The Open University of Sri Lanka
Department of Electrical and Computer Engineering
Final Examination 2008/2009
ECX5233 – Radio and Line Communication

Time: 0930 – 1230 hrs. Date: 2009-04 -02

Answer any FIVE questions

1.

(a) A periodic signal $x(t)$ has a period of oscillation T_0.

(i) Express $x(t)$ as a complex Fourier series.

(ii) Simplify the expression given in (i) if $x(t)$ is

(α) an even function ($x(t) = x(-t)$).

(β) an odd function ($x(t) = -x(-t)$).

(iii)

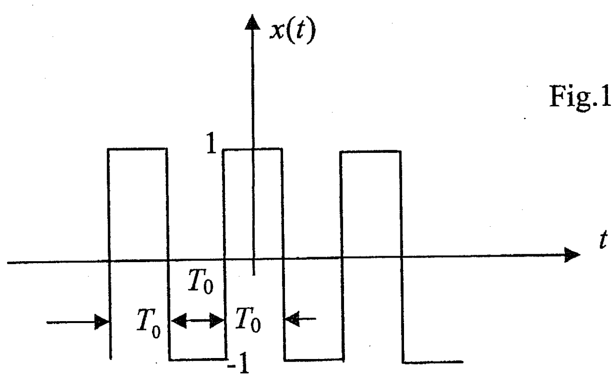


Fig.1 (a)

(α) Express the signal $x(t)$ shown in Fig.1 (a) as a Fourier series.

(β) Sketch the amplitude spectrum of $x(t)$.

(γ) What happens to the amplitude spectrum of $x(t)$ when T_0 is decreased?

(b) (i) Define the Fourier transform $Y(\omega)$ of a non periodic signal $y(t)$.

(ii) What information of the signal $y(t)$ can be extracted from $Y(\omega)$?

(c) Find the Fourier transform $X(\omega)$ of the signal $x(t)$ given in Fig.1(a).

(d) Impulse train $p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nt_0)$ is passed through a filter whose impulse response $h(t)$ is given in Fig.1(b).

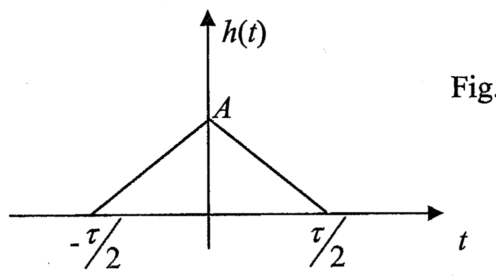


Fig.1(b)

Assuming that $T_0 > \tau$,

(i) derive an expression for the output of the filter $y(t)$.

(ii) sketch $y(t)$.

(iii) derive an expression for $Y(\omega)$ [the Fourier transform of $y(t)$].

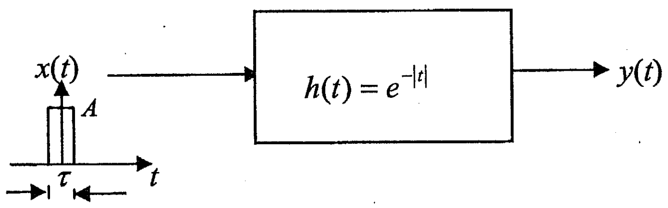
2.

(a) The deterministic signal $z(t)$ is obtained by convolving signal $x(t)$ with signal $y(t)$. ie. $z(t) = x(t) * y(t)$.

(i) Define $x(t) * y(t)$.

(ii) Show that $x(t) * y(t) = y(t) * x(t)$.

(b) A rectangular pulse $x(t)$ having a width τ and a height A is transmitted over a noise free channel whose impulse response $h(t)$ is given by $h(t) = e^{-|t|}$.



(i) Find the maximum value of the received signal $y(t)$.
(ii) Calculate $Y(\omega)$ and sketch it.

(iii) What is the impact of the channel on the spectral components of $x(t)$?

(iv) If an amplitude modulated carrier $A(1+m\cos\omega_m t)\cos\omega_c t$ is used as $x(t)$, derive an expression for $Y(\omega)$.

(v) If $x(t) = A(1+s(t))\cos\omega_c t$ where $s(t)$ is a non sinusoidal base band signal, derive an expression for $Y(\omega)$ in terms of $S(\omega)$, where $S(\omega)$ is the Fourier Transform of $s(t)$.

