THE OPEN UNIVERSITY OF SRI LANKA
Department of Civil Engineering
Bachelor of Technology - Level 5

CEX 5231 - MECHANICS OF FLUIDS

FINAL EXAMINATION - 2007/2008

Time Allowed : **Three Hours** *Index No.*

Date : 09th May, 2008 Time : 0930 - 1230

ANSWER **ALL THREE** QUESTIONS IN PART A AND **ANY TWO** QUESTIONS IN PART B. ALL QUESTIONS CARRY EQUAL MARKS.

PART A

Answer **all three** questions in this section.

1) Darcy's law for the flow of water in an unconfined aquifer can be stated as $V = -k\frac{dh}{dx}$ where **V** is the velocity of flow in the (horizontal) **x** direction, **k** is the hydraulic conductivity and **dh/dx** is the slope of the free surface in the unconfined aquifer.

a) What are the main assumptions needed to apply this equation to flow in aquifers?

A long, straight river flows in a long, straight area of deposited sediment that is bounded below and on both sides by impervious rock as shown in Figure 1. The area of deposited sediment extends for **L** m on either side of the river. The deposited sediment consists of two aquifers (consisting of uniform layers of sandy material) that are separated by a thin clay layer, as shown in the figure. The river flows in the upper layer of sandy material.

The upper aquifer is recharged at a rate of **q** due to the infiltration of rainfall. The hydraulic conductivity (coefficient of permeability) of the upper aquifer is **k**. Water leaks from the upper aquifer into the lower aquifer through the clay layer. The velocity of leakage, **w(x)**, is proportional to the water level in the upper aquifer – **w(x) = ch(x)** – where **c** is a constant.

b) Derive, from first principles, a differential equation that governs the variation of the water level, **h(x)**, in the upper aquifer.

c) List the boundary conditions you would use to solve this equation and explain their physical significance.

d) Sketch and discuss the solution for the following cases :

i) $c = 0$ and $q \neq 0$

ii) $q = 0$ and $c \neq 0$

iii) $q = ch_0$ where h_0 is the elevation of the water level in the river, as shown in Figure 1.

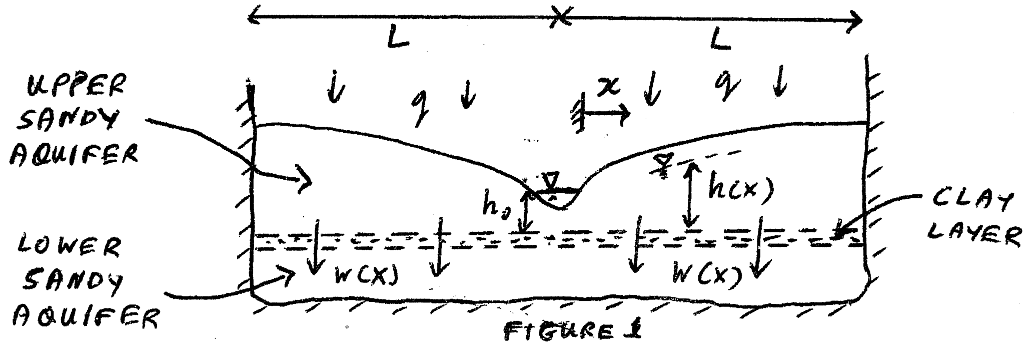

FIGURE 1

2) Figure 2 shows the pipe system in a house. Water is distributed from an overhead tank, X, to three Taps, D, E and F, through the pipes AB, BD, BC, CE and CF. The elevations of the points A, B, C, D, E and F with respect to the water level in the Tank X are shown on the figure.

The lengths of the pipes are as follows : AB = 4 m , BC = 3 m , BD = 4 m , CE = 3 m and CF = 5 m . All the pipes have a diameter of 25 mm and a friction factor of 0.01 .

a) Taps D and E are opened fully while Tap F is closed. Calculate the discharge through Taps D and E. Assume reasonable values for any coefficients not specified. Explain your calculation.

b) Explain how you would calculate the discharge in the system when Taps D, E and F are all opened fully.

