THE OPEN UNIVERSITY OF SRI LANKA
Department of Civil Engineering
Bachelor of Technology - Level 5

CEX5231 - MECHANICS OF FLUIDS
CEU3204 – FLUID MECHANICS

FINAL EXAMINATION - 2006/2007

Time Allowed : **Three Hours** *Index No.* ☐☐☐☐☐☐☐☐

Date : 10th March, 2006 Time : 1330 - 1630

ANSWER **ALL THREE** QUESTIONS IN PART A AND **ANY TWO** QUESTIONS IN PART B. ALL QUESTIONS CARRY EQUAL MARKS.

PART A

Answer **all three** questions in this section.

1) A small portion of a rigid pipe of uniform cross-section, of length δx is shown in Figure 1a. An incompressible fluid, of density ρ, enters at one end and exits at the other end. The fluid flows through the stream tube at a velocity $\mathbf{v(x,t)}$ under a pressure $\mathbf{p(x,t)}$.

a) Apply the principles of conservation of mass and momentum for unsteady flow to the element of the stream tube to derive the equation $\frac{1}{\rho}\frac{\partial p}{\partial x}+g\frac{\partial z}{\partial x}+\frac{\partial v}{\partial t}=0$.
Here $\mathbf{x}$ is the distance along the pipe while $\mathbf{z}$ is the height above a datum and $\mathbf{g}$ is the acceleration due to gravity. State all your assumptions and explain your answer.

A simple surge tank is located at the end of a long horizontal hydropower tunnel as shown in Figure 1b. The tunnel has a length of $\mathbf{L}$ and a cross-sectional area of $\mathbf{A_T}$. The surge tank has a cross-sectional area of $\mathbf{A_S}$. When there is a steady flow through the tunnel to the turbine, the water level in the surge tank lies a vertical distance $\mathbf{z_0}$ below the water level of the reservoir, as shown in the figure. If the flow to the turbine is suddenly stopped, the water flowing in the tunnel will flow into the surge tank, causing the water level in the surge tank to increase.

b) For the case of sudden closure of the turbine valve, use the equation derived in section a) and the principle of conservation of mass between the tunnel and the surge tank to derive the equation

$\frac{L}{g}\frac{A_S}{A_T}\frac{d^2z}{dt^2}+z=0$ for the variation of the water level in the surge tank. Neglect the effects of friction. State all your assumptions and explain your answer.

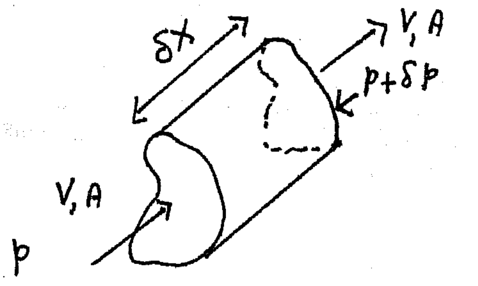

FIGURE 1a

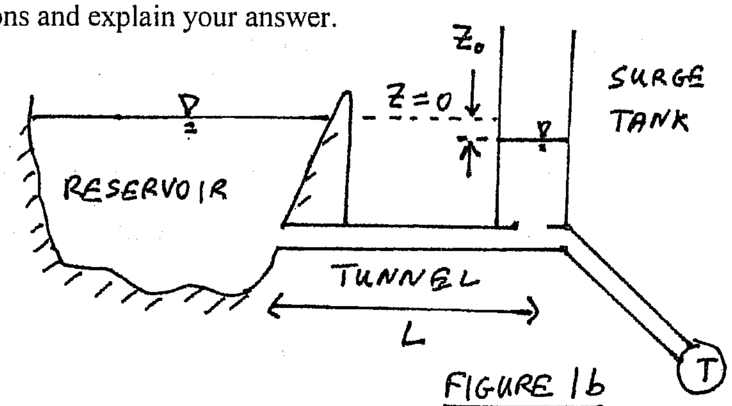

FIGURE 1b

2) A horizontal pipe of diameter 20 mm expands to a diameter of 50 mm between A and B as shown in Figure 2. Water (density 1000 kg/m^3) flows in the pipe. The average velocity at A is 1 m/s .

Two piezometer tubes are connected to A and B, with levels 125 mm at A and 165 mm at B, as shown in the figure. A U-tube mercury (density 13,600 kg/m^3) manometer is also connected to A and B, as shown in the figure.

a) Calculate the difference in the levels of the U-tube mercury manometer and sketch the levels in the two arms of the manometer.

b) In which direction is the water flowing? Explain your answer.

c) Calculate the rate of energy loss in the section AB.

d) Calculate the magnitude and direction of the force on the pipe section AB due to the flow of water.

