040

The Open University of Sri Lanka

ECX 6242 Modern Control Systems

Friday 27th March 2009, 1400 - 1700 hrs.

Three hours

Up to five questions may be attempted, selecting at least **two** questions from each section. However, full credit may be obtained for exceptionally good answers to only four questions. All questions carry equal marks.

Section A

1. Consider the following system of non-linear equations (with its origins in Biology):

$$\dot{x_1} = x_1(1-\propto x_2)$$
$$\dot{X_2} = X_2(1-\beta X_1)$$

where α and β are positive constants. Determine its equilibrium point(s), if any.

Study the stability of the system for small perturbations around the equilibrium point (s) by constructing linear model(s) of the form

$$\begin{bmatrix} \delta\dot{x_1} \\ \delta\dot{x_2} \end{bmatrix} = J \begin{bmatrix} \delta x_1 \\ \delta x_2 \end{bmatrix}$$

for small disturbances about the equilibrium point(s)

2. Consider the linear system defined by the state equations:

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \end{bmatrix} = \begin{bmatrix} 1 & -\frac{\propto}{\beta} \\ -\frac{\beta}{\alpha} & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

It is desired to stabilise the system using state feedback, such that the system with feedback represented by:

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \end{bmatrix} = \begin{bmatrix} 1 & -\frac{\propto}{\beta} \\ -\frac{\beta}{\alpha} & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} - [L] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \text{ where } [L] = \begin{bmatrix} l_1 & 0 \\ 0 & l_2 \end{bmatrix}$$

has eigen values of -1 and -1. Design the state feedback controller to achieve this objective.

3. Comment on the controllability and observability of the system governed by the following state and observation equations:

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ -0.5 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$y = \begin{bmatrix} 2 & 0.5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

4. Consider the continuous time double integrator described by the state and. observation equations:

$$\frac{d^2x}{dt^2} = u(t)$$
$$y = x$$

Obtain its equivalent discrete time system, if the sampling period is ΔT.

If we are interested in transferring the state from (2,-2) to the origin (that is 0,0) over the time interval (0,1) while minimising the cumulative square of the effort u over the time interval, that is:

$$\int_0^1 u^2 dt$$

determine what the optimum effort u* should be.

Section B

Read the passage given in the Appendix before you attempt the questions in this section.

5. State TWO of the main assumptions about the plant and its environment on which the Wiener-Hopf-Kalman optimal control theory was built and ONE reason why its problem – formulation pattern suited the problems posed by the space program.

6. What are the assumptions regarding system and observation noise made in the formulation of the standard Kalman filter that you have studied?

7. Why was the Wiener-Hopf-Kalman optimal control theory found wanting in the industrial situation? State TWO of the conditions that a satisfactory methodology had to meet in order to succeed in such an environment.

8. Why does the text refer to the Nyquist condition for stability as "not encircling the +1 point" when it is normally stated as "not encircling the -1 point"?

Appendix: From LINEAR ROBUST CONTROL by Michael Green David J.N. Limebeer

1 Introduction

1.1 Goals and origins of H_∞ optimal control

Most engineering undergraduates are taught to design proportional-integral-derivative (PID) compensators using a variety of different frequency response techniques. With the help of a little laboratory experience, students soon realize that a typical design study involves juggling with conflicting design objectives such as the gain margin and the closed-loop bandwidth until an acceptable controller is found. In many cases these "classical" controller design techniques lead to a perfectly satisfactory solution and more powerful tools hardly seem necessary. Difficulties arise when the plant dynamics are complex and poorly modelled, or when the performance specifications are particularly stringent. Even if a solution is eventually found, the process is likely to be expensive in terms of design engineer's time.

When a design team is faced with one of these more difficult problems, and no solution seems forthcoming, there are two possible courses of action. These are either to compromise the specifications to make the design task easier, or to search for more powerful design tools. In the case of the first option, reduced performance is accepted without ever knowing if the original specifications could have been satisfied, as classical control design methods do not address existence questions. In the case of the second option, more powerful design tools can only help if a solution exists.

Any progress with questions concerning achievable performance limits and the existence of satisfactory controllers is bound to involve some kind of optimization theory. If, for example, it were possible to optimize the settings of a PID regulator, the design problem would either be solved or it would become apparent that the specifications are impossible to satisfy (with a PID regulator). We believe that answering existence questions is an important component of a good design methodology. One does not want to waste time trying to solve a problem that has no solution, nor does one want to accept specification compromises without knowing that these are necessary. A further benefit of optimization is that it provides an absolute scale of merit against which any design can be measured—if a design is already all but perfect, there is little point in trying to improve it further. The aim of this book is to develop a theoretical framework within which one may address complex design problems with demanding specifications in a systematic way.

Wiener-Hopf-Kalman optimal control

The first successes with control system optimization came in the 1950s with the introduction of the Wiener-Hopf-Kalman (WHK) theory of optimal control. At roughly the same time the United States and the Soviet Union were funding a massive research program into the guidance and maneuvering of space vehicles. As it turned out, the then new optimal control theory was well suited to many of the control problems that arose from the space program. There were two main reasons for this:

1. The underlying assumptions of the WHK theory are that the plant has a known linear (and possibly time-varying) description, and that the exogenous noises and disturbances impinging on the feedback system are stochastic in nature, but have known statistical properties. Since space vehicles have dynamics that are essentially ballistic in character, it is possible to develop accurate mathematical models of their behavior. In addition, descriptions for external disturbances based on white noise are often appropriate in aerospace applications. Therefore, at least from a modelling point of view, the WHK theory and these applications are well suited to each other.

