \frac{1}{25\sqrt{2}} + \frac{1}{49\sqrt{2}} + ---\right] + 3\left[\frac{1}{\sqrt{2}} + \frac{1}{3\sqrt{2}} - \frac{1}{5\sqrt{2}} - \frac{1}{7\sqrt{2}} + ...\right]$$

$$-\left[\frac{1}{2} - \frac{1}{6} + \frac{1}{10} + ...\right]$$

$$\frac{\pi}{2} = -\frac{\sqrt{2}}{\pi}\left[1 - \frac{1}{9} - \frac{1}{25} + \frac{1}{49} + \frac{1}{81} - \frac{1}{121} + ...\right] + 3\left[\frac{1}{\sqrt{2}} + \frac{1}{3\sqrt{2}} - \frac{1}{5\sqrt{2}} - \frac{1}{7\sqrt{2}} + ---\right] \quad \text{-----(1)}$$

$$-\left[\frac{1}{2} - \frac{1}{6} + \frac{1}{10} + -----\right]$$

When $x = -\frac{\pi}{4}$

$$f\left(-\frac{\pi}{4}\right) = -\pi = -\frac{\pi}{4} - \frac{2}{\pi}\left[\frac{1}{\sqrt{2}} - \frac{1}{9\sqrt{2}} - \frac{1}{25\sqrt{2}} + \frac{1}{49\sqrt{2}} + ---\right] - 3\left[\frac{1}{\sqrt{2}} + \frac{1}{3\sqrt{2}} - \frac{1}{5\sqrt{2}} + \frac{1}{7\sqrt{2}} + ---\right]$$

$$+\frac{1}{2}\left[\frac{1}{2} - \frac{1}{6} + \frac{1}{10} + ...\right]$$

$$-\frac{3\pi}{4} = -\frac{\sqrt{2}}{\pi}\left[1 - \frac{1}{9} - \frac{1}{25} + \frac{1}{49} + ----\right] - 3\left[\frac{1}{\sqrt{2}} + \frac{1}{3\sqrt{2}} - \frac{1}{5\sqrt{2}} - \frac{1}{7\sqrt{2}} + ---\right] \quad \text{----- (2)}$$

$$+\frac{1}{2}\left[\frac{1}{2} - \frac{1}{6} + \frac{1}{10} + ...\right]$$

(1) + (2)

$$-\frac{\pi}{4} = -\frac{2\sqrt{2}}{\pi}\left[1 - \frac{1}{9} - \frac{1}{25} + \frac{1}{49} + ----\right]$$

$$\frac{\pi^2}{8\sqrt{2}} = 1 - \frac{1}{9} - \frac{1}{25} + \frac{1}{49} + ----$$

$$1 - \frac{1}{9} - \frac{1}{25} + \frac{1}{49} + \frac{1}{81} - \frac{1}{121} - \frac{1}{169} + ---- = \frac{\pi^2\sqrt{2}}{16}$$

(1) – (2) $\frac{\pi}{2} + \frac{3\pi}{4} = 6\left[\frac{1}{\sqrt{2}} + \frac{1}{3\sqrt{2}} - \frac{1}{5\sqrt{2}} - \frac{1}{7\sqrt{2}} + ---\right] - 2\left[\frac{1}{2} - \frac{1}{6} + \frac{1}{10} - \frac{1}{14} + ---\right]$

$$\frac{5\pi}{4} = \frac{2.3}{\sqrt{2}}\left(1 + \frac{1}{3} - \frac{1}{5} - \frac{1}{7} + \frac{1}{9} + \frac{1}{11} - \frac{1}{13} - \frac{1}{15} + ----\right) - \frac{2}{2}\left[1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + --\right]$$

by part (ii) $\frac{5\pi}{4} = \frac{6}{\sqrt{2}}\left[1 + \frac{1}{3} - \frac{1}{5} - \frac{1}{7} + ...\right] - \frac{\pi}{4}$

$$\frac{6\pi}{4} = \frac{6}{\sqrt{2}}\left[1+\frac{1}{3}-\frac{1}{5}+----\right]$$

$$\frac{\pi}{2\sqrt{2}} = 1+\frac{1}{3}-\frac{1}{5}-\frac{1}{7}$$

(2). $f(t) = \frac{a_0}{2} + \sum_{n=1}^{\alpha} \left(a_n \cos\left(\frac{2n\pi t}{T}\right) + b_n \sin\left(\frac{2n\pi t}{T}\right)\right)$

$$a_0 = \frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(t)dt$$

$$= \frac{2}{2}\int_{-1}^{0} -1.dt + \int_{0}^{1} 2t\, dt$$

$=-[t]_{-1}^{0} + [t^2]_0^1$

$a_0 = -1 + 1 = 0$

$$a_n = \frac{2}{T}\int_{-1}^{0} -1\cos\left(\frac{2n\pi t}{T}\right)dt + \frac{2}{T}\int_{0}^{1} 2t.\cos\left(\frac{2n\pi t}{T}\right)dt$$

$$a_n = -\frac{2}{2}\left[\sin\frac{2n\pi t}{T}\right]_{-1}^{0} \times \frac{T}{2n\pi} + \frac{4}{T}\left[\frac{2}{2n\pi}\left[t \sin\frac{2n\pi t}{2}\right]_0^1 - \frac{2}{2n\pi}\int_0^1 \sin\left(\frac{2n\pi t}{2}\right)dt\right]$$

$$a_n = \left(\frac{2}{2n\pi}\right)^2 \left[\cos\frac{2n\pi t}{2}\right]_0^1 \times \frac{4}{T}$$

$$= \frac{2}{\pi^2 n^2}\left\{(-1)^n - 1\right\}$$

$a_n = 0$ for even n

$a_n = \frac{-4}{\pi^2 n^2}$ for odd n

$$b_n = \frac{2}{T}\int_{-1}^{0} -1\sin\left(\frac{2n\pi t}{T}\right)dt + \frac{4}{T}\int_{0}^{1} t \sin\frac{2n\pi t}{T}.dt$$

