THE OPEN UNIVERSITY OF SRI LANKA
BACHELOR OF TECHNOLOGY (level 05)
ECX5239
PHYSICAL ELECTRONICS
FINAL EXAMINATION 2006

043

DATE : 5th March 2007 **TIME : 9.30 – 12.30 hours**

Answer **ONE** question from SECTION **A** and any **FOUR** questions from **SECTION B.**

SECTION A:

Answer ONE question from this section.

1. With reference to the drift-diffusion model answer following questions:

(a) Due to thermal energy, the free electrons in a semiconductor possess kinetic energy. These free electrons behaves like molecules in a gas. Using the kinetic theory of Gases derive a formula for the thermal velocity of electrons.

(b) Derive equations for
   (i) drift current density
   (ii) diffusion current density

(c) Carrier mobility could be measured and carrier sign determined by the Hall experiment.
Derive the necessary equations which would enable you to find the carrier mobility using the measured values obtained in the Hall experiment.

2. With reference to the energy band model answer following questions:

(a) According to this model, is it possible for an electron in the semiconductor to have any amount of energy? Explain you answer in point form.

(b) Why are probability functions and statistical distributions considered in this model? Give your answer in point form

(c) Derive the equations which give the free electron/hole concentration in an intrinsic semiconductor.

SECTION B:

Answer FOUR questions

The pages 1 to 7 of the article 'Current-Voltage characteristics from an asymptotic. analysis of the MOSFET equations' by Ellis Cumberbatch, Henok Abebe and Hedley Morris is attached for your reference. Read these pages and answer following questions. [Please note the Appendix A mentioned in the article is not reproduced here. Assume that all constants, variables and equations carry their usual meaning]

3. Describe in your own words (in point form)
   (i) the problem considered
   (ii) the solution proposed
   in this article.

4.
(a) In the context of the given article why is it desirable to simulate the behavior of semiconductor devices by using a program like SPICE?

(b) List the constraints that need to be overcome by SPICE in order to simulate the particular MOSFET device.

5. Briefly explain the approximations which are necessary to change the set of non-linear PDEs for the electron density n, the hole density p and the electric field E into a system of ODEs.

6.
(a) According to your knowledge of MOSFETs draw the characteristics that will be accessed by the SPICE program for necessary calculations. What is the relevance of these graphs to the ideas discussed in this article?

(b) Write the equation which gives the current-voltage relationship for the particular MOSFET considered in 6(a).

7.
(a) State the physical meaning of equations (2), (3), (4), (5), (6) and (7)

(b) List the unknown variables that need to be found by solving equations mentioned in above 7(a).

8. To find solutions for the set of differential equations given by (2), (3), (4) and (5) it is necessary to specify boundary conditions. State the type of boundary value problem solved and list the boundary conditions.

Current-Voltage Characteristics from an Asymptotic Analysis of the MOSFET Equations

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Abstract

Phenomenological formulae for the current-voltage characteristics of a MOSFET are derived by extending the asymptotic method of Ward [12]. Practical methods for combining the perturbation approximations and numerically implementing the Ward equations are developed. A detailed comparison with real MOSFET data is presented and the model is shown to be effective over a range of device geometries.

1 Introduction

A semi-conductor chip is made by fabricating up to 10^6 transistors and their inter-connections on the surface of a silicon wafer. The transistors are usually MOSFET's (metal-oxide-silicon-field-effect-transistors), drawn from a small set of designs (n-type, p-type) and of various sizes (lengths and widths). Each chip has circuit functions, and in order to design the circuits using the transistor building blocks, the VLSI industry uses a program called SPICE. SPICE is the most common program for the simulation of electronic circuits. The original SPICE was developed in 1971 at U.C. Berkeley, and updated versions exist in many forms both commercial and Public Domain. See [5] for more information. For SPICE to produce a design the software must access the current-voltage relation for *each* transistor a number of times. In order that this be effected in a reasonable time the current-voltage relationships for the transistors must have simple form.

The classical approximation governing the flow of electrons and holes in a semi-conductor relevant to the low field-strength operation of a transistor has been in use for over forty years [10],[11]. It results in a set of non-linear partial differential equations for the electron density, n, the hole density, p, and the electric field $\mathbf{E}$. Even in the time-independent case, which will be considered here and which is sufficient for many applications, the non-linear character and the non-trivial geometry of the MOSFET preclude approaches that would generate exact solutions, least of all any that have simple current-voltage relationships. Hence there has resulted a variety of approximate solutions. A typical MOSFET geometry is shown in Figure 1..

