MPZ 4230 - Assignment No. 02 – Academic Year 2008

01. (a). The number of defectives parts per day follows a Poisson probability distribution with a mean of 3(
Determine the probability that on a given day.
(i). Four defects occur
(ii). Two defect occur
(iii). Five or more defects occur

(b). A professional football term has found that 30% of the new players that it sings out of college actually play for more than one year. During a recent year 20 new players were signed to contract
Determine.
(i). The mean numbers of players who actually play for more then one year.
(ii). The variance of the number of players who play for more than one year.

(c). The product density for cereal, before sugar coating, in the Flowering Sun Cereal company is 650 grams per liter with a variance of 400. Determine the probability that the density is, (using normal distribution).
(i). Less than 600 (ii). Greater that 685 (iii). Between 615 and 690

02. The data shown below are the average wine consumption rates (in liters per person) and the number of ischemic heart disease death (per 1000 men aged 55 to 64 years old) for 18 industrialized countries.

Country	Wine Consumption	Heart disease Mortality
A	2.8	6.2
B	3.2	9,0
C	3.2	7.1
D	3.4	6.8
E	4.3	10.2
F	4.9	7.8
G	5.1	9.3
H	5.2	5.9
I	5.9	8.9
J	5.9	5.5
K	6.6	7.1
L	8.3	9.1
M	12.6	5.1
N	15.1	4.7
O	25.1	4.7
p	33.1	3.1
Q	75.9	3.2
R	75.9	2.1

Use the *ln* (mortality) vs *ln* (wine) and answer the questions.
(a). Define the simple linear regression model. Estimate the model.
(b). Comment on the model Assumptions by referring to the normality probability plot and the residu plot.
(c). Is a simple linear regression model useful? In other words, does average *ln* (mortality) depend linear on *ln* (wine)
(d). What percent of variation in *ln* (Heart disease mortality) is explained by *ln* (wine consumption).
(e). Estimate the effect of *ln* (wine) on average *ln* (mortality) with 95% confidence and interpret inte on the original scale.

03. (a). a pilot study was conducted to determine whether the diets with high fiber are successful in lower blood cholesterol. Five subjects randomly chosen from a group of 10, were prescribed high fiber. The remainders were prescribed low fiber diets. The cholesterol levels (in mg/dl) were measured three months. The data as follows.

High Fiber	Low Fiber
311	334
325	327
299	343
313	325
332	361

$\overline{X}_{High} = 316$ $\quad S^2_{High} = 165$

$\overline{X}_{low} = 338$ $\quad S^2_{Low} = 215$

(i). Is there evidence in this data that the diets with high fiber reduce blood cholesterol (use $\alpha = 0.05$) your reasoning.

(ii). Set and interpret a 95% confidence interval on the mean difference in blood cholesterol between t types of diets.

(iii). Suggest ways in watch the study could be modified to give more conclusive results.

04. Solve $\dfrac{dy}{dx} = z - x$

$\dfrac{dz}{dx} = y + x$

With $y(0) = 1$, $z(0) = 1$ to get $y(0.1)$, $z(0.1)$ & $z(0.2)$

Using

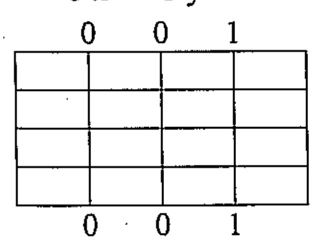
(a). Euler method

(b). Taylor's method

(c). Fourth order Runge-Kutta method.

05. (a). State the classification for a partial differential equation into elliptic, parabolic & hyperbolic

(b). Solve $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = 0$ with boundary values as shown in the figure give below.