$$= \left[\cos\frac{2n\pi t}{T}\right]_{-1}^{0} \times \frac{T}{2n\pi} + \frac{4}{T}\left[\frac{1}{n^2\pi^2}\sin n\pi t\right]_0^1 - \frac{1}{n\pi}[t \cos n\pi t]_0^1$$

$$= \frac{1}{n\pi} \frac{-(-1)^n 3}{n\pi} = \frac{1-(-1)^n.3}{n\pi}$$

$b_n = \frac{-2}{n\pi}$ for even n

$b_n = \frac{4}{n\pi}$ for odd n

$$f(x) = \sum_{\substack{1 \\ \text{odd } n.}}^{\alpha} \left\{ \frac{-4}{\pi^2 n^2} \cos\ n\pi t + \frac{4}{n\pi} \sin\ n\pi t \right\} + \sum_{\text{even } n}^{\alpha} \left(\frac{-2}{n\pi} \sin\ n\pi t \right)$$

$$= \frac{-4}{\pi^2} \left(\cos\ \pi t + \frac{1}{9} \cos\ 3\pi t + \frac{1}{5^2} \cos\ 5\pi t + \frac{1}{7^2} \cos 7\pi t + \ldots\ldots \right)$$

$$+ \frac{2}{\pi} \left(2 \sin \pi t - \frac{1}{2} \sin 2\pi t + \frac{2}{3} \sin 3\pi t - \frac{1}{4} \sin 4\pi t + \ldots\ldots \right)$$

(03). $f(x) = \dfrac{x}{2L}$

$$a_0 = \frac{2}{T} \int_0^{2L} f(x).dx = \frac{2}{2L} \int_0^{2L} \frac{x}{2L}.dx = \frac{1}{2L^2} \int_0^{2L} xdx = \frac{1}{2L^2} \left[\frac{x^2}{2} \right]_0^{2L}$$

$$= \frac{1}{2L^2} \left[\frac{4^2 L^2}{2} \right] = \underline{\underline{1}}$$

$$a_n = \frac{2}{T} \int_0^{2L} f(x).\cos\left(\frac{2\pi nx}{T} \right) dx = \frac{2}{2L} \int_0^{2L} \frac{x}{2L}.\cos\left(\frac{\pi nx}{L} \right) dx$$

$$= \frac{1}{2L^2} \left[\frac{xL}{\pi n} \left[\sin\left(\frac{xnx}{L} \right) \right]_0^{2L} - \frac{L}{\pi n} \int_0^{2L} \sin\left(\frac{\pi nx}{L} \right) dx \right]$$

$$= \frac{1}{2L^2} \left[\frac{xL}{\pi n} \sin\left(\frac{\pi nx}{L} \right) + \frac{L^2}{\pi^2 n^2} \cos\left(\frac{\pi nx}{L} \right) \right]_0^{2L}$$

$$= \frac{L}{2L^2 \pi n} \left[x \sin\left(\frac{\pi nx}{L} \right) + \frac{L}{\pi n} \cos\left(\frac{\pi nx}{L} \right) \right]_0^{2L}$$

$$= \frac{1}{2L\pi n} \left[2L \underbrace{\sin\left(\frac{2\pi n \not{L}}{\not{L}} \right)}_{=0} + \frac{L}{\pi n} \cos\left(\frac{2\pi n \not{L}}{\not{L}} \right) - \frac{L}{\pi n} \right]$$

$$= \frac{1}{2L\pi n} \left[\frac{L}{\pi n} \left(\cos(2\pi n) - 1 \right) \right]$$

$$a_n = \frac{1}{2\pi^2 n^2} \left[\underbrace{\cos(2\pi n)}_{=1} - 1 \right] = 0$$

$$b_n = \frac{2}{T} \int_0^{2L} f(x).\sin\left(\frac{2\pi nx}{T} \right).dx = \frac{\not{2}}{\not{2}L} \int_0^{2L} \frac{x}{2l}.\sin\left(\frac{\not{2}\pi nx}{\not{2}L} \right) dx$$

$$= \frac{1}{2L^2} \left[-\frac{xL}{\pi n} \cos\left(\frac{\pi nx}{L} \right) + \frac{L^2}{\pi^2 n^2} \sin\left(\frac{\pi nx}{L} \right) \right]_0^{2L}$$

$$= \frac{1}{2L\pi n}\left[-2L\cos\left(\frac{2\pi n L}{L}\right)+\frac{L}{\pi n}\underbrace{\sin\left(\frac{2\pi n L}{L}\right)}_{=0}\right]^{0}$$

$$b_n = -\frac{1}{\pi n}\cos(2\pi n)$$

$$n = 1 \quad b_n = -\frac{1}{\pi n}\cos(2\pi n)$$

$$b_1 = -\frac{1}{\pi}\cos 2\pi = -\frac{1}{\pi}$$

$$n = 2 \quad b_2 = -\frac{1}{2\pi}\cos 4x = -\frac{1}{2\pi}$$

$$n = 3 \quad b_3 = -\frac{1}{3\pi}\cos 6x = -\frac{1}{3x}$$

$$f(x) = \frac{a_0}{2} + a_1\cos x + a_2\cos 2x + a_3\cos 3x + \ldots + b_1\sin x + b_2\sin 2x + b_3\sin 3x + \ldots$$

$$= \frac{1}{2}+0-\frac{1}{\pi}\sin x-\frac{1}{2\pi}\sin 2x-\frac{1}{3\pi}\sin 3x+\ldots$$

$$f(x) = \frac{1}{2}-\sum_{n=1}^{\alpha}\frac{1}{\pi n}\sin\left(\frac{nx\pi}{L}\right)$$

