THE OPEN UNIVERSITY OF SRI LANKA

Bachelor of Technology (Civil) – Level 5

075

CEX 5233 - STRUCTURAL ANALYSIS

FINAL EXAMINATION - 2007/2008

Time Allowed : 3 hours

Date: 2008-05-06 (Tuesday) Time: 09.30 – 12.30 hrs.

The Paper consists of Eight (8) questions. Answer **Five (5)** questions

1. Analyse the frame shown in figure Q-1 using Stiffness method of structural analysis and draw the bending moment diagram. (20 marks)
(You may use Table-1 to determine stiffness coefficients)

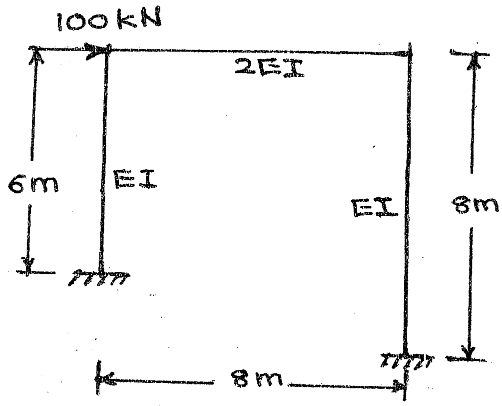


Figure Q-1

2. Analyse the continuous beam shown in figure Q-2 using Flexibility method of structural analysis and draw the bending moment diagram. (20 marks)
(You may use Table-1 to determine flexibility coefficients)

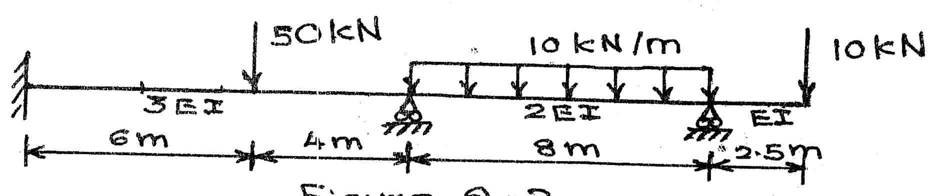


Figure Q-2

3.(a) What are the three main properties of stress tensor? (3 marks)

(b) The stress tensor at a point with reference to axes x,y,z is given by

$$\sigma_{ij} = \begin{bmatrix} 3 & 2 & -2 \\ 2 & 0 & -1 \\ -2 & -1 & 2 \end{bmatrix} Mpa$$

Show that by transformation of the axes by 30° about the z-axis in anti-clockwise direction, the new stress components are given by

$$\sigma'_{pq} = \begin{bmatrix} 3.982 & -0.299 & -2.232 \\ -0.299 & -0.982 & 0.134 \\ -2.232 & 0.134 & 2.000 \end{bmatrix} Mpa$$

(13 marks)

(c) Show that after the above transformation of axes, the stress invariants remain unchanged. (4 marks)

4.(a) What is understood by a plane stress problem? (2 marks)

(b) A cantilever beam of length L and depth 2h is in a state of plane stress. The cantilever is of unit thickness and is rigidly supported at the end x=L and is loaded as shown in the figure Q-4.

Show that the stress function $\Phi = Ax^2 + Bx^2y + Cy^3 + D(5x^2y^3 - y^5)$ is valid for the beam. (3 marks)

(c) Evaluate the constants A,B,C and D for the stress function in part (b). (12 marks)

(d) Find expressions for $\sigma_x, \sigma_y and \sigma_{xy}$. (3 marks)