2. Many of the control problems from the space program are concerned with resource management. In the 1960s, aerospace engineers were interested in minimum fuel consumption problems such as minimizing the use of retrorockets. One famous problem of this type was concerned with landing the lunar excursion module with a minimum expenditure of fuel. Performance criteria of this type are easily embedded in the WHK framework that was specially developed to minimize quadratic performance indices.

Another revolutionary feature of the WHK theory is that it offers a true synthesis procedure. Once the designer has settled on a quadratic performance index to be minimized, the WHK procedure supplies the (unique) optimal controller without any further intervention from the designer. In the euphoria that followed the introduction of optimal control theory, it was widely believed that the control system designer had finally been relieved of the burdensome task of designing by trial and error. As is well known, the reality turned out to be quite different.

The wide-spread success of the WHK theory in aerospace applications soon led to attempts to apply optimal control theory to more mundane industrial problems. In contrast to experience with aerospace applications, it soon became apparent that there was a serious mismatch between the underlying assumptions of the WHK theory and industrial control problems. Accurate models are not routinely available and most industrial plant engineers have no idea as to the statistical nature of the external disturbances impinging on their plant. After a ten year re-appraisal of the status of multivariable control theory, it became clear that an optimal control theory that deals with the question of plant modelling errors and external disturbance uncertainty was required.

Worst-case control and H_∞ optimization

H_∞ optimal control is a frequency-domain optimization and synthesis theory that was developed in response to the need for a synthesis procedure that explicitly addresses questions of modelling errors. The basic philosophy is to treat the worst case scenario:

if you don't know what you are up against, plan for the worst and optimize. For such a framework to be useful, it must have the following properties:

1. It must be capable of dealing with plant modelling errors and unknown disturbances.

2. It should represent a natural extension to existing feedback theory, as this will facilitate an easy transfer of intuition from the classical setting.

3. It must be amenable to meaningful optimization.

4. It must be able to deal with multivariable problems.

In this chapter, we will introduce the infinity norm and H_∞ optimal control with the aid of a sequence of simple single-loop examples. We have carefully selected these in order to minimize the amount of background mathematics required of the reader in these early stages of study; all that is required is a familiarity with the maximum modulus principle. Roughly speaking, this principle says that if a function f (of a complex variable) is analytic inside and on the boundary of some domain D, then the maximum modulus (magnitude) of the function f occurs on the boundary of the domain D. For example, if a feedback system is closed-loop stable, the maximum of the modulus of the closed-loop transfer function over the closed right-half of the complex plane will always occur on the imaginary axis.

To motivate the introduction of the infinity norm, we consider the question of robust stability optimization for the feedback system shown in Figure 1.1. The transfer function g represents a nominal linear, time-invariant model of an open-loop system and the transfer function k represents a linear, time-invariant controller to be designed. If the "true" system is represented by $(1+\delta)g$, we say that the modeling error is represented by a multiplicative perturbation δ at the plant output. For this introductory analysis, we assume that δ is an unknown linear, time-invariant system.

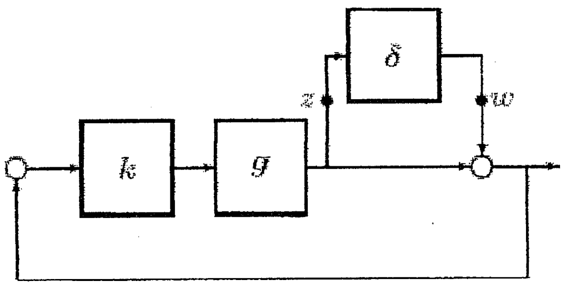

Figure 1.1: The problem of robust stability optimization.

Since

$$z = (1 - gk)^{-1} gkw,$$

the stability properties of the system given in Figure 1.1 are the same as those given in Figure 1.2, in which

$$h = (1 - gk)^{-1} gk.$$

If the perturbation δ and the nominal closed-loop system given by h are both stable,

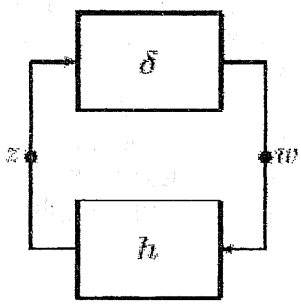

Figure 1.2: The small gain problem.

the Nyquist criterion says that the closed-loop system is stable if and only if the Nyquist diagram of hδ does not encircle the +1 point. We use the +1 point rather than the −1 point because of our positive feedback sign convention. Since the condition

$$\sup_{\omega} |h(j\omega)\delta(j\omega)| < 1. \tag{1.1.1}$$

ensures that the Nyquist diagram of hδ does not encircle the +1 point, we conclude that the closed-loop system is stable provided (1.1.1) holds.