(b). (i). $$f_m(x)=\begin{cases}\dfrac{mx}{L} & ;0\le x\le \dfrac{L}{m}\\ 1-\dfrac{m}{(m-1)}\left[x-\dfrac{L}{m}\right] & ;\dfrac{L}{m}<x<2L-\dfrac{L}{m}\\ \dfrac{m}{L}(x-2L) & ;2L-\dfrac{L}{m}\le x\le 2L\end{cases}$$

$$a_0 = \frac{2}{T}\int_0^{2L} f(t).dt$$

$$= \frac{2}{2L}\left[\int_0^{L/m}\frac{mx}{L}.dx+\int_{L/m}^{(2L-L/m)}\left[1-\frac{m}{(m-1)L}\left(x-\frac{L}{m}\right)\right]dx+\int_{\left(2L-\frac{L}{m}\right)}^{2L}-\frac{m}{L}(x-2L)dx\right]$$

$$=\frac{2}{L}\left[\frac{m}{L}\left[\frac{x^2}{2}\right]_0^{L/m}+\left[x-\frac{m}{(m-1)L}.\frac{x^2}{2}+\frac{mL}{(m-1)Lm}x\right]_{L/m}^{(2L-L/m)}+\frac{m}{L}\left[\frac{x^2}{2}-2Lx\right]_{\left(2L-\frac{L}{m}\right)}^{2L}\right]$$

$$a_0=\frac{1}{L}\left[\begin{array}{l}\frac{m}{L}\cdot\frac{L^2}{2m^2}+2L-\frac{L}{m}-\frac{L}{m}-\frac{m}{2(m-1)L}\left(4L^2-4\frac{L^2}{m}+\frac{L^2}{m^2}-\frac{L^2}{m^2}\right)+\frac{1}{(m-1))}\left(2L-\frac{L}{m}-\frac{L}{m}\right)\\+\frac{m}{L}\left\{\frac{4L^2}{2}-\frac{1}{2}\left(4L^2-\frac{4L^2}{m}+\frac{L^2}{m^2}\right)-2m\left(2L-2L+\frac{L}{m}\right)\right\}\end{array}\right]$$

$$=\frac{1}{L}\left[\frac{L}{2m}+2L-\frac{2L}{m}-\frac{2mL}{(m-1)}+\frac{2L}{(m-1)}+\frac{2L}{(m-1)}-\frac{2L}{(m-1)m}+2Lm-2mL+2L-\frac{L}{2m}-2L\right]$$

$$=\frac{1}{L}\left[2L-\frac{2L}{m}-\frac{2mL}{(m-1)}+\frac{4L}{(m-1)}-\frac{2L}{(m-1)m}\right]$$

$$=2-\frac{2}{m}-\frac{2m}{(m-1)}+\frac{4}{(m-1)}-\frac{2}{m(m-1)}$$

$$a_0=\frac{2m(m-1)-2(m-1)-2m^2+4m-2}{m(m-1)}=0$$

$$a_n=\frac{2}{T}\int_0^{2L}f(x).\cos\frac{(2\pi nx)}{T}.dx$$

$$=\begin{array}{l}\frac{2}{2L}\int_0^{L/m}\frac{mx}{L}.\cos\frac{(2\pi nx)}{2L}.dx+\int_{L/m}^{(2L-L/m)}\left\{1-\frac{m}{(m-1)L}\left(x-\frac{L}{m}\right)\right\}\cos\left(\frac{2\pi nx}{2L}\right)dx\\+\int_{\left(2L-\frac{L}{m}\right)}^{2L}\frac{m}{L}(x-2l)\cos\left(\frac{2\pi nx}{2L}\right)dx\end{array}$$

$$=\frac{1}{L}\left[\frac{m}{L}\left\{\frac{xL}{\pi n}\sin\left(\frac{\pi nx}{L}\right)+\frac{L^2}{\pi^2n^2}\cos\left(\frac{\pi nx}{L}\right)\right\}_0^{L/m}+\left\{\frac{L}{\pi n}\sin\left(\frac{\pi nx}{L}\right)\right\}_{L/m}^{(2L-L/m)}\right]$$

$$-\frac{m}{(m-1)L}\left\{\frac{xL}{\pi n}\sin\left(\frac{\pi nx}{L}\right)+\frac{L^2}{\pi^2n^2}\cos\frac{(\pi nx)}{L}\right\}_{L/m}^{(2L-L/m)}+\frac{Lm}{Lm(m-1)}\left\{\frac{L}{\pi n}\sin\left(\frac{\pi nx}{L}\right)\right\}_{L/m}^{(2L-L/m)}$$

$$+\frac{m}{L}\left\{\frac{xL}{\pi n}\sin\left(\frac{\pi nx}{L}\right)+\frac{L^2}{\pi^2n^2}\cos\left(\frac{\pi nx}{L}\right)\right\}_{\left(2L-\frac{L}{m}\right)}^{2L}-\frac{2Lm}{L}\left\{\frac{L}{\pi n}\sin\left(\frac{\pi nx}{L}\right)\right\}_{\left(2L-\frac{L}{m}\right)}^{2L}$$

$$a_n=\frac{1}{L}\left[\begin{array}{l}\frac{L}{\pi n}\sin\left(\frac{\pi n}{m}\right)+\frac{mL}{\pi^2n^2}\cos\left(\frac{\pi n}{m}\right)-\frac{mL}{\pi^2n^2}+\frac{L}{\pi n}\sin\left\{\frac{\pi n}{m}(2m-1)\right\}-\frac{L}{\pi n}\sin\left(\frac{\pi n}{m}\right)\\-\frac{(2Lm-L)}{\pi n(m-1)}\sin\left\{\frac{\pi n(2m-1)}{m}\right\}\end{array}\right]$$

$$-\frac{Lm}{x^2n^2(m-1)}\cos\left\{\frac{\pi n}{m}(2m-1)\right\}+\frac{L}{\pi n(m-1)}\sin\left(\frac{\pi n}{m}\right)+\frac{mL}{(m-1)\pi n}\cos\left(\frac{\pi n}{m}\right)+\frac{L}{(m-1)\pi n}$$