The first approximation, which allows a reduction from a PDE system to an ODE system has the designation "long channel" or "quasi-one-dimensional" (called 1-D in the sequel). As indicated, this approximation has validity when the channel length, L, is relatively large. (See Figure 1 for the coordinate system, for identification of various regions on the device, etc. Also refer to Appendices 1, 2 for variable definitions). When voltages are applied at the gate and or drain, electrons and holes are attracted or repelled from these locations. The depletion region, bereft of either electrons or holes, plays an important role; a measure of its size is $L_d(\frac{\ln\lambda}{\lambda})^{\frac{1}{2}}$ where L_D is the Debye length and λ is a measure of the doping level relative to the intrinsic level. (λ is large, typically $10^6 - 10^7$.) The ratio of these lengths, which plays the role of the aspect ratio of the device, is

$$\epsilon = \frac{L_d}{L}\sqrt{\frac{\ln\lambda}{\lambda}} \tag{1}$$

When ϵ is small most of the region under the gate is uniform in the x_2 -direction and the $1-D$ approximation is exact for the fields there; the approximation breaks down in the source-gate and drain-gate corners where the fields lose their $1-D$ character. When ϵ is large the corner regions influence the solution across the whole gate region and a two-dimensional solution is required. Ward, [12], made the $1-D$ approximation into an explicit asymptotic expansion by scaling x_2, the variable along the channel with L and scaling x_1, the perpendicular variable, with L_D. Series expansions in powers of ϵ^2 give the $1-D$ approximation as the leading term of these series. In a subsequent paper, [13], Ward obtained some solutions for the corner regions and matched them to the $1-D$ approximation in the central region under the gate. This enhanced solution would be expected to be more accurate down to smaller channel lengths, but the solution obtained is restricted to small drain voltages. For a typical device $L_d = 33$ microns, $\lambda = 10^6$, and for $L = 10$ microns, $\epsilon = 0.012$. For ϵ this small we expect the $1-D$ approximation to be accurate; this is borne out by the data. However we show that by adjustments of some of the parameters we can make the $1-D$ approximation match the data at much reduced chan-

nel lengths. At the time of writimg, the standard industry channel length is 0.25 micron, and the SPICE software, based on the 1-D approximation (but improved impirically), is still being used.

The $1-D$ approximation has the descriptor "quasi" because the ODE in the x_1 direction contains x_2 as a parameter via the quasi-Fermi potential, φ. The electric potential, ψ, satisfies the second-order non-linear ODE in x_1, which contains φ, and therefore its solution depends on x_2 parametrically. The form of $\varphi(x_2)$ is determined subsequently when the current flowing across the device is calculated. The ODE allows one integration in the case of constant doping. The current can then be expressed as a double integral called the Pao-Sah integral, see [6]. An approximation allows reduction to a single integral [7]. Further approximations valid over different regimes of the applied voltages V_{ds}, V_{gs} then yield explicit formulae for $I_{ds}(V_{ds}, V_{gs})$. These latter approximations are motivated by neglect of various physical effects in one or other regimes of operation. A comprehensive survey of these approximations and their inter-relations is found in [11] and [4]. These separate formulae, valid over distinct ranges of voltages, were found to be accurate for long channel use. As the need for denser packing of transistors on chips has intensified, and the technology has been able to satisfy those needs by reducing lengths and widths of transistors, these formulae have been adapted in various ways for use at the reduced sizes. Often these adaptations are little more than "fudge factors," which have parameters valid only over limited ranges of size and voltage regimes. Hence these parameters are changed regime to regime ("binning"). Current technology uses of the order of 150-200 parameters for $I_{ds}(V_{ds}, V_{gs})$ formulae for use in SPICE. Most of these parameters are not universal constants, and they must be "identified" for each fabrication batch. This means considerable testing and data collection, followed by numerical optimization for parameter identification. All of this is expensive, both in time and money. There are pressures to improve the models so that they are less cumbersome and based more on physics. The work reported here, done by the Claremont Graduate University Mathematics Clinic for the Information Sciences Institute MOSIS Program, has these more efficient models as its goal. Related goals were (a) comparison of the asymptotic results with data to determine the domain of validity of the asymptotic approximations, and (b) improvement of solution "blending" by which solutions valid over different voltage regimes are patched together.

