



Time: 0930 – 1230 hrs.

Date: 2007-04 -09

**Answer any FIVE questions**

1.

(a)

Any periodic function  $f(t)$  can be represented by a *Fourier series* as given below:

$$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + \sum_{n=1}^{\infty} b_n \sin(n\omega_0 t) \text{ where } a_0, a_n, b_n \text{ and } \omega_0 \text{ are constants.}$$

- (i) If the period of  $f(t)$  is  $T$ , explain how would you calculate the values of  $a_0, a_n, b_n$  and  $\omega_0$ .
- (ii) If the function  $f(t)$  is an even function ( $f(t) = f(-t)$ ) how would you simplify the above series for  $f(t)$ ?

(b)

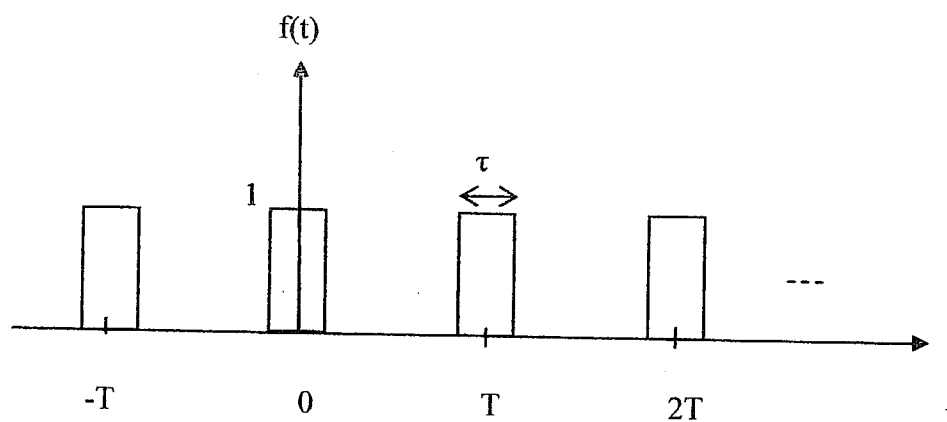


Fig.1

A repetitive waveform is shown in Fig.1.

- (i) Express the waveform as a *Fourier series* and calculate the values of  $a_0$ ,  $a_n$ ,  $b_n$  and  $\omega_0$ .
- (ii) Sketch the Fourier coefficient  $c_n = \sqrt{a_n^2 + b_n^2}$  vs.  $n\omega_0$
- (iii) Redo (ii) when  $T \rightarrow \infty$ .
- (iv) At what important conclusion can you arrive from (iii) regarding the frequency content of a non-repetitive rectangular pulse? Justify your conclusion.

2.

- (a) What is understood by *convolution* of two signals?  $f(t)$  and  $g(t)$  are two non-periodic signals. The function  $h(t)$  represents the convolution of  $f(t)$  with  $g(t)$ .

ie.  $h(t) = f(t) * g(t)$

If the geometrical shapes of  $f(t)$  and  $g(t)$  are known, explain how would you find the shape of  $h(t)$ .

- (b) If  $f(t) = \delta(t - T)$  and  $g(t)$  is known, sketch  $f(t)$ ,  $g(t)$  and  $h(t)$ .
- (c) Due to a technical problem in the modulator, a transmitter outputs a constant dc signal when it is switched on. At the receiver the received signal is monitored in the time domain. If the impulse response  $g(t)$  of the channel is given by  $g(t) = e^{-t}$  for  $t > 0$  ( $g(t) = 0$  for  $t \leq 0$ ), find the received signal  $y(t)$  and sketch it.

3.

- (a) The Fourier Transform of a function  $f(t)$  is given by  $F(\omega)$ . Write down the relationship between  $F(\omega)$  and  $f(t)$ .

- (b) Write down the equation for the Fourier Transform of

- (i)  $f(t - t_0)$ , where  $t_0$  is a constant.

- (ii)  $\frac{df(t)}{dt}$

- (iii)  $f(at)$ , where  $a$  is a constant

in terms of  $F(\omega)$ .

- (c) The high frequency components of a rectangular pulse  $s(t)$  are filtered using an ideal low-pass filter whose lower-cutoff frequency is  $f_L$ .