3.

(a) A signal $x(t)$ is sampled using an impulse train $s(t) = \sum_{n=-\infty}^{\infty} \delta(t-nT_0)$.

Sampled signal $x_s(t) = x(t).s(t)$. The process is *natural* sampling.

(i) Sketch $x_s(t)$ for any arbitrary signal $x(t)$.

(ii) $x(t)$ is now passed through an ideal low pass filter whose lower cut-off frequency is f_L. The filtered signal $x_p(t)$ is then sampled using the same impulse train $s(t)$. If no information of $x_p(t)$ is lost due to sampling,

(α) find the maximum allowable value ($T_0^{\max}$) of T_0.

(β) sketch the sampled $x_p(t)$ assuming that $x(t)$ is a triangular pulse.

(δ) derive an expression for $X_{ps}(\omega)$, the Fourier transform of the sampled $x_{ps}(t)$ in terms of $X_p(\omega)$, where $X_p(\omega)$ is the Fourier transform of $x_p(t)$.

(γ) sketch $X_{ps}(\omega)$ if $T_0 = T_0^{\max}$.

(b) Signal $y(t)$ is sampled using a pulse train $r(t) = \sum_{n=-\infty}^{\infty} p(t-nT_0)$, where $p(t)$ is a rectangular pulse having a width τ and height 1.

free

ie. $$p(t) = \begin{cases} 1, \text{ for } |t| \le \tau/2 \\ 0 \quad \text{otherwise} \end{cases}$$

$y(t)$ and $y_s(t)$ (sampled $y(t)$) are shown in Fig.3(b).

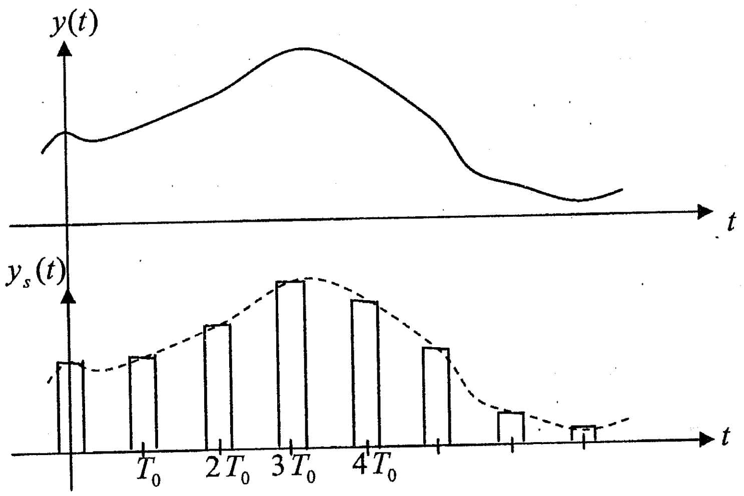


Fig.3(b)

Sampling method used here is *flat top sampling.*
[$y_s(t)$ can also be obtained by *pulse amplitude modulating* $y(t)$ using $r(t)$.]

(i) Write an expression for the $y_s(t)$ in terms of $s(t)$, $y(t)$ and $p(t)$.
[$s(t)$ is defined in Q.3(a)].

(ii) Derive an expression for $Y_s(\omega)$, the Fourier transform of $y_s(t)$.

(iii) Sketch $Y_s(\omega)$ if $y(t)= x_p(t)$.

4.

(a) Define *Auto Correlation function* $\Re_{xx}(\tau)$, of a random process $X(t)$.

(b) Define *Cross Correlation function* $\Re_{xy}(\tau)$, of two random processes $X(t)$ and $Y(t)$.

(c) What is understood by two *independent* random processes $X(t)$ and $Y(t)$?

(d) $X(t)$ and $Y(t)$ are two random processes. A random process $Z(t)$ is defined by

$Z(t)=X(t)+Y(t)$.
Derive an expression for the autocorrelation function $\Re_{zz}(\tau)$ of $Z(t)$ in terms $\Re_{xx}(\tau)$, $\Re_{yy}(\tau)$, $\Re_{xy}(\tau)$ and $\Re_{yx}(\tau)$.

where $\Re_{xx}(\tau)=$ *Auto Correlation function* of random process $X(t)$.
$\Re_{yy}(\tau)=$ *Auto Correlation function* of random process $Y(t)$.
$\Re_{xy}(\tau)=$ *Cross Correlation function* of random possesses $X(t)$ and $Y(t)$.
$\Re_{yx}(\tau)=$ *Cross Correlation function* of random possesses $Y(t)$ and $X(t)$.