The work presented in here generates explicit $I_{ds}(V_{ds}, V_{gs})$ formulae based on the $1-D$ approximation. It takes further the asymptotic results for large λ derived by Ward in (Ward 1990, 1992). These results in turn were based on the asymptotic scheme originated by Please, [8], for the $p-n$ junction. This scheme, based on the matched asymptotic expension concept, [1], obtains solutions valid in different regions and matches them across overlapping regions to provide a composite expansion valid over the whole region (to the order com-

puted). In the $1-D$ MOSFET approximation there are three main regions: the inversion layer, a thin boundary layer containing a profusion of holes or electrons under some voltage regimes, the depletion layer, and the substrate which is determined by conditions far from the channel. The separate solutions in these regions are, by themselves, not difficult to obtain. However, matching them, and satisfying the boundary condition at the gate, are non-trivial exercises. Ward successfully accomplished these, in [12] for constant doping and for variable doping of constant signature, and in [13] for variable doping of opposite signature.

The asymptotic solution achieved by Ward requires numerical work (to solve non-linear transcendental equations) and the $I_{ds}(V_{ds}, V_{gs})$ formula remains an integral to be computed numerically. Hence its use in the semi-conductor industry (via SPICE) is precluded. We briefly outline Ward's asymptotic results, and in Section 3 we show how the Ward analysis may be taken further (in the constant doping case) to achieve an *explicit* $I_{DS}(V_{ds}, V_{gs})$ formula. Part of our analysis is justifiable as a continuation of the asymptotics and it agrees with the numerical solution of the exact ODE/BC system. A second approximation is not justifiable asymptotically. It approximates by a constant a function that is varying inside an integral; with this, though, the integral can be evaluated analytically, and the two approximations combined to produces a formula for the current.

2 Equations, Boundary Conditions, and the Quasi-1-D Asymptotic Approximation

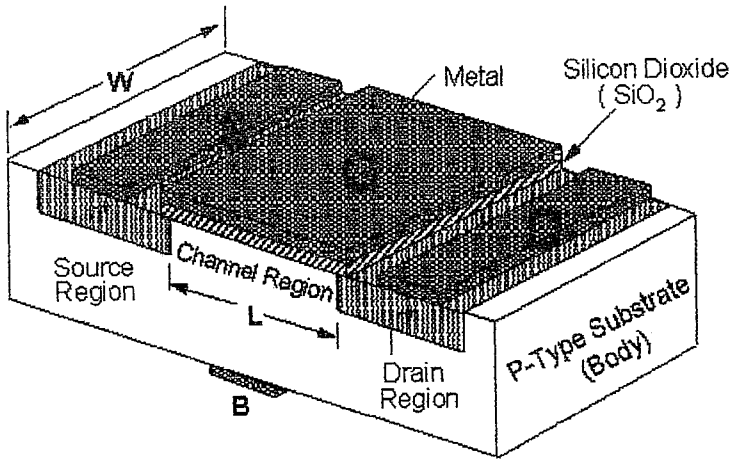


Fig 1. A schematic diagram of a MOSFET.

Voltages may be applied at the source, drain, gate and substrate contacts (S,D,G,B in Figure 1); we shall assume that the substrate and source are at the same voltage. The voltages drain-to-source and gate-to-source are denoted by V_{ds}, V_{gs}, respectively. We shall treat an n-channel device (in inversion the channel is profuse in electrons). Here the silicon has been doped with acceptor atoms (typically the number ionized is $N_A^- \sim 10^{16}-10^{17}$). Source, drain regions have been overlaid with donor atoms ($N_D^+ \sim 10^{19}$). An insulated layer (silicon dioxide) separates the gate contact from the silicon. As the gate voltage is raised, holes are repelled from the region under the gate, creating a depleted region. Further increase in V_{gs} attracts electrons to the insulator-substrate interface, creating the "channel". Voltages at the drain then cause a current to flow from source to drain. The aim of analysis of the device is to obtain the current created by this flow of electrons, I_{ds}, as a function of the applied voltages V_{ds} and V_{gs}. In addition, I_{ds} also depends on the parameters of the device: its size, the doping levels, the mobility of electrons, etc.