$$\sin\left\{\frac{\pi n(2m-1)}{m}\right\}-\frac{L}{\pi n(m-1)}\sin\left(\frac{\pi n}{m}\right)$$

$$+\frac{2mL}{\pi n}\sin(2\pi n)-\frac{(2Lm-L)}{\pi n}\sin\left\{\frac{\pi n}{m}(2m-1)\right\}+\frac{mL}{\pi^2n^2}\cos(2\pi n)-\frac{mL}{\pi^2n^2}\cos\left\{\frac{\pi n}{m}(2m-1)\right\}$$

$$-\frac{2Lm}{\pi n}\sin(2\pi n)+\frac{2Lm}{\pi n}\sin\left(\frac{\pi n}{m}(2m-1)\right)$$

$$a_n=\frac{1}{L}\left[\begin{array}{l}\left\{\frac{L}{\pi n}-\frac{L}{\pi n}+\frac{L}{\pi n(m-1)}-\frac{L}{\pi n(m-1)}\right\}\sin\left(\frac{\pi n}{m}\right)-\left\{\frac{mL}{\pi^2n^2}+\frac{mL}{(m-1)\pi^2n^2}\right\}\\ \cos\frac{\pi n}{m}+\left\{\frac{L}{\pi n}-\frac{(2Lm-L)}{\pi n(m-1)}+\frac{L}{(m-1)\pi n}-\left(\frac{2Lm-L}{\pi n}\right)+\frac{2Lm}{\pi n}\right\}\sin\left(\frac{\pi n(2m-1)}{m}\right)\\ +\left\{\frac{-Lm}{\pi^2n^2(m-1)}-\frac{mL}{\pi^2n^2}\right\}\cos\left(\frac{\pi n(2m-1)}{m}\right)+\left\{\frac{2mL}{\pi n}-\frac{2L^2m}{\pi n}\right\}\\ \underbrace{\sin(2\pi n)}_{=0}+\frac{mL}{\pi^2n^2}\underbrace{\cos(2\pi n)}_{=1}-\frac{mL}{\pi^2n^2}\end{array}\right]$$

$$a_n=\frac{1}{L}\left[\begin{array}{l}\left\{\frac{mL(m-1)-mL}{\pi^2n^2(m-1)}\right\}\cos\left(\frac{\pi n}{m}\right)+\left\{\frac{L(m-1)-(2m-1)L+L(2m-1)L(m-1)+2Lm(m-1)}{\pi n(m-1)}\right.\\ \sin\left(\frac{\pi n(2m-1)}{m}\right)-\left\{\frac{Lm+Lm(m-1)}{\pi^2n^2(m-1)}\right\}\cos\left(\frac{\pi n(2m-1)}{m}\right)\end{array}\right]$$

$$a_n=\frac{1}{L}\left[\frac{m^2L}{\pi^2n^2(m-1)}\cos\left(\frac{\pi n}{m}\right)-\frac{m^2L}{\pi^2n^2(m-1)}\cos\left(\frac{\pi n}{m}(2m-1)\right)\right]$$

$$a_n \text{ ஶ } \frac{1}{L}\left[\frac{m^2L}{\pi^2n^2(m-1)}\left\{\cos\left(\frac{\pi n}{m}\right)-\cos\left(\frac{\pi n}{m}(2m-1)\right)\right\}\right]$$

$$a_n=\frac{1}{L}\left[\frac{m^2L}{\pi^2n^2(m-1)}\left\{\cos\left(\frac{\pi n}{m}\right)-\cos\left(\frac{\pi n}{m}(2m-1)\right)\right\}\right]$$

$$=\frac{m^2}{\pi^2n^2(m-1)}\left\{2\sin\left(\frac{\pi n}{m}(2m-1)-\frac{\pi n}{m}\right)\frac{1}{2}-\sin\left(\frac{\pi n}{m}+\frac{\pi n}{m}(2m-1)\right)\frac{1}{2}\right\}$$

$$a_n=0$$

$$b_n = \frac{2}{T}\int_0^{2L} f(x).\sin\frac{(2\pi nx)}{T}.dx$$

$$b_n = \frac{2}{2L}\left[\underbrace{\int_0^{L/m} \frac{mx}{L}\sin\left(\frac{2\pi nx}{2L}\right).dx}_{(A)} + \underbrace{\int_{L/m}^{(2L-L/m)} \left\{1-\frac{m}{(m-1)L}\left(x-\frac{L}{m}\right)\right\}\sin\left(\frac{2\pi nx}{2L}\right)dx}_{(B)} + \underbrace{\int_{\left(2L-\frac{L}{m}\right)}^{2L} \frac{m}{L}(x-2L)\sin\left(\frac{2\pi nx}{2L}\right)dx}_{(C)}\right]$$

(A)

$$\frac{1}{L}\int_0^{L/m} \frac{mx}{L}\sin\left(\frac{\pi nx}{L}\right)dx = \frac{m}{L^2}\left[-\left\{\frac{xL}{\pi n}\cos\left(\frac{\pi nx}{L}\right)+\frac{L^2}{\pi^2n^2}\sin\left(\frac{\pi nx}{L}\right)\right\}\right]_0^{L/m}$$

$$= \frac{m}{L^2}\left[-\frac{L}{\pi n}\frac{L}{m}\cos\left(\frac{\pi nL}{mL}\right)+\frac{L^2}{\pi^2n^2}\sin\left(\frac{\pi nL}{mL}\right)\right]$$

$$= -\frac{1}{\pi n}\cos\left(\frac{\pi n}{m}\right)+\frac{m}{\pi^2n^2}\sin\left(\frac{\pi n}{m}\right) \qquad \text{-------- (A)}$$