(e) Simplify the equation for $\Re_{zz}(\tau)$ if the processes $X(t)$ and $Y(t)$ are independent.

(f) If $Y(t)$ has a mean value of zero, $\Re_{xx}(\tau) = \frac{\eta}{2}\delta(\tau)$ and $\Re_{yy}(\tau) = \cos(\omega_c\tau)$

(i) evaluate $\Re_{zz}(\tau)$.
(ii) sketch $\Re_{zz}(\tau)$.
(iii) find the power of $Z(t)$.

($X(t)$ and $Y(t)$ are independent signals)

5.

(a) Consider a source m emitting messages $m_1, m_2, \ldots, m_i \ldots m_n$ with probabilities $P_1, P_2, \ldots P_i \ldots, P_n$ respectively.

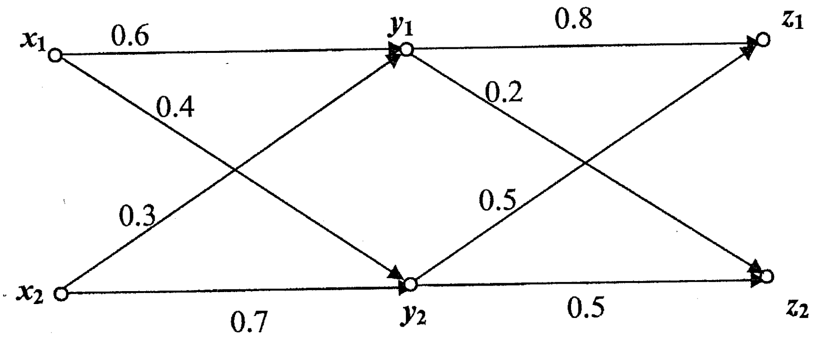
(i) Write an expression for the information content $I(m_i)$ of the message m_i.
(ii) Write an expression for the average information content $H(m)$.

(b) Consider a binary memoryless source X which emits two symbols x_1 and x_2.

Show that the average information content $H(X)$ is a maximum when both x_1 and x_2 are equiprobable.

(c) What is understood by

(i) a *lossless channel*?
(ii) a *deterministic channel*?
(iii) a *noiseless channel*?



(d) Two binary channels are cascaded as shown in the diagram.

(i) Find the overall channel matrix of the resultant channel.
(ii) Draw the resultant equivalent channel diagram.
(iii) If the probability of occurring x_1 is twice that of the probability of occurring x_2, find $p(z_1)$ and $p(z_2)$.

$p(z_i)$ = the probability of occurring z_i.

6.

A sinusoidal carrier $A\cos(\omega_c t)$ is phase-modulated using a sinusoidal modulating signal $\sin(\omega_m t)$. The phase deviation constant of the modulated signal is β.

(a) Write an expression for the phase modulated carrier $x_{cp}(t)$.

(b) Show that $x_{cp}(t)$ can be expressed in the form $x_{cp}(t) = \mathrm{Re}\left[Ae^{j\omega_c t}\sum_{n=-\infty}^{\infty} C_n e^{jn\omega_m t}\right]$

Express C_n as a finite integral.

(c) Show that $x_{cp}(t) = A\sum_{n=-\infty}^{\infty} J_n(\beta)\cos(\omega_c + n\omega_m)t$, where $J_n(\beta)$ is a first kind *Bessel* function of n^{th} order and argument β and given by the equation

$$J_n(\beta) = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j(\beta\sin\lambda - n\lambda)}\,d\lambda$$

Some selected values of $J_n(\beta)$ are tabulated below:

n \ β	0.1	0.2	0.5	1	2	5	8	10
0	0.997	0.990	0.938	0.765	0.224	-0.178	-0.172	-0.246
1	0.050	0.100	0.242	0.440	0.577	-0.178	0.172	-0.246
2	0.001	0.005	0.031	0.115	0.353	0.047	-0.113	0.255
3			0.003	0.020	0.129	0.365	-0.291	0.058
4				0.002	0.034	0.391	-0.105	-0.220
5					0.007	0.261	0.286	-0.234

(d) If A = 2 V, $f_m = \frac{\omega_m}{2\pi} = 1\,kHz$ and $\beta = 0.5$, find the power of the fundamental and the first 3 harmonics of the modulated signal.