The standard model for the MOSFET comprises Gauss' equation for charge

$$\epsilon_s \nabla \cdot \mathbf{E} = -\rho = q(p - n + N) \tag{2}$$

together with the drift-diffusion model for the motion of electrons

$$\mathbf{J}_n = q\mu_n\left(\frac{kT}{q}\nabla n + n\mathbf{E}\right) \tag{3}$$

and the static conservation equation with no recombination/generation for the flux of electrons

$$\nabla \cdot \mathbf{J}_n = 0. \tag{4}$$

The assumption that the holes remain in thermal equilibrium allows the hole density to be obtained as

$$p = n_i e^{-q\frac{\psi}{kT}} \tag{5}$$

It is convenient to write

$$n = n_i e^{q\frac{\psi - \varphi_n}{kT}} \tag{6}$$

with φ_n replacing n as a primary variable; it is called the quasi-Fermi potential. For other notation and definitions see Appendix A.

The boundary conditions comprise

(i) specification of the electric potential at the gate contact:

$$\psi = V_{gs} - V_{fb} \text{ at } \mathrm{x_1} = -\mathrm{t} \tag{7}$$

where V_{fb} is the "flat-band voltage" which is a voltage that results from terminal contacts with the substrate [10].

(ii) continuity of electrical potential and electric displacement at the oxide-substrate interface:

$$\psi^- = \psi^+ \text{ and } \epsilon_{\mathrm{i}} \frac{\partial \psi^-}{\partial \mathrm{x_1}} = \epsilon_{\mathrm{s}} \frac{\partial \psi^+}{\partial \mathrm{x_1}} \text{at } \mathrm{x_1} = 0 \tag{8}$$

(iii) specification of the electric and quasi-Fermi potentials at the source-substrate and drain-substrate boundaries. These are given by Ward, [12], and will not be written out here. They are required for the specification of ψ, φ_n at these boundaries, and they will be introduced when needed subsequently.

(iv) current flow occurs only across the source-substrate and drain-substrate boundaries. A consequence of this is that that the total current flowing across a plane $x_2 = \text{const}$ does not vary with x_2 for $0 \leq x_2 \leq L$.

The independent and dependent variables are scaled, see Appendix B. The differential scaling of the independent variables introduces the parameter ϵ, equation (1), into the field equations (2) and (4). The quasi-one-dimensional approximation is obtained for the solution in the mid-channel region (away from the source and drain corners) by the asymptotic expansion for the scaled electric and quasi-Fermi potentials

$$w(x,y) = w^0(x,y) + \varepsilon^2 w^1(x,y) + ... \tag{9}$$

$$\varphi(x,y) = \varphi^0(x,y) + \varepsilon^2 \varphi^1(x,y) + ... \tag{10}$$

The electron mobility in (3) is not necessarily constant: it may vary spatially and with field strength. The Ward assumption is

$$\mu_n = \mu_n(x, \frac{\partial \varphi^0}{\partial y}) > 0 \tag{11}$$

To leading order

$$\frac{\partial^2 w^0}{\partial x^2} = \frac{1}{\lambda}(\exp((w^0 - \varphi^0)\ln(\lambda)) - \exp(-w^0 \ln(\lambda)) + d(x) \tag{12}$$

$$\frac{\partial^2 \varphi^0}{\partial x^2} = -(\frac{1}{\mu_n}\frac{\partial \mu_n}{\partial x} + \ln(\lambda)(\frac{\partial w^0}{\partial x} - \frac{\partial \varphi^0}{\partial x}))\frac{\partial \varphi^0}{\partial x} \tag{13}$$

The equation for φ^0 can be integrated, and with the no-flux boundary condition it follows that

$$\frac{\partial \varphi^0}{\partial x} = 0 \text{ and } \varphi^0 = \varphi(\mathrm{y}) \tag{14}$$

where we have dropped the zero superscript for convenience.