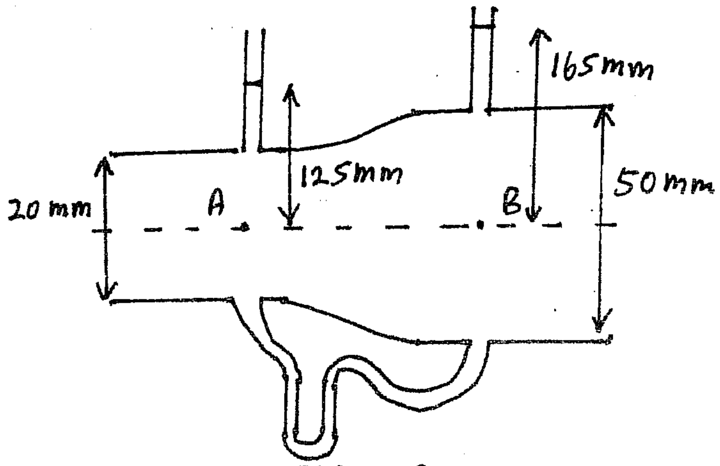

FIGURE 2

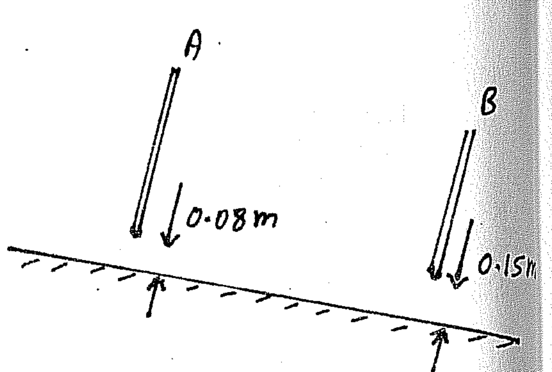

FIGURE 3

3) A laboratory open channel has a width of 0.2 m and a Manning's coefficient of 0.015 . Two gates are placed at points A and B in the channel as shown in Figure 3. The channel slope is set to 0.005 and a discharge of 0.05 m^3/s is sent through the channel. The opening of the gate at A is set to 0.08 m while the opening of the gate at B is set to 0.15 m . The flow depth upstream of the gate at A is 0.56 m.

a) Show that there is a free hydraulic jump between the gates at A and B. Explain your answer.

b) Sketch the variation of the free surface from upstream of the gate at A to downstream of the gate at B and identify the surface profile elements (from M1, M2, M3, S1, S2, S3, C1, C2, C3, etc.). Explain your answer.

c) If the slope of the channel is now **increased slightly**, while keeping the discharge and the gate openings as they were, what will happen to the location of the jump? Explain your answer.

d) The slope of the channel is increased to 0.05, **so that the uniform depth is now 0.12 m**, while keeping the discharge and the gate openings as they were. Sketch the variation of the free surface from upstream of the gate at A to downstream of the gate at B and identify the surface profile elements (from M1, M2, M3, S1, S2, S3, C1, C2, C3, etc.). Explain your answer.

(Note : The equations $\frac{dy}{dx}=\frac{S_0-S_f}{1-F_r^2}$ and $\frac{y_2}{y_1}=\frac{1}{2}\left(\sqrt{1+8F_{r1}^2}-1\right)$ may be useful.)

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PART B

Answer any **two** of the **five** questions in this section.

4) a) Define the stream function and the potential function in two-dimensional ideal fluid flow and show that stream lines and equipotential lines always intersect at right angles.

A solid body is modelled by considering a source and sink of equal strength, **m**, placed a distance **2r** apart in a uniform flow of velocity, **U**, as shown in Figure 4. The line joining the source and sink is parallel to the direction of the flow.

b) Obtain the complex potential for this flow.

c) Locate the stagnation points and obtain an expression for the dividing streamline (this is the streamline that passes through the stagnation points).

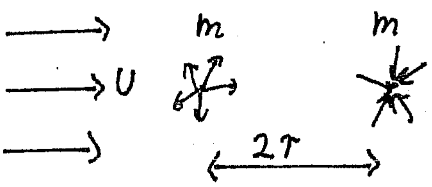

FIGURE 4

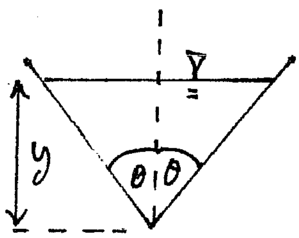

FIGURE 5

5) a) Define specific energy, **E**, in open channel flow.

b) What is the definition of critical depth in open channel flow?

