The Open University of Sri Lanka
Department of Electrical and Computer Engineering
Final Examination 2007/2008
ECX5233 – Radio and Line Communications

Time: 0930 – 1230 hrs. Date: 2008-04 -26

Answer any FIVE questions

1.
(a) $\delta(t-T)=0$ is defined as $\delta(t-T)=0\ ,\ t\neq T$

$$\int_{-\infty}^{\infty}\delta(t-T)\,dt=0$$

$f(t)=\sum_{n=-\infty}^{\infty}\delta(t-nT)$ represents a transmitting signal.

(i) Sketch $f(t)$.

(ii) If $f(t)=\sum_{n=-\infty}^{\infty}\delta(t-nT)=\sum_{n=0}^{\infty}A_n\cos n\omega_0 t+B_n\sin n\omega_0 t$, find the values of ω_0, A_n and B_n.

(b) A communication medium has low pass characteristics given by the frequency response

$$H(\omega)=e^{-\left(\frac{\omega}{\omega_B}\right)^2}\ ,|\omega|\leq 2\omega_B$$
$$=0\ \text{ otherwise}$$

(i) Sketch $H(\omega)$.
(ii) If $f(t)$ given in (a) is transmitted over the above communication medium, sketch the frequency spectrum of the received signal.
(iii) Calculate the power of the received signal. Assume that $\omega_B=4\omega_0$.

(c) If $f(t)$ is transmitted over a medium whose impulse response is given by

$$g(t)=1,\ |t|\leq\frac{T}{4}$$
$$=0\ \text{ otherwise,}$$

calculate and sketch the time signal at the receiving end.

2.

A phase modulated signal is given by $x(t) = A(\cos\omega_c t + \beta\sin\omega_m t)$

If narrow band modulation ($(\beta \langle\langle 1)$ is used,

(a) find the frequency spectrum of $x(t)$.
(b) sketch the frequency spectrum of $x(t)$.
(c) draw a phasor diagram of $x(t)$.
(the phasors of unmodulated carrier and the modulation components with the sense of rotation, should be included in your phasor diagram.)

(d) A noise signal $K\cos\phi(t)$ has a constant amplitude and a random phase. If this is added to $x(t)$,

(i) draw a phasor diagram for the composite signal

$$x'(t) = x(t) + K\cos\phi(t).$$

(ii) find the maximum phase distortion of the phase modulated carrier due to noise.

3.

A signal $x(t) = 2 + \sin\omega_0 t$ is pulse amplitude modulated using a pulse train

$$s(t) = \sum_{n=-\infty}^{\infty} p(t - nT_s) \text{ where } \begin{aligned} p(t) &= 1, \ |t| < \frac{T}{2} \\ &= 0 \ \text{otherwise} \end{aligned}$$

$$\omega_0 = \frac{\pi}{T_s}, \quad T << T_s$$

(a) Sketch the pulse amplitude modulated signal $y(t)$.
(b) Write an expression for $y(t)$ in terms of $x(t)$ and $s(t)$.
(c) Suppose $x(t)$ is sampled using the pulse train

$$h(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

The sampled signal $x'(t)$ is given by

$$x'(t) = \sum_{n=-\infty}^{\infty} x(t)\delta(t - nT_s)$$

(i) Sketch $x'(t)$.

(ii) Write the relationship between $x'(t)$, $p(t)$ and $y(t)$.

(d) Derive an expression for $Y(\omega)$ and sketch $Y(\omega)$.

4.

(a) Define the Fourier transform of $f(t)$.

(b) If the Fourier transform of $f(t)$ is $F(\omega)$, show that the Fourier transform of $\frac{df(t)}{dt}$ is $j\omega F(\omega)$.

(c)

$$p(t) = 1, \ |t| < \frac{T}{2}$$
$$= 0 \ \text{otherwise}$$

(i) Find $P(\omega)$, the Fourier transform of $p(t)$.

(ii) A triangular pulse $x(t)$ has an amplitude A and a pulse width $2T$ as shown in Fig.1.

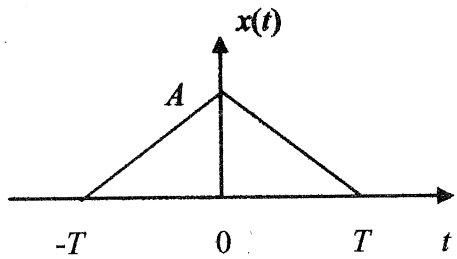

Fig. 1

Find the relationship between $p(t)$ and $x(t)$.