MPZ 4230 – Model Answer 02
Academic Year 2008

(01). Let x denotes the number of defectives with mean 3.6

(i). $p(x=4) = \dfrac{e^{-\mu}\,\mu^{x}}{x!}$

$= \dfrac{e^{-3.6} \times 3.6^{4}}{4!}$

$= 0.1912$

(ii). $p(x=2) = \dfrac{e^{-3.6} \times 3.6^{2}}{2!}$

$= 0.1770$

(iii). $p(x \geq 5) = 1 - [p\ (x=1) + p\ (x=2) + p\ (x=3) + p\ (x=4)\ ]$

$= 1 - [e^{-3.6} \times 3.6 + \dfrac{e^{-3.6} \times 3.6^{2}}{2!} + \dfrac{e^{-3.6}\,3.6^{3}}{3!} + \dfrac{e^{-3.6}\,3.6^{4}}{4!}]$

$= 0.3562$

(b). Let x denotes the number of players actually play for more than one year in sample size 20 by assuming binomial distribution.

(i). $\mu = n\pi$

$= 20 \times 0.3$

$= 6$

(ii). $\sigma^{2} = n\,\pi\,(1\text{-}\pi)$

$= 20 \times 0.3 \times 0.7$

$= 4.2$

(c). Let x denotes the density for serial with mean 650 and variance of 400

(i). $p(x < 600) = p\left(\dfrac{x-\mu}{\sigma} < \dfrac{600-650}{20}\right)$

$= p\ (z < -2.5)$

$= p\ (z > 2.5)$

$= 1 - 0.9938$

$= 0.0062$

(ii). $p(x > 685) = p\left(\dfrac{x-\mu}{\sigma} > \dfrac{685-650}{20}\right)$

$= p\ (z > 1.75)$

$= 1 - 0.9599$

$= 0.0401$

(iii). $p(615 < x < 690) = p(x < 690) - p(x < 615)$

$$= p\left(\frac{x-\mu}{\sigma} < \frac{690-650}{20}\right) - p\left(\frac{x-\mu}{\sigma} < \frac{615-650}{20}\right)$$

$= p(z < 2) - p(z < -1.75)$

$= p(z < 2) - p(z > 1.75)$

$= 0.9772 - [1 - p(z < 1.75)]$

$= 0.9772 - (1 - 0.9599)$

$= 0.9772 - 0.0401$

$= 0.9371$

2)

Country	wine consumption (x)	X=ln x	Heart disease mortality (y)	Y=ln y	XY	X^2	Y^2
A	2.8	1.030	6.2	1.825	1.879	1.060	3.329
B	3.2	1.163	9	2.197	2.556	1.353	4.828
C	3.2	1.163	7.1	1.960	2.280	1.353	3.842
D	3.4	1.224	6.8	1.917	2.346	1.498	3.675
E	4.3	1.459	10.2	2.322	3.387	2.128	5.393
F	4.9	1.589	7.8	2.054	3.264	2.526	4.219
G	5.1	1.629	9.3	2.230	3.633	2.654	4.973
H	5.2	1.649	5.9	1.775	2.926	2.718	3.150
I	5.9	1.775	8.9	2.186	3.880	3.150	4.779
J	5.9	1.775	5.5	1.705	3.026	3.150	2.906
K	6.6	1.887	7.1	1.960	3.699	3.561	3.842
L	8.3	2.116	9.1	2.208	4.673	4.479	4.876
M	12.6	2.534	5.1	1.629	4.128	6.420	2.654
N	15.1	2.715	4.7	1.548	4.201	7.370	2.395
O	25.1	3.223	4.7	1.548	4.988	10.387	2.395
P	33.1	3.500	3.1	1.131	3.959	12.247	1.280
Q	75.9	4.329	3.2	1.163	5.036	18.744	1.353
R	75.9	4.329	2.1	0.742	3.212	18.744	0.550
Σ		39.088		32.100	63.074	103.540	60.441