Figure 5 shows an open channel that has a triangular cross-section. The flow depth is y.

c) Obtain a relationship between the critical depth, y_c, the discahrge, Q and the angle θ.

d) Sketch a graph of y/y_c against E/y_c for this channel.

A triangular open channel has a semi-vertex angle, θ, of 45 degrees. The discharge is $0.1\ m^3/s$ and the flow depth is $0.5\ m$. The channel has a smooth transition where the semi-vertex angle reduces to 30 degrees and then increases back to 45 degrees.

e) Sketch the variation of the flow depth through this transition. Explain your answer using the graph sketched above.

6) a) Sketch the Shields' curve and define the quantities on the two axes.

b) Explain what the Shields' curve is used for.

A simple equation for the critical shear stress at the initiation of motion for the flow of water over a uniform sand bed is $\tau_{0cr} = 900d$. Here d is the grain size in metres and τ_{0cr} is the critical shear stress in Pascals.

c) To what part of the Shields' curve does this equation correspond?

(continued ....)

6) (……. continued)
An open channel of rectangular cross-section is to be designed to convey a flow of 1.5 m^3/s of water that is diverted from a river. The channel slope is to be 0.0005 while the channel is to be constructed of finished concrete with a Manning's coefficient of 0.025 . The water from the river contains sediment of a maximum grain size of 0.15 mm . The channel is to be designed to avoid any deposition of sediment.

d) Select suitable cross-sectional dimensions for this channel. Explain your answer.

7) Three identical centrifugal pumps are to be used in combination to move water from Reservoir A to Reservoir B as shown in Figure 7. Two of the pumps are in parallel, while the third is in series with the first two, as shown in the figure.

The pipeline JB has a length of 2.5 km, a diameter of 200 mm and a friction factor of 0.02 . The minor losses in the pipeline total $10v^2/2g$, where **v** is the average velocity in the pipeline. The elevation difference between the free surfaces of the reservoirs is 40 m . One of these pumps was tested at its operating speed of 1500 r.p.m. The results of the test are given in Table 7.

Discharge (l/s)	0	10	20	30	40
Head (m)	30	27.5	23.5	17.0	7.5
Efficiency	-	45	80	50	30

Table 7

a) Explain the variation of the pump head and efficiency with discharge as shown by the test results.

b) Calculate the discharge in the pipeline. Explain your method and state all your assumptions. Neglect the losses in the pipes between A and J.

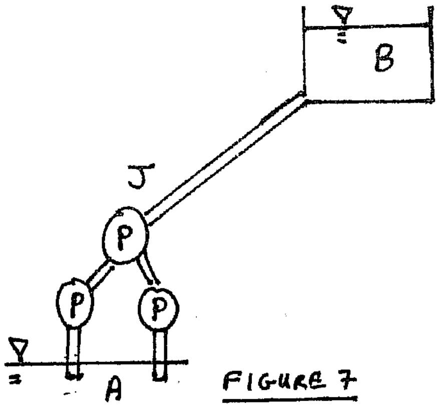

FIGURE 7

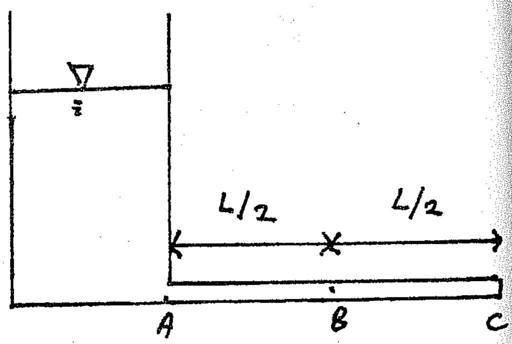

FIGURE 8

8) Water (of density ρ and dynamic viscosity μ) is discharged steadily from a large tank to the atmosphere through a long pipe, ABC, as shown in Figure 8. A valve is located at C, as shown. The average velocity of the flow is **V** . The pipe diameter is **d** and the length of the pipe is **L** . The pipe wall thickness is **T** while the Young's modulus of the pipe material is **E** . The bulk modulus of water, κ , is defined by $\frac{1}{\kappa} = \frac{1}{\rho}\frac{\partial \rho}{\partial p}$ where **p** is the pressure.

a) Explain what would happen if the valve at C was closed suddenly at a time t = 0 .
b) Sketch, on the same graph, the variation of the pressure in the pipe at the points, A, B and C. Explain your answer.
c) Obtain a non-dimensional relationship for the maximum pressure in the pipeline as a function of the other relevant variables.