The order ϵ^2 terms give for φ^1

$$\frac{\partial^2 \varphi^1}{\partial x^2} + (\frac{1}{\mu_n}\frac{\partial \mu_n}{\partial x} + \ln(\lambda)\frac{\partial w^0}{\partial x})\frac{\partial \varphi^1}{\partial x} = -(\varphi'' + (\frac{1}{\mu_n}\frac{\partial \mu_n}{\partial y} + \ln(\lambda)(\frac{\partial w^0}{\partial y} - \varphi'))\varphi') \tag{15}$$

where primes represent derivatives with respect to y. By means of an integrating factor

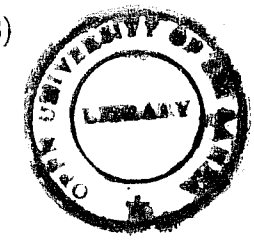
$$\frac{\partial}{\partial x}(\mu_n \exp(\ln(\lambda)w^0)\frac{\partial \varphi^1}{\partial x}) = -\mu_n \exp(\ln(\lambda)w^0)(\varphi'' + (\frac{1}{\mu_n}\frac{\partial \mu_n}{\partial y} + \ln(\lambda)(\frac{\partial w^0}{\partial y} - \varphi'))\varphi') \tag{16}$$

If we integrate each side of (16) from $x = x^*$ (x^* is a point in the substrate where there is no current flux across $x = x^*$) we obtain for the left-hand side

$$[(\mu_n \exp(\ln(\lambda)w^0)\frac{\partial \varphi^1}{\partial x})]_0^{x^*} = 0 \tag{17}$$

as $\frac{\partial \varphi^1}{\partial x} = 0$ when $x = 0$ and $x = x^*$. It follows that we must have

$$\varphi'' + \frac{\partial}{\partial y}\ln(\int_0^{x^*} \mu_n \exp(\ln(\lambda)(w^0 - \varphi))dx)\varphi' = 0 \tag{18}$$



The notation

$$\sigma_m = \int_0^{x^*} \mu_n \exp(\ln(\lambda)(w^0 - \varphi))dx \tag{19}$$

allows the equation determining the y dependence of φ to be integrated once to give

$$\varphi' \sigma_m \ln(\lambda) = C \tag{20}$$

$e = 1.6 \times 10^{-19}$ C

$m_e = 9.108 \times 10^{-31}$ kg

$h = 6.625 \times 10^{-34}$ Js

$c = 3 \times 10^{8}$ ms^{-2}

$k = 1.38 \times 10^{-23}$ J/k

1 eV $= 1.6 \times 10^{-19}$ J

$\varepsilon o = 8.854 \times 10^{-12}$ F/m

$P_n = nh/2\pi r$ $\quad \lambda = h/p$ $\quad \Delta x.\Delta p \geq h$ $\quad \Delta E.\Delta t \geq h$

$I = N_- n_- e + N_+ n_+ e$

$R = \rho.l/A$ $\quad \rho = 1/\sigma$ $\quad J = \sigma E$

$\sqrt{\overline{c^2}} = \sqrt{(3KT/m_e)}$ $\quad J = -ne\overline{v}$

$\overline{v} = e\tau E/m_e$

$R_H = -(1/ne) = (1/pe)$

$p_n = n_i^2/N_D$ $\quad n_p = n_i^2/N_A$

$\mu_e = e\tau/m_e$ $\quad \mu_h = e\tau/m_h$

$\lambda = \tau\sqrt{\overline{c^2}}$ $\quad D = (\lambda/3).\sqrt{\overline{c^2}}$

$D_e/\mu_e = KT/e$ $\quad E = hc/\lambda$

$\sigma = \mu_e ne = \mu_h pe$

$J_e = ne\mu_e E + eD_e\,(dn/dx)$

$J_h = pe\mu_h E - eD_h\,(dp/dx)$

$n = N_c \exp -(E_c - E_F)/KT$

$p = N_v \exp -(E_F - E_c)/KT$

$n_i = \sqrt{N_c N_v} \exp -(Eg/2KT)$

$V_0 = (KT/e).\ln(N_D N_A/n_i^2)$

$V_0 - V = (D_e/\mu_e).\ln(n_n/n_p)$

$I = I_s\,[\exp(eV/KT) - 1]$

$I_s = A\,[(D_e e n_p/L_p) + (D_h e p_n/L_n)]$

$W_n^2 = W_{n0}^2\,(1 - V/V_0)$ $\quad W = W_n + W_p$

$W_{n0}^2 = (2\varepsilon V_0/e).[N_A/(N_A N_D + N_D^2)]$

$W_{p0}^2 = (2\varepsilon V_0/e).[N_D/(N_A N_D + N_A^2)]$

10