(B)

$$\frac{1}{L}\int_{L/m}^{(2L-L/m)} \left\{1-\frac{m}{(m-1)L}\left(x-\frac{L}{m}\right)\right\}\sin\left(\frac{\pi nx}{L}\right)dx$$

$$= \frac{1}{L}\left[-\frac{L}{\pi n}\cos\left(\frac{\pi nx}{L}\right)-\frac{m}{(m-1)L}\left\{-\frac{xL}{\pi n}\cos\left(\frac{\pi nx}{L}\right)+\frac{L^2}{\pi^2n^2}\sin\left(\frac{\pi nx}{L}\right)\right\}-\frac{1}{(m-1)}\frac{L}{\pi n}\cos\left(\frac{\pi nx}{L}\right)\right]_{L/m}^{2L-L/m}$$

$$= \frac{1}{L}\left[-\frac{L}{\pi n}\cos\left(\frac{\pi n}{m}(2m-1)\right)+\frac{L}{\pi n}\cos\left(\frac{\pi n}{m}\right)+\frac{L(2m-1)}{(m-1)\pi n}\cos\left(\frac{\pi n}{m}(2m-1)\right)\right.$$

$$-\frac{L}{\pi n(m-1)}\cos\left(\frac{\pi n}{m}\right)-\frac{mL}{\pi^2n^2(m-1)}\sin\left(\frac{\pi n}{m}(2m-1)\right)+\frac{mL}{\pi^2n^2(m-1)}\sin\left(\frac{\pi n}{m}\right)$$

$$\left.-\frac{1}{(m-1)}\frac{L}{\pi n}\cos\left(\frac{\pi n}{m}(2m-1)\right)+\frac{L}{\pi n(m-1)}\cos\left(\frac{\pi n}{m}\right)\right]$$

$$= \frac{1}{L}\left[\left(-\frac{L}{\pi n}+\frac{L(2m-1)}{\pi n(m-1)}-\frac{L}{\pi n(m-1)}\right)\cos\left(\frac{\pi n}{m}(2m-1)\right)+\left(\frac{L}{\pi n}-\frac{L}{\pi n(m-1)}+\frac{L}{\pi n(m-1)}\right)\cos\left(\frac{\pi n}{m}\right)\right]$$

$$+\left(-\frac{mL}{\pi^2n^2(m-1)}\right)\sin\left(\frac{\pi n}{m}(2m-1)\right)+\frac{mL}{\pi^2n^2(m-1)}\sin\left(\frac{n\pi}{m}\right)$$

$$=\left(\frac{-(m-1)+(2m-1)-1}{\pi n(m-1)}\right)\cos\left(\frac{\pi n}{m}(2m-1)\right)+\frac{L}{\pi n}\cos\left(\frac{\pi n}{m}\right)$$

$$-\frac{m}{\pi^2 n^2(m-1)}\sin\left(\frac{\pi n}{m}(2m-1)\right)+\frac{m}{\pi^2 n^2(m-1)}\sin\left(\frac{\pi n}{m}\right)$$

$$=\frac{1}{\pi n}\left[\cos\left(\frac{\pi n}{m}(2m-1)\right)+\cos\left(\frac{\pi n}{m}\right)\right]+\frac{m}{\pi^2 n^2(m-1)}\left[\sin\left(\frac{\pi n}{m}\right)-\sin\left(\frac{\pi n}{m}\right)-\sin\left(\frac{\pi n}{m}(2m-1)\right)\right]$$

------- (B)

(c)

$$\frac{1}{L}\int_{2L-\frac{L}{m}}^{2L}\frac{m}{L}(x-2L)\sin\left(\frac{\pi nx}{L}\right)dx=\frac{m}{L^2}\left[-\frac{xL}{\pi n}\cos\left(\frac{\pi nx}{L}\right)+\frac{L^2}{\pi^2 n^2}\sin\left(\frac{\pi nx}{L}\right)+\frac{2L^2}{\pi n^2}\cos\left(\frac{\pi nx}{L}\right)\right]_{\left(2L-\frac{L}{m}\right)}^{2L}$$

$$=\frac{m}{L^2}\left[\begin{aligned}&-\frac{2L^2}{\pi n}\underbrace{\cos(2\pi n)}_{=1}+\frac{L^2(2m-1)}{\pi nm}\cos\left(\frac{\pi n}{m}(2m-1)\right)+\frac{L^2}{\pi^2 n^2}\underbrace{\sin(2\pi-1)}_{=0}-\frac{L^2}{\pi^2 n^2}\sin\left(\frac{\pi n}{m}(2m-1)\right)\\&+\frac{2L^2}{\pi n}\underbrace{\cos(2\pi n)}_{=1}-\frac{2L^2}{\pi n}\cos\left(\frac{\pi n}{m}(2m-1)\right)\end{aligned}\right]$$

$$=-\frac{2m}{\pi n}+\frac{2m}{\pi n}+\left[\frac{(2m-1)}{\pi n}-\frac{2m}{\pi n}\right]\cos\left(\frac{\pi n}{m}(2m-1)\right)-\frac{m}{\pi^2 n^2}\sin\left(\frac{\pi n}{m}(2m-1)\right)$$

$$=-\frac{1}{\pi n}\cos\left(\frac{\pi n}{m}(2m-1)\right)-\frac{m}{\pi^2 n^2}\sin\left(\frac{\pi n}{m}(2m-1)\right)$$ ------------------- (c)