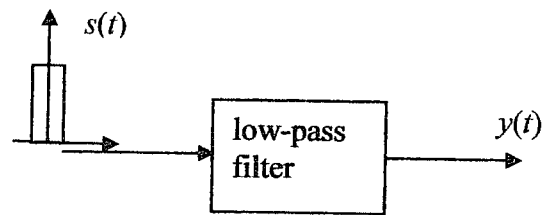


Fig.3

- (i) Calculate and sketch *the frequency spectrum* of the filtered signal  $y(t)$ .
- (ii) Comment on the shape of  $y(t)$ . Sketch the approximate shape of  $y(t)$ .

4.

(a)

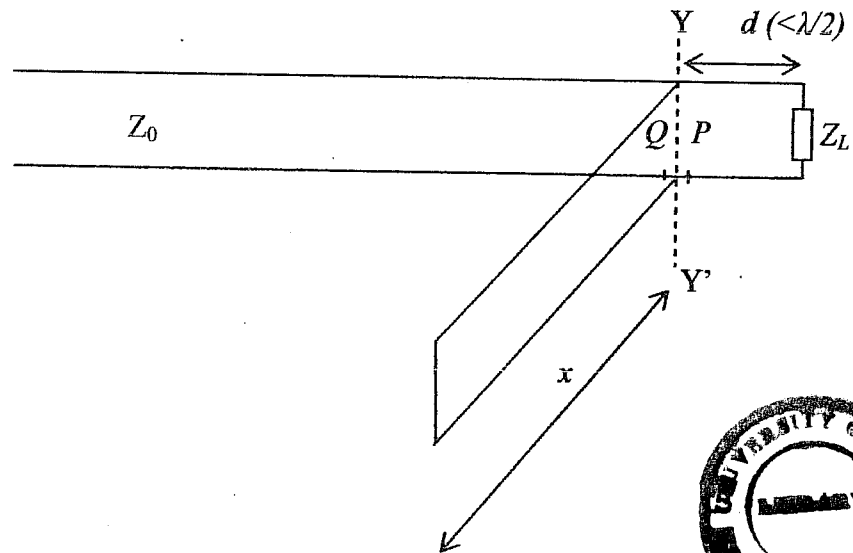


Fig. 4

A transmission line having a characteristic impedance  $Z_0$  (pure resistive), is terminated with a load impedance  $Z_L$ .

- (i) A student wishes to match the load  $Z_L$  to the line using a variable short circuit stub. He connects the stub at a *fixed distance*  $d$  and attempts to match  $Z_L$  to the line by varying the stub length  $x$ . Does he arrive at a matching point? Justify your answer.

- (ii) Sketch a Smith chart and indicate the load point on it as  $L_0$ . Now we move from the load point towards the generator. Sketch the variation of the normalized admittance on your diagram for a distance  $\lambda/2$ . Clearly indicate points P (a point just before the stub is added) and Q (a point immediately after the stub) on it.  
(When you sketch the Smith chart sketch the constant admittance- and susceptance curves that are relevant to the problem only).
- (iii) A load is matched to a line using “double stub method”. The distance between the two stubs is  $\lambda/4$  and the distance between the load and the first stub is  $\lambda/8$ . With the help of a diagram explain how the load is matched to the line.

5.

A non-repetitive signal  $s(t)$  has a Fourier transform  $S(\omega)$ .

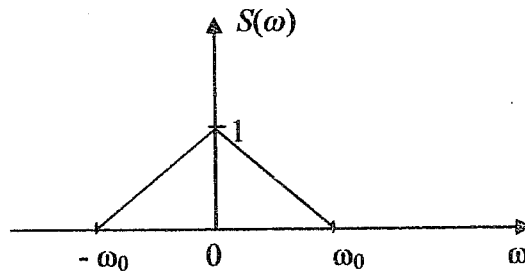


Fig. 5

The signal  $s(t)$  is sampled using a pulse train  $p(t) = \sum_{n=0}^{\infty} \delta(t - nT)$ . The sampler output  $y(t)$  is given by the product  $p(t) \cdot s(t)$ .