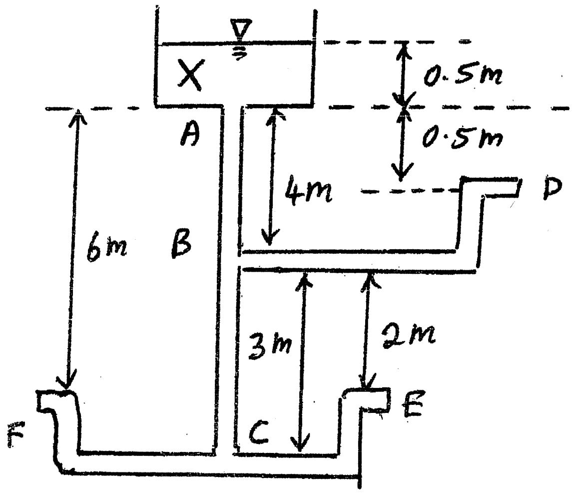

FIGURE 2

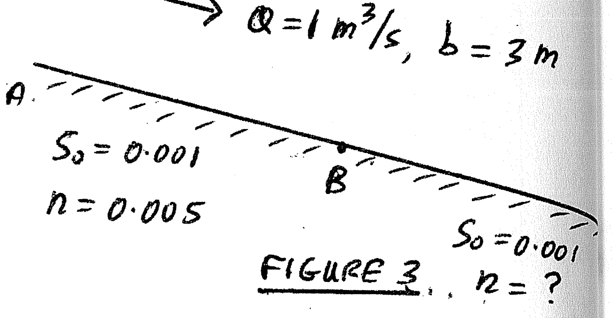

FIGURE 3

3) An equation used in the computation of flow profiles in open channel flows is $\dfrac{dy}{dx} = \dfrac{S_0 - S_f}{1 - F_r^2}$.

a) Identify the variables **dy/dx** , $\mathbf{S_0}$, $\mathbf{S_f}$ and $\mathbf{F_r}$ in this equation using a neat diagram.

b) List the important assumptions that have been made to derive this equation.

c) Explain why this equation cannot be used when the flow depth is close to the critical depth.

A rectangular open channel of **constant width and slope** has two long sections – AB and BC – of different Manning's coefficients – as shown in Figure 3. The flow far upstream of B and far downstream of B is at the uniform depth. The channel width is 3 m and the channel slope is 0.001 . The channel carries a discharge of 1 m^3/s .

The Manning's coefficient of section AB is 0.005 . The Manning's coefficient of section BC is not known but it is observed that the flow depth far downstream of B is 0.5 m .

d) Show that there is a hydraulic jump near B.

e) Determine whether the hydraulic jump is upstream of B or downstream of B. Explain your answer.

f) Classify the surface profiles between A and C (from M1, M2, M3, S1, S2, S3, C1, C2, C3, etc.). Explain your answer.

(Note : The equation $\dfrac{y_2}{y_1} = \dfrac{1}{2}\left(\sqrt{1 + 8F_{r1}^2} - 1\right)$ may be useful.)

PART B

Answer any **two** of the **five** questions in this section.

4) An open channel has a rectangular cross-section of width 1 m . The channel carries a discharge of 0.5 m^3/s at an average velocity of 1 m/s .

a) What is critical depth in open channel flow?

b) Show that the flow depth, **y** , the critical depth, $\mathbf{y_c}$, and the specific energy, **E** , in an open channel of rectangular cross-section are related by the expression $\frac{E}{y_c} = \frac{y}{y_c} + \frac{1}{2}\left(\frac{y}{y_c}\right)^{-2}$

The channel has a smooth transition where the width is reduced by **x** m and the bed level is raised by a smooth step of height **x** m , as shown in Figure 4.

c) Calculate the depth of flow in the channel after the transition for x = 0.05 m .

d) Calculate the magnitude and direction of the force on the transition when x = 0.05 m

e) What will happen when x = 0.2 m ?

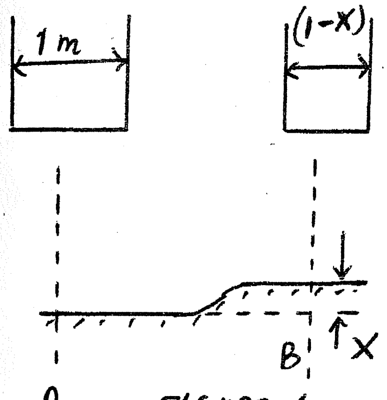

FIGURE 4

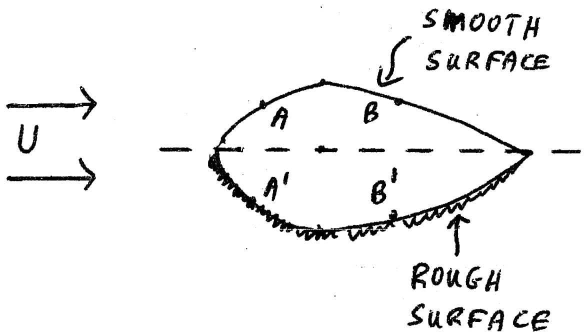

FIGURE 5

5) A two-dimensional streamlined body is held in a uniform flow of air, as shown in Figure 5. The cross-section of the body has a plane of symmetry that is parallel to the direction of the flow, as shown in the figure. However, the top surface of the body is smooth, while the bottom surface of the body is rough.

a) Compare the velocity profiles in the boundary layer around the body at the locations A and A' when the Reynolds number of the flow is small. Explain your answer.

b) Compare the velocity profiles in the boundary layer around the body at the locations A and A' when the Reynolds number of the flow is large. Explain your answer.

c) Compare the velocity profiles in the boundary layer around the body at the locations B and B' when the Reynolds number of the flow is large. Explain your answer.

d) Sketch the pattern of streamlines around the body at large and small Reynolds numbers and explain the differences.