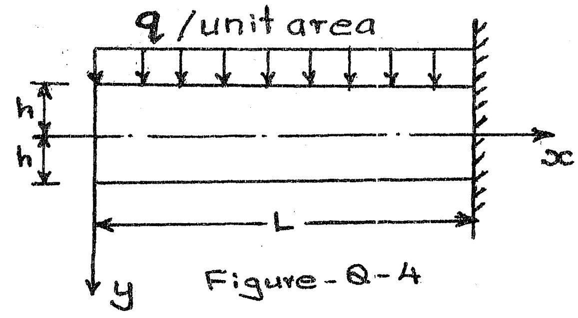


Figure - Q - 4

5.(a) Explain how the criteria of equilibrium, mechanism and yield are related to the concept of upper and lower bounds in plastic analysis. (6 marks)

(b) Determine the value of **α** (α<1), the ratio of the plastic moment of the end span to that of the center span for the continuous beam shown in figure Q-5, for the case, when ultimate strength is realized simultaneously in all the spans.

(14 marks)

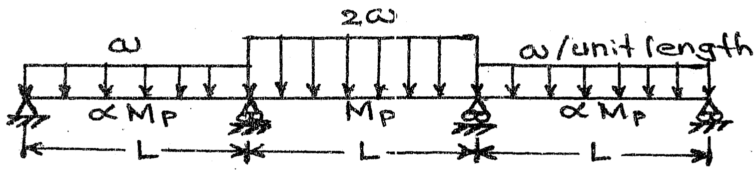


Figure -Q-5

6.(a) Compare the **safety factor** in elastic design with the **load factor** in plastic design. (4 marks)

(b) A plane frame with varying plastic moment capacity is shown in figure Q-6. Sketch all possible collapse mechanisms and determine the plastic moment capacity M_p.

(12 marks)

(c) Sketch the collapse bending moment diagram, showing the principal values. (4 marks)

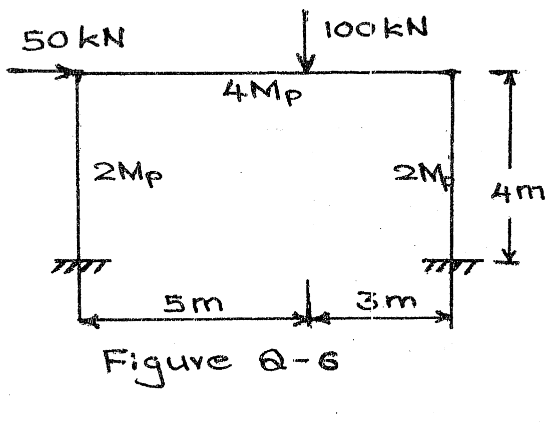


Figure Q-6

7. Determine the membrane stress resultants N_x, N_θ and $N_{\theta x}$ in a circular cylindrical section, subtending an angle 2α at the axis, under a udl of ω per unit surface area (similar to dead load).
The length of the cylinder is **L**. The sides of the cylinder are covered by diaphragm walls. The cylinder is supported on edge beams.
Assume that the origin of x coordinate is at the mid section of the cylinder.
(20 marks)

The governing equations for a cylindrical shell are given by;

$$\frac{\partial N_x}{\partial x} + \frac{1}{a}\frac{\partial N_{\theta x}}{\partial \theta} + P_x = 0$$

$$\frac{\partial N_{x\theta}}{\partial x} + \frac{1}{a}\frac{\partial N_\theta}{\partial \theta} + P_\theta = 0$$

$$\frac{N_\theta}{a} + P_z = 0$$

$$N_{\theta x} = N_{x\theta}$$

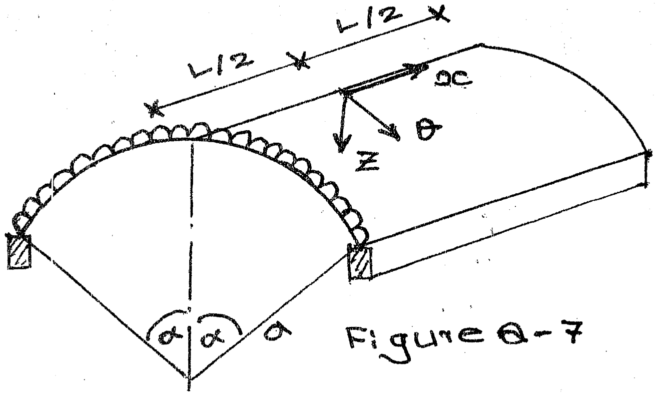


Figure Q-7

8. An isotropic rectangular thin plate OABC shown in figure Q-8 is simply supported in all four edges. The plate is loaded with a point load P. Show that the lateral deflection of the plate is given by