(iii) Find the Fourier transform of $x(t-T)$ (use $x(t)$ given in Fig.1).

(iv) Find the frequency spectrum of the signal $q(t)$ given in Fig.2.
[Hint: Use the answer of (iii)]

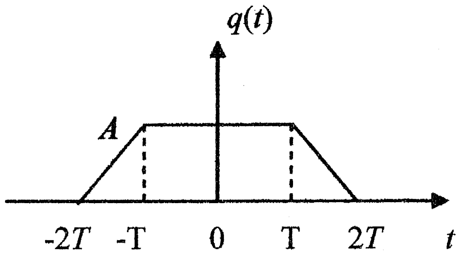

Fig. 2

(e) If $r(t) = q(t)\cos\omega_0 t$,

(i) calculate $R(\omega)$ the Fourier transform of $r(t)$.

(ii) sketch $R(\omega)$.

(Refer to $q(t)$ given in Fig.2. Assume that $\omega_0 >> \frac{\pi}{T}$.)

5.

(a) Two random signals X and Y are defined by $X = \cos\theta$ and $Y = \sin\theta$. θ is a random variable uniformly distributed over $[0, 2\pi]$. Show that X and Y are uncorrelated.

(b) The spectral density $S(\omega)$ of an ergodic signal $x(t)$ is given by

$$S(\omega) = A\delta(\omega) + B\delta(\omega - \omega_0) + B\delta(\omega + \omega_0)$$

(i) Calculate the autocorrelation function $\Re(\tau)$.

(ii) Calculate the signal power.

(c) What is understood by a stationary random process?

6.

(a) A transmitter sends three independent digital signals. The signals have amplitudes -1 V, +1 V and +2 V. The probabilities of sending these signals are given below:

$$P(\text{-1 } V) = \frac{1}{4}, P(\text{+1 } V) = \frac{1}{2} \text{ and } P(\text{+2 } V) = \frac{1}{4}.$$

Above signals propagate in a noisy medium. The noise $n(t)$ has a probability density function $P_{n(t)}$ shown in Fig.3.

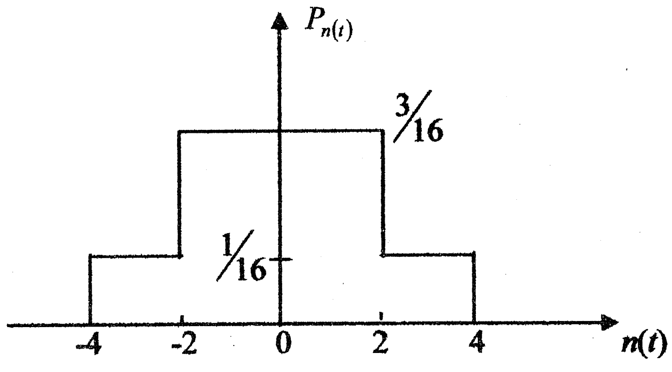

Fig.3

The received signal $x(t)$ is analyzed and the corresponding transmitted signal $s(t)$ is identified according to the following table:

observation	*decision*
$x(t) > +3\ V$	$s(t) = +2\ V$
$+3\ V > x(t) > +2\ V$	$s(t) = +1\ V$
$+2V > x(t)$	$s(t) = -1\ V$

(i) Calculate the power of the received signal (with noise).

(ii) Find the probability that '+1 *V*' sent by the transmitter is erroneously detected at the receiver.

(iii) Find the probability that '-1 *V*' sent by the transmitter is detected as '+2 *V*' at the receiver.

(b) Two independent noise signals $n_1(t)$ and $n_2(t)$ have probability density functions $P_{n_1(t)}$ and $P_{n_2(t)}$ respectively. What is the probability density function of the composite noise signal $n(t) = n_1(t) + n_2(t)$?

7.

(a) A memoryless source emits two messages m_1 and m_2. The probability of emitting m_1 is P.

(i) Write an expression for the entropy (H) of the source.

(ii) Show that H becomes maximum when $P = \frac{1}{2}$.

(iii) Find the maximum entropy of the source.

(b) A black and white TV picture consists of 1.6×10^6 picture elements. Picture frames appear in the TV screen at the rate of 32 frames per second. Each picture element can have 16 brightness levels. If all brightness levels have equal likelihood of occurrence, calculate the average rate of information conveyed by the TV.

8.

Briefly explain the following:

(i) Nyquist first criterion for zero inter-symbol interference.
(ii) Power spectral density of a random process.
(iii) Wide sense stationary process.
(iv) Eye diagram and its use.
(v) Hilbert transform.