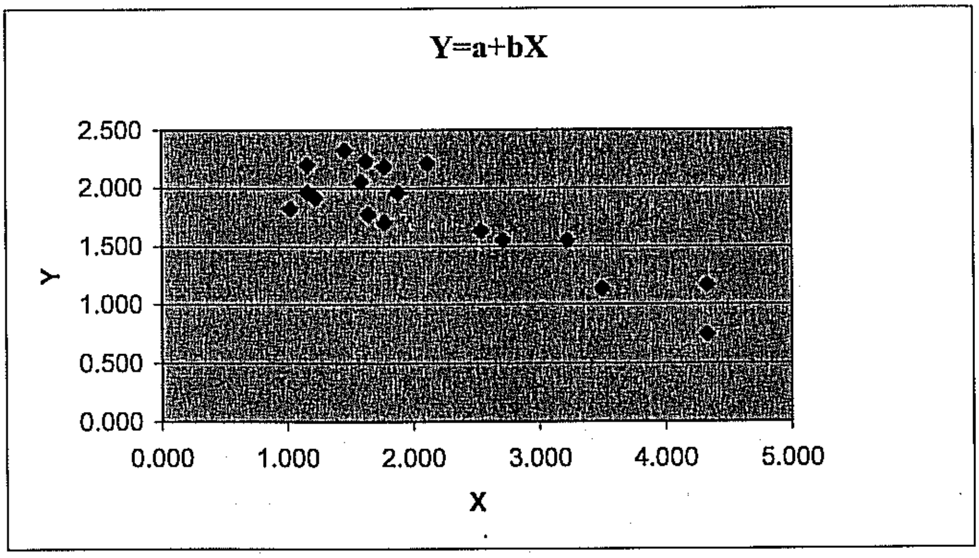
Regression Equation is $Y = a + bX$

$$b = \frac{n\,\Sigma XY - \Sigma X\,\Sigma Y}{n\,\Sigma X^2 - (\Sigma X)^2} \qquad a = \frac{\Sigma Y - b\Sigma X}{n}$$

$$= \frac{1135.326 - 1254.746}{1863.725 - 1527.895} \qquad = \frac{32.100 + 13.9}{18}$$