(e) Derive an expression for the power (P_t) of the frequency modulated carrier.

(f) What percentage of total power P_t is distributed among the fundamental, the first-, the second and the third harmonic?

(g) How does the value calculated in (f) depend on β?

(h) (i) What is narrow band *phase modulation*?
(ii) Show that a narrow band *phase modulated* signal can be approximated to *an amplitude modulated* signal.

7.

(a) A data source emits a random digital signal $x(t)$ whose probability density function $f_x(x)$ is given by $f_x(x) = A\delta(x) + 0.25\delta(x-1) + 0.25\delta(x+1)$.

(i) Find the value of the constant A.

(ii) Give all the amplitude values of $x(t)$. Also find the probabilities of occurrence of these amplitude values.

(b) A transmitter emits a binary data signal $s(t)$ which can take two values -2 or +2. The probability of occurrence of -2 and +2 are $\frac{1}{3}$ and $\frac{2}{3}$ respectively.

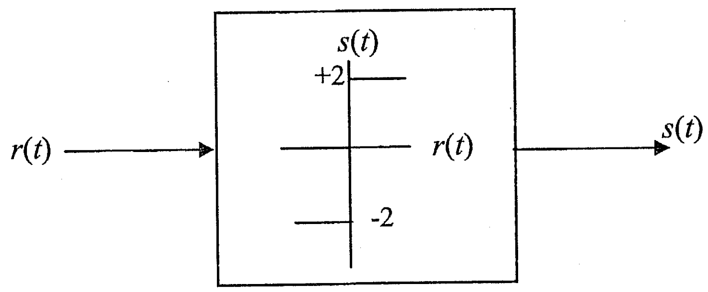
During the transmission a noise signal $n(t) = \frac{e^{\frac{-n^2}{2}}}{\sqrt{2\pi}}$ is added to $s(t)$.

(i) Find the noise power.

(ii) Find the power of the received signal $r(t) = s(t) + n(t)$ assuming that the data signal and the noise signal are totally independent.

At the receiver, the value of $s(t)$ is estimated using *a threshold detector*. The detection criterion is as follows:

If $r(t) > 0$ then $s(t)$ is +2.
Otherwise $s(t)$ is –2.



Threshold detector

(iii) Find the error probability.

(iv) Find the probability that -2 transmitted at the transmitter is received error free at the receiver.

Gaussian distribution $\left(\text{value of } \int_0^x \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}} dx \right)$

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.0000	.0040	.0080	.0120	.0159	.0199	.0239	.0279	.0319	.0359
.1	.0398	.0438	.0477	.0517	.0557	.0596	.0635	.0675	.0714	.0753
.2	.0792	.0832	.0871	.0909	.0948	.0987	.1026	.1064	.1102	.1141
.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
.4	.1554	.1591	.1627	.1664	.1700	.1736	.1772	.1808	.1844	.1879
.5	.1915	.1950	.1985	.2019	.2054	.2088	.2122	.2156	.2190	.2224
.6	.2257	.2291	.2324	.2356	.2389	.2421	.2454	.2486	.2517	.2549
.7	.2580	.2611	.2642	.2673	.2703	.2734	.2764	.2793	.2823	.2852
.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3364	.3389
1.0	.3413	.3437	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3707	.3728	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3868	.3888	.3906	.3925	.3943	.3962	.3979	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4146	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4278	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4648	.4656	.4664	.4671	.4678	.4685	.4692	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4874	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4895	.4898	.4901	.4903	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4924	.4926	.4928	.4930	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4944	.4946	.4948	.4949	.4950	.4952
2.6	.4953	.4955	.4956	.4957	.4958	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4983	.4984	.4984	.4985	.4985	.4986
3.0	.4986	.4987	.4987	.4988	.4988	.4988	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4991	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4993	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4996
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998
3.6	.4998	.4998	.4998	.4998	.4999	.4999	.4999	.4999	.4999	.4999
3.7	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999
3.8	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999
3.9	~.5000									

8.
Briefly explain the following:

(a) Vestigial side band modulation and it's advantage over single side band modulation (SSB).

(b) Central limit theorem.

(c) Eye diagram and its uses.

(d) Prediction of the nature of a random process from it's auto correlation function.

(e) Coloured noise and noise whitening filter.