$\therefore$ From (A) + (B) + (C) ;

$$b_n=-\frac{1}{\pi n}\cos\left(\frac{\pi n}{m}\right)+\frac{m}{\pi^2 n^2}\sin\left(\frac{\pi n}{m}\right)+\frac{1}{\pi n}\cos\left(\frac{\pi n}{m}(2m-1)\right)+\frac{1}{\pi n}\cos\left(\frac{\pi n}{m}\right)$$

$$+\frac{m}{\pi^2 n^2(m-1)}\sin\left(\frac{\pi n}{m}\right)-\frac{m}{\pi^2 n^2(m-1)}\sin\left(\frac{\pi n}{m}(2m-1)\right)$$

$$-\frac{1}{\pi n}\cos\left(\frac{\pi n}{m}(2m-1)\right)-\frac{m}{\pi^2 n^2}\sin\left(\frac{\pi n}{m}(2m-1)\right)$$

$$=\left[\frac{m}{\pi^2 n^2}+\frac{m}{\pi^2 n^2(m-1)}\right]\sin\left(\frac{\pi n}{m}\right)-\left[\frac{m}{\pi^2 n^2(m-1)}+\frac{m}{\pi^2 n^2}\right]\sin\left(\frac{\pi n}{m}(2m-1)\right)$$

$$=\left[\frac{m(m-1)+m}{\pi^2 n^2(m-1)}\right]\left[\sin\left(\frac{\pi n}{m}\right)-\sin\left(\frac{\pi n}{m}(2m-1)\right)\right]$$

$$=\frac{m^2}{\pi^2 n^2(m-1)}\left[2\cos\left[\left(\frac{\pi n}{m}+\frac{\pi n}{m}(2m-1)\right)\frac{1}{2}\right]\sin\left[\left(\frac{\pi n}{m}-\frac{\pi n}{m}(2m-1)\right)\frac{1}{2}\right]\right]$$

$$= \frac{2m^2}{\pi^2 n^2 (m-1)}\left[\underbrace{\cos(2\pi n)}_{=1}.\sin\left(-\frac{2\pi n}{m}(m-1)\right)\frac{1}{2}\right]$$

$$b_n = -\frac{2m^2}{\pi^2 n^2 (m-1)}\sin\left(\frac{2\pi n}{m}(m-1)\right)$$

$$\therefore\ b_n = -\frac{2m^2(-1)^n}{\pi^2 n^2 (m-1)}\sin\left(\frac{\pi n}{m}(m-1)\right)$$

$f(x) = \frac{a_0}{2} + a_1 \cos x + a_2 \cos 2x + \ldots.. + a_n \cos nx + \ldots\ldots + b_1 \sin x + b_2 \sin 2x + \ldots.. + b_n$ $\sin nx + \ldots\ldots.$

$$= 0 - \left(\frac{2m^2(-1)}{\pi^2(m-1)}\sin\left(\frac{\pi}{m}(m-1)\right) + \frac{2m^2(-1)^2}{\pi^2 2^2(m-1)}\sin\left(\frac{2\pi}{m}(m-1)\right) + \ldots..\right)$$

$$f(x) = -\frac{2m^2}{\pi^2(m-1)}\sum_{n=1}^{\alpha}\frac{(-1)^n}{n^2}\sin\left(\frac{\pi n}{m}(m-1)\ \right)x$$

$$f(x) = \frac{2m^2}{\pi^2(m-1)}\sum_{n=1}^{\alpha}\frac{(-1)^{n-1}}{n^2}\sin\left(\frac{\pi n}{m}(m-1)\right)x$$

(ii). $m = 2$

$$f(x) = \frac{2m^2}{\pi^2(m-1)}\sum_{n=1}^{\alpha}\frac{(-1)^{n-1}}{n^2}\sin\left(\frac{\pi n}{m}(m-1)\right)x$$

$$f(x) = \frac{8}{\pi^2}\sum_{n=1,3,5}^{\alpha}\frac{(-1)^{\frac{(n-1)}{2}}}{n^2}\sin\left(\frac{\pi n}{2}\right)x$$

---

4). Following data are given

$C(0)$ = Concentration of the pesticide in lake at time $t(0) = 35 \times 10^{-6}$ ppg.

$V(0)$ = Volume of the lake at time $t(0) = 100 \times 10^6$ galloons

r_{in} = Input water flow = $300\ gm^{-1}$

r_{out} = Output water flow = $400\ gm^{-1}$

C_{in} = concentration of pesticide in input water = 5×10^{-6} ppg

Consider

$C(t)$ = concentration of the pesticide in lake at any time t

$P(t)$ = amount of pesticide in the lake at any time t

$V(t)$ = volume of the lake at any time t

Then $\quad c(t) = \dfrac{p(t)}{v(t)}$ --------- (1)

Considerer the amount of pesticide in the lake in small time Δt

$\Delta p = C_{in}\, r_{in}\, \Delta t - C_{out}\, r_{out}\, \Delta t$

$$\frac{\Delta p}{\Delta t} = C_{in}\, r_{in} - C_{out}\, r_{out}$$

by (1) $\frac{dp}{dt} = C_{in}\, r_{in} - r_{out} \quad \frac{p(t)}{v(t)}$ ---- (2)

but $v(t) = v(0) + (r_{in} - r_{out})\, t$ ----- (3)

(3) in to (2) $\frac{dp}{dt} = C_{in} r_{in} - \frac{r_{out}}{v_0 + (r_{in} - r_{out})t}$

$$\frac{dp(t)}{dt} = c_{in} r_{in} - \frac{r_{out} p(t)}{v_0 + (r_{in} - r_{out})t}$$

$$= 5 \times 10^{-6} \times 300 - \frac{400p(t)}{100 \times 10^6 + (300 - 400)t}$$

$$= 15 \times 10^{-4} - \frac{400p(t)}{100(10^6 - t)}$$

$$\frac{dp(t)}{dt} + \frac{4p(t)}{(10^6 - t)} = 15 \times 10^{-4}$$

I.F. $(10^6 - t)^{-4} \frac{dp(t)}{dt} + 4p(t) = (10^6 - t)^{-4} \times 15 \times 10^{-4}$

$$\int d(10^6 - t)^{-4}\, p(t) = \int 15 \times 10^{-4} (10^6 - t)^{-4}\, dt + c$$

$$p(t)\,(10^6 - t)^{-4} = 15 \times 10^{-4} (10^6 - t)^{-3} + c$$

$$p(t) = 15 \times 10^{-4} (10^6 - t) + c\,(10^6 - t)^{-4}$$

$$p(t) = c(t)\,.\,v(t)$$

$$p(0) = c_0\, v_0$$

$t = 0 \quad p(0) = 35 \times 10^{-6} \times 100 \times 10^6$

$$= 3500$$

$$c = 3000 \times 10^{-24}$$

$$c = 3 \times 10^{-21}$$

$p(t) = vc\,(t)$

$= v_0 + (r_{in} - r_{out}) \times c(t)$

$= [100 \times 10^6 + (300 - 400)\, t\,] \times 5 \times 10^{-6}$

$= (10^6 - t) \times 5 \times 10^{-4}$

$(10^{-6} - t)\, 5 \times 10^{-4} = 5 \times 10^{-4} (10^6 - t) + 3 \times 10^{-21} (10^6 - t)^{-4}$

