THE OPEN UNIVERSITY OF SRI LANKA

BACHELOR OF TECHNOLOGY – LEVEL 04

FINAL EXAMINATION - 2006

MPZ 4230 – ENGINEERING MATHEMATICS II

DURATION : THREE (03) HOURS

391

Index No. ....

Date : 18th March 2007 Time: 1330 – 1630 hrs.

Instructions:

- Answer only six (06) questions.
- State any assumption you required.
- Do not spend more than 30 minutes for one problem.
- Show all your workings.
- All symbols are in standard notation.
- Request statistical tables.

01. a) If $u = f(x,y)$, $x = r\cos\theta$, $y = r\sin\theta$

Show that,

$$\left(\frac{\partial u}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial u}{\partial \theta}\right)^2 = \left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2$$

b) Find and classify the stationary points of the function.

$f(x,y) = x^2 - 4xy + y^3 + 4y$

c) The two sides forming the right angle are denoted by a and b. The hypotenuse is h. If there are possible error of $\pm 0.5\%$ in measuring a and b, find the maximum possible error in calculating

i. The area of the triangle
ii. The length of h

If the measured value of h is 10 units in what range will the actual value lie?

02. Show that the force $\underline{F}$ defined by $\underline{F} = 3x^2y\underline{i} + (x^3+1)\,\underline{j} + 9z^2\underline{k}$ represents a conservative field of force. Find a scalar potential ϕ such that $\underline{F} = \nabla\phi$. Hence find the work done in moving a particle of unit mass under this field of force from the point (0,0,0) to the point (1,1,1).

03. i. If $f(z) = u + iv$ analytic at $z = z_0$. Then show that u,v satisfy Laplace's equation at $z = z_0$.

$$\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial x^2} = 0$$

ii. Show that

$$u = xe^x \cos y - ye^x \sin y \text{ function is harmonic.}$$

iii. For above harmonic function find the conjugate harmonic function v and express u+ iv as an analytic function of z.

iv. Evaluate $\oint \bar{z}\,dz$ around the square at z=0,2 , 2+2i, 2i

04. a) Classify the following function according as they are even, odd or neither even or odd.

$f(x) = x(10-x), \qquad 0 < x < 10$
with period = 10

b) An alternating current after passing through a rectifier has the form

$$i = \begin{cases} I_0 \sin x & , \quad 0 < x < \pi \\ 0 & , \quad \pi < x < 2\pi \end{cases}$$

Where I_0 is the maximum current and the period is 2π, Express i as a fourier series.

05. A Mechanical system satisfies the following differential equations.

$$2\frac{d^2x}{dt^2} + 3\frac{d^2y}{dt^2} = 4 \quad \text{and} \qquad 2\frac{d^2y}{dt^2} - 3\frac{dx}{dt} = 0$$

Use laplace transforms to obtain x and y given that $x, y, \frac{dx}{dt}, \frac{dy}{dt}$ all vanish at t=0.

06. i. Using the Taylor series method find y=(0.1), y(0.2) for the differential equation $y'=xy+y^2$ given that y(0)=1.

ii. The fourth order Runge-Kutta formulae for solving the differential equation $\frac{dy}{dx}=f(x,y)$ are:

$$k_1 = hf(x_r, y_r)$$

$$k_2 = hf\left(x_r+\frac{h}{2}, y_r+\frac{k_1}{2}\right)$$

$$k_3 = hf\left(x_r+\frac{h}{2}, y_r+\frac{k_2}{2}\right)$$

$$k_1 = hf(x_r+h, y_r+k_3)$$

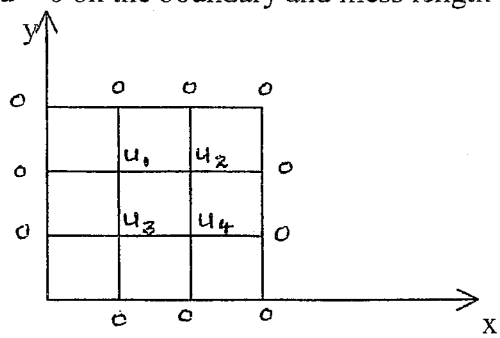
and $y_{r+1} = y_r + \frac{1}{6}(k_1+2k_2+2k_3+k_4)$

a) Calculate an estimate of y(0.45) for the initial value problem $\frac{dy}{dx}=x+y+xy, y(0)=1$, using step size h = 0.15

b) Calculate an estimate of y (1.30) for the initial value problem $\frac{dy}{dx}=\frac{1}{x+y}, y(1)=2$, using step size h=0.10

(State any assumptions you may use)

07. Solve $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=-10(x^2+y^2+10)$ over the square with x = 0 = y, x = 3 = y with u = 0 on the boundary and mess length = 1



(Five point scheme for the Laplace's equation is $u_{i+1,j}+u_{i,j+1}-u_{i-1,j}-u_{i,j-1}=4u_{i,j}$)

08. **Answer only three parts**

i. Show that set of vectors (4,1,-3), (3,1,-2) and $(3,1,-\frac{3}{2})$ is a linearly independent set.

ii. Find the Kernal of the linear transformation $L(x_1,x_2) = (x_1,x_2)\begin{pmatrix} Cos\theta & -Sin\theta \\ Sin\theta & Cos\theta \end{pmatrix}$ Explain the geometrical explanation of the transformation.

iii. It is given that $A = \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \end{pmatrix}$ and $\underline{b} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$
Show that set of solutions of $A\underline{x} = \underline{b}$ is not a subspace of R^3

iv. A is matrix is said to be orthogonal if it is square and its columns are orthonormal. Two vectors $\underline{a}$ and $\underline{b}$ are orthonormal if they are orthogonal and in addition $|\underline{a}| = |\underline{b}| = 1$. A matrix A is given below. Show that it's columns are orthonormal.