$$\omega = \frac{4P}{\pi^4 abD}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{1}{\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^2} Sin\frac{m\pi x_0}{a}\, Sin\frac{n\pi y_0}{b}\, Sin\frac{m\pi x}{a}\, Sin\frac{n\pi y}{b}$$

(16 marks)

Find the maximum deflection of the plate when the load **P** acts in the center.
(4 marks)

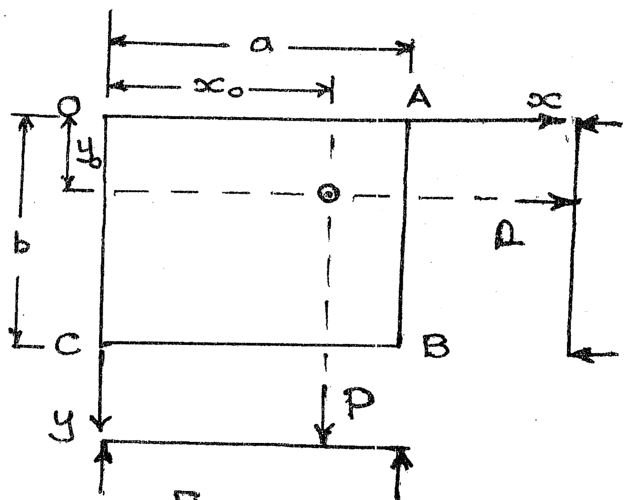
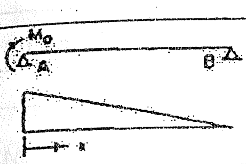
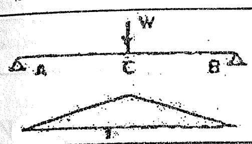
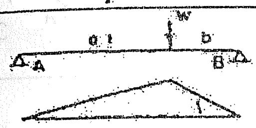
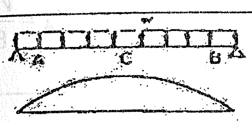
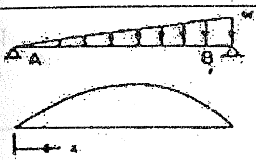
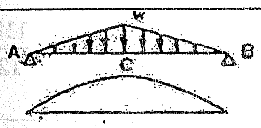
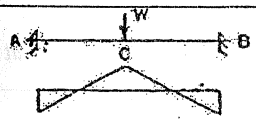
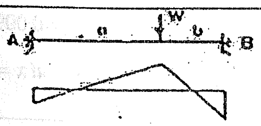
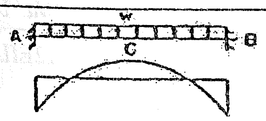
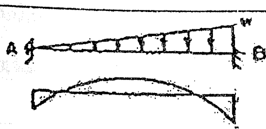
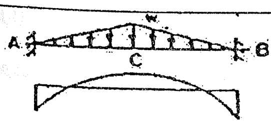