$t = 10^6$ min

b). $\frac{dp(t)}{dt} + \frac{p(t) r_{out}}{v_0 + (r_{in} - r_{out})t} = c_{in} r_{in}$

$$\frac{dp(t)}{dt} + \frac{p(t)450}{100 \times 10^6 + (400 - 450)t} = 5 \times 10^{-6} \times 400$$

$$\frac{dp(t)}{dt}+\frac{9p(t)}{\left(2\times10^{6}-t\right)}=2\times10^{-3}$$

$$\text{I.F. } \int_{e} \frac{9}{\left(2\times10^{-6}-t\right)}=\left(2\times10^{6}-t\right)^{-9}$$

$$(2\times10^{6}-t)^{-9}\,p(t)=2\times10^{-3}\,\frac{\left(2\times10^{-6}-t\right)^{-9}}{9}+c$$

at $t=0$ $p(0)=3500$

$3500=2\times10^{-3}(2\times10^{-6})+c(2\times10^{-6})^{9}$

$$3500=\frac{1}{2}\times1000+c(2\times10^{-6})^{9}$$

$$\frac{3000}{\left(2\times10^{-6}\right)}=c$$

$$p(t)=\frac{2\times10^{-3}\left(2\times10^{6}-t\right)}{8}+\frac{3000}{\left(2\times10^{6}\right)^{9}}\,(2\times10^{6}-t)^{9}$$

$t=450000$

$$p(t)=2\times10^{-3}(2\times10^{6}-0.45\times10^{6})+\frac{3000}{2\times10^{6}}\left(2\times10^{-6}-t\right)^{9}$$

$$=\frac{2\times1.55\times10^{3}}{8}+\frac{3000}{2^{9}.10^{54}}\times1.55^{9}$$

$$=\frac{3.1\times10^{3}}{8}$$

$=30387.5$

$$c(t)=\frac{p(t)}{v(t)}=\frac{387.5}{55\times10}6$$

05).

$$Ri+L.\frac{di}{dt}=v(t)$$

$$L.\frac{d^2i}{dt^2}+R.\frac{di}{dt}=\frac{dv}{dt}$$

$$L\left(\frac{di}{dt}\right)_{t=0}+Ri(o)=v(0)$$

$t=0$ $i=6$

$i(0)=6$

$$\frac{di}{dt}=0$$

$Li^{1}(0)+Ri(0)=v(0)$

$$i^{1}(0)=\frac{v(0)-6R}{L}$$

$0.5\ \frac{d^2i}{dt^2}+\frac{10di}{dt}=6\cos 2t$

$\frac{d^2i}{dt^2}+20.\frac{di}{dt}=12\ \cos\ 2t$ ----------(1)

$m^2+20\ m=0$

$m(m+20)=0$

$m=0$ or $m+20=0$

$m=-20$

$i_c=Ae^{0t}+Be^{-20t}$

$i_p=\alpha\ \sin\ 2t+\beta\cos\ 2t$

$\frac{d_ip}{dt}=2(\alpha\cos 2t-\beta\sin 2t)$

$\frac{d_i^2p}{dt^2}=4\ (-\alpha\ \sin\ 2t-\beta\ \cos\ 2t)$

From eq^n (1) ;

$4(-\alpha\ \sin\ 2t-\beta\cos\ 2t)+20\times 2\ (\alpha\ \cos\ 2t-\beta\ \sin\ 2t)=12\ \cos\ 2t$

$-\quad 4\alpha\ \sin\ 2t-4\beta\ \cos\ 2t+40\ \alpha\ \cos\ 2t-40\ \beta\ \sin\ 2t=12\ \cos\ 2t$

$\text{Sin}\ 2t\ (-4\alpha-40\beta)+\cos\ 2t(-4\beta+40\alpha)=12\ \cos\ 2t$

$-4\beta+40\ \alpha=12$

$-4\alpha-40\beta=0$

$4\alpha=-40\beta$

$\alpha=-10\beta$

$-4\beta+(40\times -10\beta)=12$

$-404\beta=12$

$\beta=-\frac{12}{404}=-\frac{3}{101}$

Therefore $\alpha=-10\times\frac{-3}{101}=\frac{30}{101}$

The general solution is

$i(t)=i_c+ip$

$=Ae^{-0t}+Be^{-20t}+\alpha\ \sin\ 2t+\beta\ \cos\ 2t$

$=Ae^{-0t}+Be^{-20t}+\frac{30}{101}\sin\ 2t-\frac{3}{101}\cos\ 2t$

$i(0)=A+B-\frac{3}{101}=6$ ------- (2)