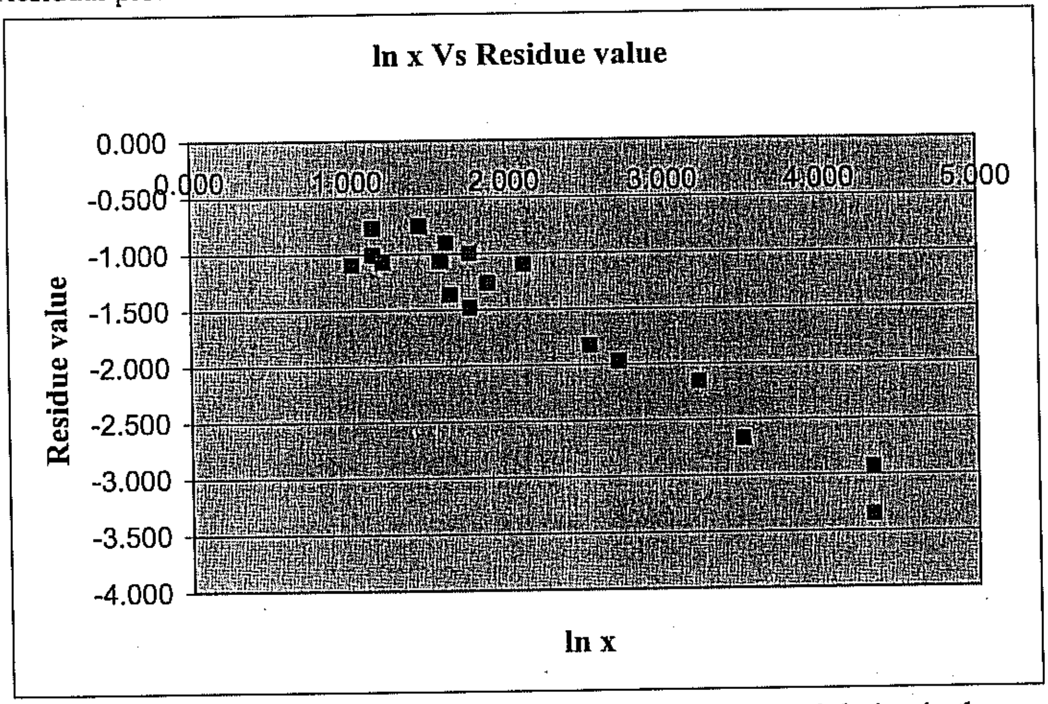
$= -0.356$ $\qquad = 2.556$

ln (mort) $= 2.556 - 0.356\ ln$ (wine) (b). Normal probability plot



Country	Wine consumption (x)	X=ln x	Y=ln y	Y=a+bX	Residue value
A	2.8	1.030	1.825	2.923	-1.098
B	3.2	1.163	2.197	2.970	-0.773
C	3.2	1.163	1.960	2.970	-1.010
D	3.4	1.224	1.917	2.992	-1.075
E	4.3	1.459	2.322	3.075	-0.753
F	4.9	1.589	2.054	3.122	-1.068
G	5.1	1.629	2.230	3.136	-0.906
H	5.2	1.649	1.775	3.143	-1.368
I	5.9	1.775	2.186	3.188	-1.002
J	5.9	1.775	1.705	3.188	-1.483
K	6.6	1.887	1.960	3.228	-1.268
L	8.3	2.116	2.208	3.309	-1.101
M	12.6	2.534	1.629	3.458	-1.829
N	15.1	2.715	1.548	3.522	-1.975
O	25.1	3.223	1.548	3.703	-2.156
P	33.1	3.500	1.131	3.802	-2.670
Q	75.9	4.329	1.163	4.097	-2.934
R	75.9	4.329	0.742	4.097	-3.355
∑		39.088	32.100	16.471	15.629

Residual plot



From normal probability plot there does not appear to be any violation in the normality assumption. From the residual plot there doesn't appear to be any serve violation in the equal variability assumption.

(c). $H_0: b = 0$

$H_a: b_1 \neq 0$

$$t_0 = \frac{b_1}{s_b} = \frac{-0.356}{0.053} = -6.721$$

p-value $= 2p\ (t_{16} > -6.721) \approx 0.0000$

With a p-value roughly 0.0000 there is very strong evidence to indicate that average of *ln*(mortality) does depend linearly on *ln* (wine consumption)

$$s_b = \frac{s_{Y/X}}{\sqrt{\Sigma X^2 - \frac{(\Sigma X)^2}{n}}}$$

$=0.053$

$$s_{Y/X} = \sqrt{\frac{(\Sigma Y^2 - a\Sigma Y - b\Sigma XY)}{n-2}}$$

$$= \frac{\sqrt{60.44 - 82.034 + 22.429}}{18.2}$$

$= 0.229$

(d). $r = \dfrac{n\Sigma XY - (\Sigma X)(\Sigma Y)}{\sqrt{(n\Sigma X^2 - (\Sigma X)^2)\ (n\Sigma Y^2 - (\Sigma Y)^2)}}$

$r^2 = 0.738$

73.8% of variation in *ln* (Heart disease mortality) is explained by *ln* (wine consumption)

(e). $S_b = 0.053$

Confidence interval $= b \pm t\, s_b$

$= -0.356 \pm 2.120\ (0.053)$

$= (-0.46836, -0.24364)$

On the original scale, we can estimate the multiplicative change in wine consumption. Specifically, it is estimated with 95% confidence that a multiplicative change in wine consumption by a factor of k is associated with a multiplicative change in the median mortality between $(k^{-0.46836}, k^{-0.24364})$.

For example it is estimated with 95% confidence that a doubling in wine consumption is associated with a multiplicative change in median mortality between $(2^{-0.46836}, 2^{-0.24364}) \Rightarrow$ (0.723, 0.845). This is a reduction in mortality between 15.5% and 27.7%.

(03)

(a).

(i). This is the two – sample test and data sets are independence from each other. Therefore we use pooled comparison test

Also the sample size is less than 30. ie it is a small sample. Therefore the t – test was used

Let μ_1, μ_2 are the means of the blood cholesterol of high fiber diets and low fiber diets respectively.

Test the null hypothesis is $H_0 : \mu_1 < \mu_2$ against the alternative hypothesis is

$H_0 : \mu_1 < \mu_2$

ie. high fiber cause to low blood cholesterol.