Figure Q-8

Table 1 Formulas for Beams

Structure	Shear	Moment	Slope	Deflection
Simply supported Beam				
	$S_A = -\frac{M_o}{L}$	M_o	$\theta_A = \frac{M_o L}{3EI}$ $\theta_B = -\frac{M_o L}{6EI}$	$Y_{max} = 0.062\frac{M_o L^2}{EI}$ $at\ x = 0.422L$
	$S_A = \frac{W}{2}$	$M_o = \frac{WL}{4}$	$\theta_A = -\theta_B = \frac{WL^2}{16EI}$	$Y_c = \frac{WL^3}{48EI}$
	$S_A = \frac{Wb}{L}$ $S_B = \frac{Wa}{L}$	$M_o = \frac{Wab}{L}$	$\theta_A = \frac{Wab}{6EIL}(L+b)$ $\theta_B = -\frac{Wab}{6EIL}(L+a)$	$Y_o = \frac{Wa^2b^2}{3EIL}$
	$S_A = \frac{WL}{2}$	$M_c = \frac{WL^2}{8}$	$\theta_A = -\theta_B = \frac{W\ L^3}{24EI}$	$Y_c = \frac{5WL^4}{384EI}$
	$S_A = \frac{WL}{6}$ $S_B = \frac{WL}{3}$	$M_{max} = 0.064Wl^2$ $at\ x = 0.577L$	$\theta_A = \frac{7WL^3}{360EI}$ $\theta_B = -\frac{8WL^3}{360EI}$	$Y_{max} = 0.00652\frac{WL^4}{EI}$ $at\ x = 0.519L$
	$S_A = \frac{WL}{4}$	$M_c = \frac{WL^2}{12}$	$\theta_A = -\theta_B = \frac{5W\ L^3}{192EI}$	$Y_c = \frac{WL^4}{120EI}$
Fixed Beams				
	$S_A = \frac{W}{2}$	$M_c = \frac{WL}{8}$	$\theta_A = \theta_B = 0$	$Y_c = \frac{WL^3}{192EI}$
	$S_A = \frac{Wb^2}{L^3}(3a+b)$ $S_B = \frac{Wa^2}{L^3}(3b+a)$	$M_A = -\frac{Wab^2}{L^2}$ $M_B = -\frac{Wba^2}{L^2}$	$\theta_A = \theta_B = 0$	$Y_o = \frac{Wa^3b^3}{3EIL^3}$
	$S_A = \frac{WL}{2}$	$M_A = M_B = -\frac{WL^2}{12}$	$\theta_A = \theta_B = 0$	$Y_c = \frac{WL^4}{384EI}$
	$S_A = \frac{3WL}{20}$ $S_B = -\frac{7WL}{20}$	$M_A = -\frac{WL^2}{30}$ $M_B = -\frac{WL^2}{20}$	$\theta_A = \theta_B = 0$	$Y_{max} = 0.00131\frac{WL^4}{EI}$ $at\ x = 0.525L$
	$S_A = \frac{WL}{4}$	$M_A = M_B = -\frac{5WL^2}{96}$	$\theta_A = \theta_B = 0$	$Y_c = \frac{0.7WL^4}{384EI}$

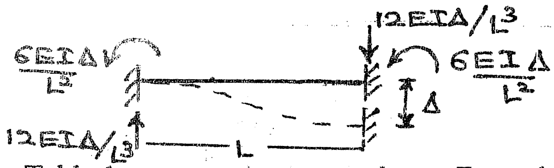
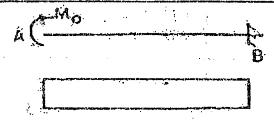
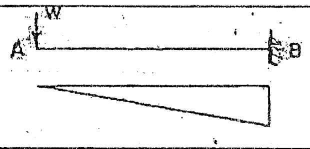
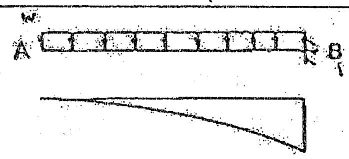
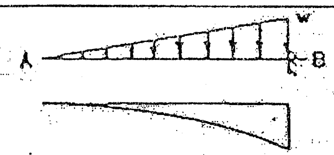
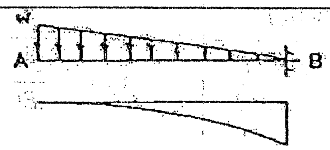
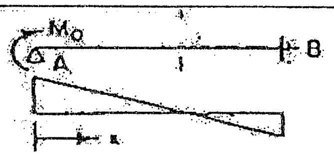
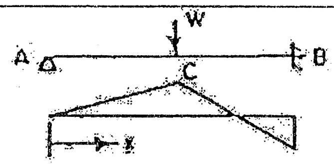
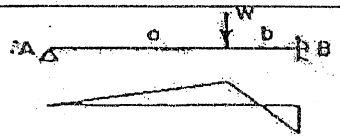
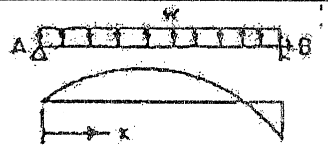
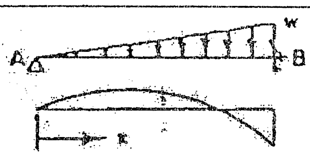
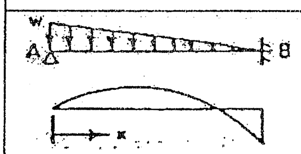