$i(t)=\frac{v(0)-6R}{L}$

$i'(t)=-20B\ e^{-20t}+\frac{60}{101}\cos\ 2t+\frac{6}{101}\sin\ 2t$

$i'(0)-20B+\frac{60}{101}=0$

$$20B = \frac{60}{101}$$

$$B = \frac{3}{101}$$

$$A + B = \frac{3}{101} + 6$$

$$A = \frac{609}{101} - \frac{3}{101} = \frac{606}{101} = 6$$

$$i(t) = 6\,e^{0} + \frac{3.e^{-20t}}{101} + \frac{30}{101}\sin 2t - \frac{3}{101}\cos 2t$$

$$i(t) = \frac{3}{101}\left[-\frac{3}{2}e^{0} + \frac{1}{2}e^{-20t} + 10\sin 2t - \cos 2t\right]$$

$$= 6 + \frac{3}{101}e^{-20t} + \frac{30}{101}\sin 2t - \frac{3}{101}\cos 2t$$

06). (a). (i). Using ration test

$$\frac{\left|(n+1)^{st}\text{ term}\right|}{\left|n^{th}\text{term}\right|} = \frac{(n+1)|x|^{n+1}}{2^{n+2}}.\frac{2^{n+1}}{n|x|^{n}}$$

$$= \frac{(n+1)|x|}{n.2}$$

$$\underset{n\to\infty}{} \frac{(n+1)|x|}{n.2} \Rightarrow \frac{|x|}{2}$$

Thus the given series divergence if $|x| > 2$ and converges absolutely if $|x| < 2$. Hence it has radius of converge 2.

ii). Using ration test

$$\frac{\left|(n+1)^{St}\text{ term}\right|}{\left|n^{th}\text{term}\right|} = \frac{(n+1)!|x|^{n+1}}{(n+1)^{n+1}}\frac{n^{n}}{n!|x|^{n}}$$

$$= (n+1)\,|x|.\frac{n^{n}}{(n+1)^{n+1}}$$

$$= \frac{n^{n}}{(n+1)^{n}}\,|x|$$

$$= \left(\frac{n}{n+1}\right)^{n}|x|$$

$$\lim_{n\to\infty}\left(\frac{n}{n+1}\right)^n |x| = \frac{|x|}{\lim_{n\to\infty}\left(\frac{n+1}{n}\right)^n}$$

$$= \frac{|x|}{e}$$

The given series divergence if $|x| > e$ and converges absolutely if $|x| < e$. Hence it has radius of convergence e.

b). $y'' - x(y')^2 + y^2 = 0$; $y(0) = 1$ $y'(0) = 0$ ------- (1)

Take $y' = p \Rightarrow y'' = p'$

Substitute above value to eq^n (1)

$p' - xp^2 + y^2 = 0$

$y(0) = 1$

$p(0) = 0$

Taylor series for eq^n (2) about $x = x_0$ &

when $x = x_1 \Rightarrow y = y_1$

$$y_1 = y_0 + h y_0^1 + \frac{h^2}{2!} y_0'' + \frac{h^3}{3!} y_0''' + \frac{h^4}{4!} y_0^{iv} + \text{-----} \ (3)$$

using eq^n (1) ie $y'' - x(y')^2 + y^2 = 0$

when $x = x_0 = 0 \Rightarrow y = y_0 = 1$

$y_0'' - x_0(y_0') + y_0^2 = 0$

$y_0'' - 0 + 1 = 0 \Rightarrow y' = -1$

eq^n (1) d.w.r. to x

$y''' - (x_0\ 2y'y'' + (y')^2) + 2yy^2 = 0$ ------- (4)

$y_0''' - \left[2x_0 y_0' y_0'' + (y_0')\right] + 2y_0 y_0' = 0$

$y_0''' - [0 + 0] + 0 = 0$

$y_0''' = 0$

eq^n (4) d.w.r. to x

$$y^{iv} - \left[2x\left\{y'y''' + (y'')^2\right\} - 2y'y'' - 2y'y'' + 2yy'' + 2(y')^2\right] = 0$$

$$y^{iv} - 2x_0\left\{y_0'y_0''' + (y_0'')^2\right\} - 2y_0'y_0'' - 2y_0'y_0'' + 2y_0y'' + 2(y_0')^2 = 0$$

$y_0^{iv} - 2 = 0$

$y_0^{iv} = 0$

Taylor's series for eq^n (2) about $x = x_0$ & when $x = x_1 \Rightarrow y = y$

$$y_1 = 1 + (0.1) . 0 + \frac{(0.1)}{2!}(-1) + \frac{(0.1)^3}{3!}0 + \frac{(0.1)^4}{4!}2$$

$y_{(0.1)} = 0.995008333$

$$y_2 = y_1 + hp_1 + \frac{h^2}{2!}p_1^{\prime} + \frac{h^3}{3!}p_1'' + \frac{h^4}{4!}p_1''' + \ldots\ldots$$

using eq^n (3)

Taylor's series for eq^n (3) about $x = x_0$ & $x = x_1 \Rightarrow p = p_1$

$$p_1 = p_0 + h p_0^{\prime} + \frac{h^2 p_0''}{2!} + \frac{h^3}{3!}p_0''' + \frac{h^4}{4!}p_0^{iv}$$

eq^n (3)

$$p_0' - x_0 p_0^2 + y_0 = 0$$

$$p_0^{\prime} - 0 + 1 = 0 \Rightarrow p_1^{\prime} = -1$$

(3) d.w.r. to x

eq^n $\quad p'' - \{x2p\, p'' + p^2\} + 2yy' = 0$ ------- (5)

$$p_0'' - \{2x_0 p_0 p_0'' + p_o'^2\} + 2y_0 y_0^{\prime} = 0$$

$$p_0'' - [0 + 0] + 0 = 0 \Rightarrow p_0'' = 0$$

eq^n (5) d.w.r. to x

$$p_0''' - 2xpp' - 2x(p')^2 - 2pp' - 2p'p + 2(y')^2 + 2yy'' = 0 \qquad \text{--------- (6)}$$

$$p_0''' - 2 = 0$$

$$p_0''' = 2$$

$$y_2 = y_1 + hp_1 + \frac{h^2}{2!}p_1^{\prime} + \frac{h^3}{3!}p_1'' + \frac{h^4}{4!}p_1''' + \ldots\ldots.$$

$$y_2 = 0.995 + \frac{(0.1)^2}{2!}(-0.0997)$$

$y_2 = 0.9801$