$H_0 : \mu_1 < \mu_2$

$H_a : \mu_1 < \mu_2$

t = test statistic

$$t_{cal} = \frac{(\bar{x}_1 - \bar{x}_2)}{S_p\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}}$$

$$t_{cal} = \frac{\bar{x}_1 - \bar{x}_2}{S_P\sqrt{2/n}} (\because\ n_1 = n_2 = n)$$

$$S_p = \sqrt{\frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2}}$$
$$= \sqrt{\frac{s_1^2+s_2^2}{2}} \quad (\because\ n_1 = n_2)$$

$$S_p = \sqrt{\frac{165+215}{2}} = \sqrt{\frac{380}{2}} = \sqrt{190} = 13.784$$
$$t_{cal} = \frac{316-338}{13.784\sqrt{2/5}} = -2.523$$

The table value $t_{tab} = t_{\alpha,\ n1+n2\ -2}$ $(\because$ one tail$)$
$= t_{0.05,\,8}$
$= 1.86$

-2.523 < -1.86
$\Rightarrow t_{cal} < -t_{tab}$
$\Rightarrow H_0$ is rejected
$\Rightarrow H_a$ is Accepted
$\Rightarrow \mu_1 < \mu_2$

High fiber reduce blood cholesterol

(ii).

confidence inteval on the mean difference in blood cholesrol $\} = (\bar{x}_1 - \bar{x}_2) \pm t_{\alpha, n_1+n_2-2} = S_p\sqrt{\frac{1}{n}}$

$$95\%\ CI = (316-338) \pm 1.86 \times 13.784\sqrt{\frac{2}{5}}$$
$= -22 \pm 16.215$
$= [-3.8215, -5.785]$

(4). Given that

$$\frac{dy}{dx} = z - x \quad \text{------------------ (1)}$$

Integration (1)

$$y = zx - \frac{x^2}{2} + c_1$$

when $x = 0$, $y = 1$ & $z = 1$

$c_1 = 1$

Then y becomes $y = zx - \frac{x^2}{z} + 1$

$$\frac{dz}{dx} = zx - \frac{x^2}{2} + 1 + x \quad \text{-------------------- (A)}$$

Given that

$$\frac{dz}{dx} = y + x$$

Integrating (2)