Table 1 Formulas for Beams

Structure	Shear	Moment	Slope	Deflection ↓
Cantilever Beam				
	0	M_o	$\theta_A=\frac{M_oL}{EI}$	$Y_A=\frac{M_oL^2}{2EI}$
	W	$M_B=-WL$	$\theta_A=-\frac{WL^2}{2EI}$	$Y_A=\frac{WL^3}{3EI}$
	$S_B=-WL$	$M_B=-\frac{WL^2}{2}$	$\theta_A=-\frac{WL^3}{6EI}$	$Y_A=\frac{WL^4}{8EI}$
	$S_B=-\frac{WL}{2}$	$M_B=-\frac{WL^2}{6}$	$\theta_A=-\frac{WL^3}{24EI}$	$Y_A=\frac{WL^4}{8EI}$
	$S_B=-\frac{WL}{2}$	$M_B=-\frac{WL^2}{2}$	$\theta_A=-\frac{WL^3}{8EI}$	$Y_A=\frac{11WL^4}{120EI}$
Propped Cantilever				
	$S_A=-\frac{3M_o}{2L}$	$M_B=-\frac{M_o}{2}$	$\theta_A=-\frac{M_oL}{4EI}$	$Y_{max}=\frac{M_oL^2}{27EI}$ at $x=\frac{L}{3}$
	$S_A=-\frac{3M_o}{2L}$	$M_B=-\frac{3WL}{16}$ $M_c=\frac{5WL}{32}$	$\theta_A=\frac{WL^2}{32EI}$	$Y_{max}=0.00962\frac{WL^3}{ELI}$ at $x=0.4471$
	$S_A=\frac{Wb^2}{2L^3}(a+2L)$ $S_B=-\frac{Wa}{2L^3}(3L^2-a^2)$	$M_B=-\frac{Wab}{L^2}(a+\frac{b}{2})$	$\theta_A=\frac{Wab^2}{4EIL}$	$Yo=\frac{Wa^2b^2}{12EIL^3}(3L+$
	$S_A=+\frac{3WL}{8}$	$M_B=-\frac{WL^2}{8}$	$\theta_A=\frac{WL^3}{48EI}$	$Y_{max}=0.0054\frac{WL^4}{ELI}$ at $x=0.42$
	$S_A=+\frac{WL}{10}$	$M_{max}=0.03WL^2$ at $x=0.447L$ $M_B=-\frac{WL^2}{15}$	$\theta_A=\frac{WL^3}{120EI}$	$Y_{max}=0.00239\frac{WL}{EI}$ at $x=0.447$
	$S_A=+\frac{11WL}{40}$	$M_{max}=0.0423WL^2$ at $x=0.329L$ $M_B=-\frac{7WL^2}{120}$	$\theta_A=\frac{WL^3}{80EI}$	$Y_{max}=0.00305\frac{WL}{EI}$ at $x=0.40$