6) a) A wide open channel is a channel that has a width much greater than the flow depth. Show that for uniform flow in a wide channel the bottom shear stress, τ_0, is given by the equation $\tau_0 = \rho g h S_0$, where **h** is the depth of flow and $\mathbf{S_0}$ the channel slope.

b) Use the Manning's equation for a wide open channel to show that the bottom shear stress in a wide open channel is related to the discharge, **Q**, by the equation $\tau_0 = \rho g \left(\frac{Qn}{b}\right)^{\frac{3}{5}} S_0^{\frac{7}{10}}$ when the flow is uniform. Here **b** is the width of the channel and **n** the Manning's coefficient.

The Meyer-Peter-Muller formula for bed load transport is given by $\frac{q_s \rho^{1/2}}{(\rho(s-1)gd)^{3/2}} = 8\left(\frac{\rho u_*^2}{\rho(s-1)gd} - 0.047\right)$ where the shear velocity $u_* = \sqrt{\frac{\tau_0}{\rho}}$, $\mathbf{q_s}$ is the rate of sediment transport, ρ the density of water, **s** the specific gravity of the sediment, **d** the grain diameter.

c) What are the dimension and units of the rate of sediment transport $\mathbf{q_s}$?

A long straight river is 100 m wide, has a uniform cross-section and has a slope of 0.0001. The river bottom consists of fine sand of diameter 0.1 mm and specific gravity 2.65 .

d) Use the Meyer-Peter-Muller formula and the result of section b) to estimate the discharge for which the sediment just begins to move ($q_s = 0$).

e) Calculate the rate of sediment transport in the river when the discharge is twice the value estimated in section d).

7) The drag force, **F**, on a ship is a function of the velocity of the ship, **V**, the length of the ship, **L**, the width of the ship, **B**, the total weight of the ship, **W**, the density of water, ρ, the dynamic viscosity of water, μ, and the acceleration due to gravity, **g**.

a) What does Buckingham's Π-theorem say about the relationship between the number of physical variables describing a problem, the number of dimensions contained in these variables and the number of non-dimensional variables needed to describe the problem?

b) Derive a relationship between the non-dimensional drag force on the ship and other non-dimensional quantities.

A container ship has a length of 200m, a width of 25 m and moves at 1 m/s . The fully loaded ship displaces 100,000 metric tons of sea water. The drag force on the fully loaded ship at this speed is to be estimated by measuring the force needed to tow a 1:50 scale model of the ship in a towing tank. The towing tank is filled with sea water.

c) Assuming that Reynolds Number effects can be neglected, calculate the weight of water that should be displaced by the model and the velocity at which the model of the ship should be towed.

8) Two identical pumps are connected in series to pump water from Reservoir X to Tank Y, as shown in Figure 8. The pipeline AB has a length of 500 m , a diameter of 50 mm and a friction factor of 0.01. Tank Y has a uniform cross-sectional area of 5 m^2 and a maximum depth of 30 m . The bottom of Tank Y is 15 m above the water level of Reservoir X and the Tank Y is empty when pumping begins.

One of the pumps is tested at its operating speed and the results are given in Table 8.

Discharge (l/s)	0	10	20	30	40
Head (m)	20	18	15	10	2

Table 8

a) What is the maximum water level that can be achieved in Tank Y? Explain your answer.

b) Calculate the discharge in the pipeline when Tank Y is empty. Explain your answer.

c) Calculate the water level in Tank Y when the **discharge in the pipeline** is half the value calculated in section b). Explain your answer.

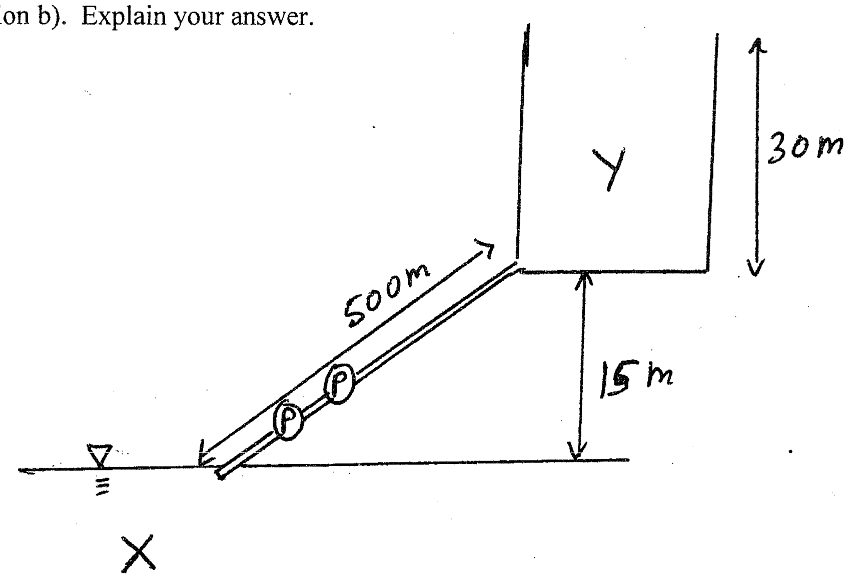

FIGURE 8