Hence deduce that A is orthogonal and find inverse of A.

A matrix is given as ;

$$A = \begin{pmatrix} \frac{3}{\sqrt{11}} & \frac{-1}{\sqrt{6}} & \frac{-1}{\sqrt{66}} \\ \frac{1}{\sqrt{11}} & \frac{2}{\sqrt{6}} & \frac{-4}{\sqrt{66}} \\ \frac{1}{\sqrt{11}} & \frac{1}{\sqrt{6}} & \frac{7}{\sqrt{66}} \end{pmatrix}$$

09. Prove that eigen values of a 3x3 diagonal matrix are the diagonal elements of the matrix . A matrix is given as

$$A = \begin{pmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{pmatrix}$$

Find

a) Eigen values of A

b) Eigen vectors of A

c) Obtain a matrix P such that $P^{-1}AP$ is a diagonal matrix.

d) If $S=P^{-1}AP$, state the special property of S and find S^{-1}.

e) Using above results reduce the quadratic form $Q(x)$ to form $Q(y)$
Where

$$Q(x) = 3x_1^2 + 3x_3^2 + 4x_1x_2 + 4x_2x_3 + 8x_1x_3$$

and $Q(y) = a_1y_1^2 + a_2y_2^2 + a_3y_3^2$

f) Obtain the relationship between $x = (x_1, x_2, x_3)'$ and $(y = y_1, y_2, y_3)'$

10. i) A machine produces components of mean diameter 1.630 cm and standard deviation 0.004 cm. The diameters are assumed to be normally distributed.

a) If all components with diameters outside the range 1.620cm to 1.635 cm are rejected, what proportion of components are rejected.
b) Find the probability that if 4 components are selected at least three will be rejected.
c) If 30% of components have diameters less than x, find the value of x.

ii) The probability that a particular automobile parties defective is known to be 0.001. 3000 parts are required in the assembly of a car.

a) Use the binomial probability distribution to calculate the probability that there are no defectives. Now calculate the approximate probability using the Poisson distribution.

b) Calculate the probability of exactly 5 defectives. Try using both the binomial distribution and the Poisson approximation to obtain your answer.

11. i. A low-noise transistor for use in computing products is being developed. It is claimed that the mean noise level will be below the 2.5-dB level of products currently in use. (*Journal of Electronic Engineering,* March 1983, p.17)

a) Set up the appropriate null and alternative hypotheses for verifying the claim.
b) A sample of 16 transistor yields $\bar{x} = 1.8$ with $s = 0.8$. Find the P value for the test. Do you think that H_0 should be rejected? What assumption are you making concerning the distribution of the random variable X, the noise level of a transistor?
c) Explain, in the context of this problem, what conclusion can be drawn concerning the noise level of these transistors.

ii. As part of an industrial training program, some trainees are instructed by Method A which is straight teaching-machine instruction, and some are instructed by Method *B* which also involves the personal attention of an instructor. If random samples of size 10 are taken from large groups of trainees instructed by each of these two methods, and the scores which they obtained in an appropriate achievement test are:

Method A : 71 75 65 69 73 66 68 71 74 68
Method B : 72 77 84 78 69 70 77 73 65 75

Test the null hypothesis μ_1-μ_2=1.5 against the alternative hypothesis $\mu_1 - \mu_2 > 1.5$ at the level of significance α=0.05. Assume that the populations are normal and have the same variance.

12. Connectors used in computers are subject to simultaneous multidimensional stresses such as high temperatures and mechanical stresses. A study is conducted to identify and quantify interface stresses. Experiments are conducted to investigate the relationship between pitch and connector length. These data are obtained:

Connector length, x (inches)	Pitch, y (millimeters)
.150	3.81
.100	2.54
.098	2.50
.079	2.00
.050	1.27
.040	1.02
.039	1.00
.032	0.80
.020	0.50
.016	0.40
.010	0.25
.005	0.13

(Based on information from "Interaction of Multichip Module Substrates with High Density Connectors," Yakov Belopolsy, *IEEE MICRO*, April 1993, pp 36-43)

a) Plot the scatter diagram for these data on a graph paper.

b) Estimate the regression line.

c) Find the estimate of the error variance.

d) Test whether the slope parameter is zero.

e) Find 99.5% confidence internal for the slope parameter

and

f) From your regression equation, find the pitch for connector length 0.085.