$$z = xy + \frac{x^2}{2} + c_2$$

when $x = 0$, $y = 1$ & $z = 1$

Then $c_2 = 1$

Then z becomes $z = xy + \frac{x^2}{2} + 1$

$$\frac{dy}{dx} = xy + \frac{x^2}{2} + 1 - x \quad \text{------------- (B)}$$

(a). Euler method

$$y_{k+1} = y_k + hf(x_k, y_k)$$

$$y_1 = y_0 + h(x_0 y_0 + \frac{x_0^2}{2} - x + 1)$$

$$y_1 = y(0.1) = 1.1$$

$$z_{k+1} = z_k + hf(z_k x_k + \frac{x_k^2}{2} - x_k + 1)$$

$$z_1 = z_0 + h(x_0 z_0 + \frac{x_0^2}{2} - x_0 + 1)$$

$z_2 = z_1 + h(x_1 y_1 + \frac{x_1^2}{2} - x_1 + 1)$

$z_1 = z(0.1) = 1.1$

$z_2 = z(0.2) = 1.2205$

(b). Taylors ,method

$$y_{n+1} = y_n + hy'_n + \frac{h^2 y''_n}{2!} + \frac{h^3 y'''_n}{3!} + ...$$

$y' = xy + \frac{x^2}{2} - x + 1$

$y'' = x y' + y + x - 1$

$y''' = xy'' + y' + y' + 1 \qquad = xy'' + 2y' + 1$

$y^{iv} = xy''' + y'' + y'' + y'' \qquad = xy''' + 3y''$

When $x_0 = 0$ and $y_0 = 1$

$y'_0 = 1 \qquad y''_0 = 0 \qquad y'''_0 = 3 \qquad y^{iv}_0 = 0$

$y_1 = y(0.1) = 1.1005$

$$z_{n+1} = z_n + h\ z'_n + \frac{h^2 z''_n}{21} + \frac{h^3 z'''_n}{3!} + ...$$

$z' = zx - \frac{x^2}{2} + x + 1$

$z'' = z + x z' - \frac{2x}{2} + 1$

$z''' = z' + x z'' + z' - 1 \qquad = x z'' + 2 z' - 1$

$z^{iv} = z'' + x z''' + z'' + z'' \qquad = x z''' + 3 z''$

When $x_0 = 0 \quad z_0 = 1$

$z_0' = 1 \qquad z_0'' = 2$

$z_0''' = 1 \qquad z_0^{iv} = 6$

$z_1 = z(0.1) = 1.110192$

when $x_1 = 0.1$ & $z_1 = 1.110192$

$z_1' = 1.206019 \qquad z_1'' = 2.130794$

$z_1''' = 1.625118 \qquad z_1^{iv} = 6.554893$

$z_2 = z(0.2) = 0.117411$

c) R – K method

$$k_1 = hf(x_0, y_0)$$

$$k_2 = hf(x_0 + \frac{1}{2}h, y_0 + \frac{1}{2} k_1)$$

$$k_3 = hf(x_0 + \frac{1}{2}h, y_0 + \frac{1}{2}k_2)$$

$$k_4 = hf(x_0 + h, y_0 + k_3)$$

$$y_1 = y_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$k_1 = 0.1 \qquad k_2 = 0.100375$

$k_3 = 0.100376 \qquad k_4 = 0.091504$

$y_1 = y(0.1) = 1.098834$

$k_1 = hf(x_0, z_0)$

$k_2 = hg(x_0 + ½ h, z_0 \; ½ k_1)$

$k_3 = hf(x_0 + ½ h,\ z_0 + ½ k_2)$

$k_4 = hf(x_0 + h_1,\ z_0 + k_3)$

$$z_1 = z_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$k_1 = 0.1$ $k_2 = 0.110125$

$k_3 = 0.11015$ $k_4 = 0.110602$

$z_1 = z(0.1) = 1.108525$

$k_1 = hf(x_1, z_1)$

$k_2 = hf(x_1 + ½ h,\ z_1 + ½ k_1)$

$k_3 = hf(x_1 + ½ h,\ z_1 + ½ k_2)$

$k_4 = hf(x_1 + h,\ z_1 + k_3)$

$$z_2 = z_1 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$k_1 = 0.120585$ $k_2 = 0.131407$

$k_3 = 0.131407$ $k_4 = 0.12063$

$z_2 = z(0.2) = 1.23636$

(5). $\frac{\partial^2 u}{\partial x^2} = \frac{u_{i+1,j} + u_{i-1,j} - 2u_{i,j}}{h^2}$

$$\frac{\partial^2 u}{\partial y^2} = \frac{u_{i,j+1} + u_{i,j-1} - 2u_{i,j}}{k^2}$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{u_{i+1,j} + u_{i-1,j} - 2u_{i,j}}{h^2} + \frac{u_{i,j+1} + u_{i,j-1} - 2u_{i,j}}{k^2}$$

We assume $h = k$ then

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = u_{i+1,j} + u_{i-1,j} + u_{i,j+1} + u_{i,j-1} - 4u_{i,j} = 0$$

Consider following points;

u_1; $u_4 + u_2 - 4u_1 = 0$

u_2; $u_1 + u_5 + u_3 - 4u_2 = 0$

u_3 ; $u_2 + u_6 + 2 + 1 - 4u_3 = 0$

u_4; $u_1 + u_5 + u_7 - 4u_4 = 0$

u_5; $u_2 + u_4 + u_6 + u_8 - 4u_5 = 0$

u_6 ; $u_3 + u_5 + u_9 - 4u_6 + 2 = 0$

u_7 ; $u_4 + u_3 - 4u_7 = 0$

u_8 ; $u_5 + u_7 + u_9 - 4u_8 = 0$

u_9 ; $u_6 + u_8 + (1 + 2) - 4u_9 = 0$

Solving $eq^{\underline{n}}$ we can get

$u_1 = u_7 = 0.188$	$u_2 = u_8 = 0.5$	$u_3 = u_9 = 1.188$
$u_4 = 0.25$	$u_5 = 0.625$	$u_6 = 1.25$