- (a)
- Find the *Fourier series* of  $p(t)$ .
  - Find  $Y(\omega)$ , the *Fourier transform* of  $y(t)$ . Sketch  $Y(\omega)$ .
  - In order to extract  $s(t)$ ,  $y(t)$  is low-pass filtered. What relationship should exist between  $T$  and  $\omega_0$  so that a distortionless signal appears at the filter output?
  - It was found that due to bad sampling frequency,  $y(t)$  is distorted. Redo (ii).
- (b) If the pulse train  $f(t)$  given in Fig.1 is used as the sampling signal (instead of  $p(t)$ ) sketch  $Y(\omega)$ .

6.

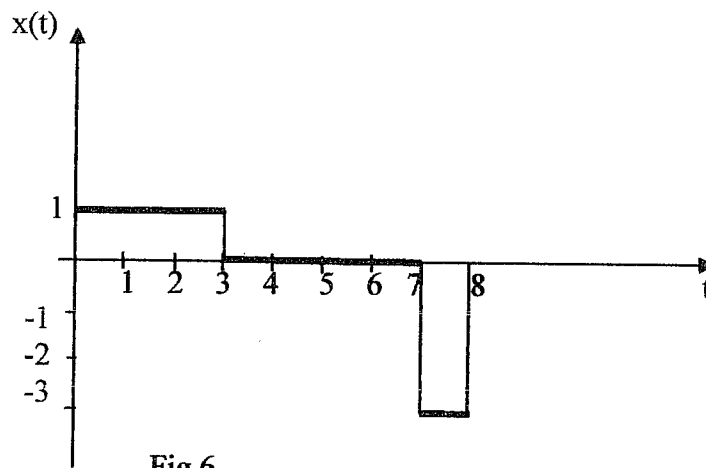


Fig.6

A  $T$  periodic random process is given in Fig.6. Assume that  $T = 8$  units.

- Calculate and sketch the time autocorrelation function  $\overline{x(t)x(t+\tau)}$
- Calculate the average power of the signal.
- How would you calculate the power spectral density function of the signal? (Actual calculation is not required).

7.

- Noise is added to an unmodulated sinusoidal carrier signal. The amplitude of the carrier is 10 V. A noise signal with constant amplitude of 5 V and variable phase is added to the carrier during the transmission.
  - Calculate the maximum possible phase error in the unmodulated carrier wave due to the noise signal.
  - Find the percentage change of the carrier amplitude when the phase error is at a maximum.
- If the above carrier is amplitude modulated using a sinusoidal modulating signal of 3 V, find the maximum possible phase error.

8.

- In a digital transmission system 8-bit words are transmitted. Each word consists of 7 data bits and 1 parity bit (It is assumed that the parity bit never undergoes an error). At the transmitter parity bit is set to 1, if odd number of 1's are present in data. Otherwise the parity bit remains at '0'. At the receiver a parity detector (even parity) detects whether even number of 1's are present in the 8-bit word. If there are odd number of 1's in the word, the detector assumes that one of the bit has changed its sign

during the transmission and asks the receiver to resend that word. If the bit error probability  $P_e$  (the probability that any bit can change its value during the transmission) is  $10^{-5}$ , calculate the probability of an undetectable error.

- (b) A transmitter transmits either a '1' (+2 V) or a '0' (-1 V) with equal probability. Due to channel noise '1' level and '0' level are shifted during the transmission. A level detector takes a decision on the received bit as follows:

If the received voltage value ( $V_r$ ) is greater than 3 V then the received level is '1'

If the received voltage value is less than or equal to 3 V then the received level is '0'

The probability density function of channel noise is given below:

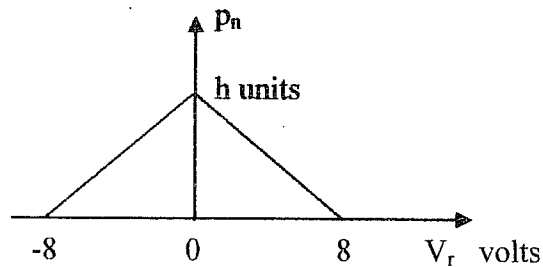


Fig. 8

- (i) Calculate the value of  $h$  and indicate its units.
- (ii) Calculate the error probability.

