THE OPEN UNIVERSITY OF SRI LANKA

BACHELOR OF TECHNOLOGY – LEVEL 06

FINAL EXAMINATION – 2008/2009

MPZ 6231 – DISCRETE MATHEMATICS (ESSAY TYPE PAPER)

DURATION : THREE (03) HOURS

Date : 13th March 2009 **Time: 0930 - 1230 hrs.**

Instructions:

- **Answer only six questions**
- **State any assumption you required**
- **Do not spend more than 30 minutes for one problem**
- **Show all your workings**
- **All symbols are in standard notation.**

01. a) How do you explain the following propositions for the propositions *p,q and r*,
   i. associative
   ii. distributive
   iii. De Morgan's law

b) Let p,q and r be three propositions.
Let P denotes the propositional statement $(p \vee q)$ and Q denotes the prepositional statement $[(p \to r) \wedge (q \to r)]$.

   i. Draw a truth table to calculate the truth values of P and Q.
   ii. State whether P and Q are logically equivalent. Justify your answer.
   iii. Using above truth table show that the propositional statement $[(p \vee q) \wedge (p \to r) \wedge (q \to r)] \to r$ is a tautology.

c) Identify the relation between $p \oplus r$ and $\neg(p \leftrightarrow q)$

02. a) Prove or disprove
   i. If x and y are real numbers. Then $(x^2 = y^2) \Leftrightarrow (x = y)$
   ii. $\forall\, x \in \mathbb{R}, \quad x^3 > x^2$
   iii. There exists $x \in \mathbb{N}$, there exists $y \in \mathbb{N}$ such that $x^3 - y^3 = 50$

b) Using mathematical induction prove that
   i. $\sum_{k=1}^{n} k^3 = \left[\frac{n(n+1)}{2}\right]^2$ for every positive integer n.
   ii. $n^3 + 2n$ divisible by 3 for all positive integer n.

03. a) i. Define $A \Delta B$ and $A \setminus B$ for sets A and B

ii. Without using Venn diagram show that $A \Delta B = (A \setminus B) \cup (B \setminus A)$

b) If $A \cap B \subseteq C$ and $A' \cap B \subseteq C$, then show that $B \subseteq C$

c) There are 50 elements in each of sets *A, B* and *C*. Find the number of elements in $(A \cup B \cup C)$ in each of following cases.

i. The sets are pair wise disjoint.

ii. There are 20 common elements in each pair of sets and no elements common to all three sets.

iii. There are 20 common elements in each pair of sets and 10 common elements in all three sets.

04. a) Define $f : \mathbb{N} \to \mathbb{N}$ by

$$f(n) = \begin{cases} n-2 & ;\text{if } n \geq 6 \\ f(f(n+4)) & ;\text{if } n < 6 \end{cases}$$

i. Find $f(1), f(3), f(7)$

ii. Is f a one – to - one function? Justify your answer.

iii. Is f a onto function? Justify your answer.

b) Let $f : \mathbb{R} \to \mathbb{R}$ and $f(x) = 2x$ for all $x \in \mathbb{R}$

and $g : \mathbb{R} \to \mathbb{R}$ and $g(x) = x^2 + 2$ for all $x \in \mathbb{R}$

Find

i. $fog(x)$ and $gof(x)$

ii. $x_0 \in \mathbb{R}$ such that $fog(x_0) = gof(x_0)$

iii. $x_0' \in \mathbb{R}$ such that $fog(x_0') = gof(x_0')$

c) Find $f^{-1}(x)$ for the following functions if it exists.

i. $f(x) = \sec x \quad ;0 \leq x \leq \pi$

ii. $f(x) = cosec\ x \quad ;\frac{-\pi}{2} \leq x \leq \frac{\pi}{2}$

iii. $f(x) = |x-1| \quad ;x \in \mathbb{R}$

05. a) Let $P = \{(x, y) \in \mathbb{R} \times \mathbb{R} \;;\; x^2 + y^2 = 4\}$

Does P have the reflexive, symmetric and transitive properties? Justify your answer.

b) Let $A = \{1,2,3\}$

i. Give an examples of a relation define on A, that is reflexive, symmetric but not transitive.

ii. List all equivalent relation on set A.

c) Let A be the set of books for sale in certain book store and assume the among these are books with the following properties.

Book	Price in Rs.	No. of pages
P	50	100
Q	125	125
R	80	150
S	50	200
T	25	100

Suppose $(a, b) \in R$ if and only if the price of book a is higher than or equal to the price of book b and the length of book a is greater than or equal to the length of book b.

i. List the elements of R

ii. Is R reflexive, symmetric, transitive?

06. a) Determine whether the following are binary operation

i. for $x,y \in \mathbb{R}$, $x * y = x^y$

ii. for non zero integers m & n, $m * n = m \div n$

b) Determine whether the binary operation * defined below are commutative or associative.

i. * defined on $\mathbb{R}$ by $x * y = y$

ii. * defined on $\mathbb{R}$ by $x*y = xy+1$

c) Let $M_2(\mathbb{R})$ denote the set of all 2×2 real matrices. Then matrix multiplication is a binary operation on $M_2(\mathbb{R})$

Let $T = \left\{A \in M_2(\mathbb{R}) : A = \begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix} \text{ for some } a \in \mathbb{R}\right\}$

i. Verify that matrix multiplication is a binary operation on T.

ii. Show that the matrix multiplication is non commutative in $M_2(\mathbb{R})$, but it is commutative in T.

iii. Show that both M_2 ($\mathbb{R}$) and T contain identities for matrix multiplication, but the identities are not the same.

07. Consider the following recursive relations

$X_{n+1} = \lambda X_n(1 - X_n)$ -------(1)

$Y_{n+1} = \lambda Y_n$ -------(2)

a) i. Iterate the given recursive relations for λ= 2.7 and indicate the initial condition $x_0 = y_0 = 0.8$ (at least 5 iteration steps are necessary).

ii. Mathematically explain why did you use the equation $X_{n+1} = \lambda X_n(1 - X_n)$ for represent to the population growth in a limited eco system without using $Y_{n+1} = \lambda Y_n$

b) i. Graphically show that $X_{n+1} = \lambda X_n(1 - X_n)$ for λ=2.8 when n tends to $\propto$, x tends to some x * value, taking $x_0 = 0.8$ and x_0=0.1.

ii. Do you get the same x* value for $X_{n+1} = \lambda X_n(1 - X_n)$ when $x_0 = 0.8$ & $x_0 = -0.8$.

c) Draw the bifurcation diagram and comment on the diagram when λ varies from 0 to 3.5. Using the diagram how many solutions have part (b).

08. A three dimensional system is governed by the following 3 differential equations.

$$\frac{dx}{dt} = x + 3y + 3z$$

$$\frac{dy}{dt} = -3x - 5y - 3z$$

$$\frac{dz}{dt} = 3x + 3y + z$$

At $t = 0$, $(x,y,z) = (1,1,1)$

Find the phase space value (x_n, y_n, z_n) for $n = 1, 2, 3$

09. a) A graph is to have 6 vertices and 9 edges and each vertex is to be of degree 2,3 or 4. What are the possibilities for the number of vertices of each degree.

b) Show that a tree with n vertices have exactly n – 1 edges.

c) Is there a simple bipartite graph with degree sequence 3, 2, 2, 2, 1, 1, 1.