The Open University of Sri Lanka
Department of Electrical and Computer Engineering

083

ECX 6241 – Field Theory
Final Examination – 2008/2009

Date: 09.03.2009 Time: 0930 - 1230

This paper contains EIGHT questions. Answer any FIVE questions. All questions carry equal marks. Show your work clearly.

Q1)

a. What is the "Poynting's Vector" and its physical meaning?

b. Calculate the total instantaneous power flow W leaving a closed surface S by using

$$W = \oint (\underline{E} \times \underline{H}).dS$$

Use the vector identity

$div(\underline{E} \times \underline{H}) = \underline{H}.curl\underline{E} - \underline{E}.curl\underline{H}$,to show where the power leaving a close surface goes, in terms of stored energy and ohmic dissipation.

c. An antenna is centered at the origin of a spherical coordinate system as shown in figure 01. The fields produced by the antenna at coordinate (r, θ, ϕ) are given by

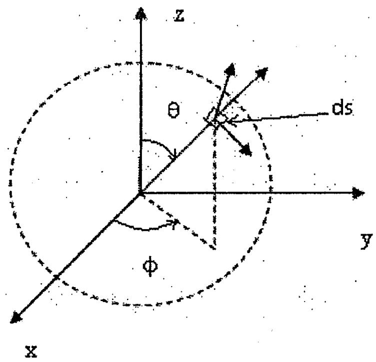

Figure 01

$$\underline{E} = \frac{E_0}{r}\sin\theta \sin\omega\left(t - \frac{r}{u_0}\right)\underline{u_\theta}$$

$$\underline{H} = \frac{E_0}{r\sqrt{\mu_0/\varepsilon_0}}\sin\theta \sin\omega\left(t - \frac{r}{u_0}\right)\underline{u_\phi}$$

i. Derive an equation for the total instantaneous power leaving the surface of the sphere with radius R (assume σ = 0)

ii. Hence show that the total average power leaving (radiated by) the antenna is

$$\frac{4\pi}{3}\frac{E_0^2}{\sqrt{\mu_0/\varepsilon_0}}$$

Q2)

"Plane wave propagation involves sending signals through open space"

As shown in figure 02, a plane wave is originated in the lossy medium 0, where $\sigma \neq 0$, $\rho = 0$, and propagating through the **dielectric mediums 1& 2** respectively in the z direction.

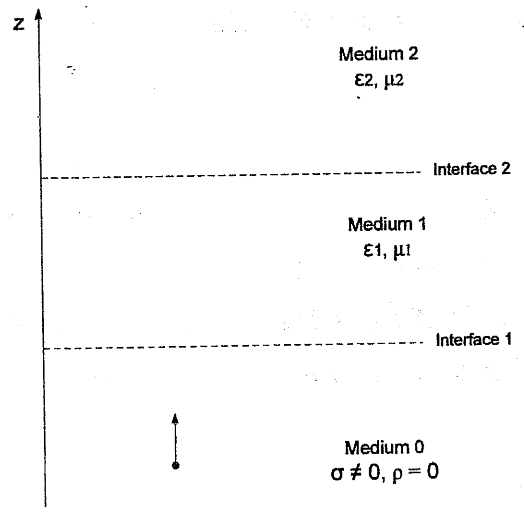

Figure 02

a. Derive the wave (governing) equation for the Electric field (*E*), when the plane wave is in the medium 0.
b. If the variation of Electric field (*E*) with time is sinusoidal, modify the above equation for sinusoidal Electric field. Hence write down the equation for propagation constant γ
c. Show that the Electric field mentioned above has two components
   i. Forward wave
   ii. Backward wave
d. Now the wave enters into the medium 1. Derive equation for the attenuation constant α and phase constant β in terms of ω, μ_1, and ε_1.
e. Considering the interface 2 (dielectric-dielectric interface), derive equations for transmission coefficient τ in this interface (assume that the plan wave perpendicularly meets the interface 2).

Q3)

a. State Coulomb's law for electrostatic field.

b. A ring C, $x^2 + y^2 = a^2$, has line charge density $\left(x^3 \underline{u_x} + y^3 \underline{u_y}\right)$. Show that the total charge on the ring is given by the expression $3\pi a^4$.

c. Now the ring C is uniformly charged with q C. Find the electric field at a field point P at distance D above the axis of the ring.

d. Using the result derived in part (c), show that the electric field caused at a distance D from center of the charged shell of radius R and charge q is given by

$$\underline{E} = \frac{q}{4\pi\varepsilon D^2}\underline{u_r} \qquad D > R$$

$$\underline{E} = 0 \qquad D < R$$

Q4)

a. Write down Maxwell's Equations.

b. Show that the fields

$$E = E_m \sin x \sin t \ \underline{u_y}$$

$$H = \frac{E_m}{\eta_0} \cos x \cos t \ \underline{u_z}$$ in the free space are not valid solutions of Maxwell's Equations.

($\underline{u_x}$, $\underline{u_y}$ and $\underline{u_z}$ are the unit vectors in x, y and z directions respectively)

c. Two plane waves with the same frequency have amplitudes E_{ox} and E_{oy} at right angle to one another, with phases φ_x, φ_y. Sketch how the real part of the sum of the wave will vary on x-y plot, if $E_{ox} = 2\,E_{oy}$ and $\varphi_x - \varphi_y = 180^0$

d. A plane wave with angular frequency ω travels in the direction of the vector $x\,\underline{u_x} + y\,\underline{u_y}$ with the constant velocity v.

   i. Express the electric field vector in the terms of ω, β, x, y and t, where β is the phase constant.

   ii. Find the magnetic field of the plane wave.

Q5)

a. The phasor voltage and phasor current distributions on a infinitely long transmission line (i.e: no reflection) can be represented as,

$$V(z,t) = V_0 e^{(j\omega t - \gamma z)}$$

$$I(z,t) = \frac{V_0}{Z_0} e^{(j\omega t - \gamma z)}$$

Where Z_0 $(R_0 + jX_0)$ is the characteristic impedance of the line.

γ $(\alpha + j\beta)$ is the propagation constant.

i. Determine the average power of the wave on the transmission line.

ii. Hence prove that the attenuation coefficient α can be written as $\alpha = \frac{W_L}{2W_T}$, where W_L is the power loss with distance on the line, and W_T is the power being transported along the line.

b. Prove that the input impedance Z_i at point $z = 0$ on a transmission line of characteristics impedance Z_0 terminated at $z = l$ with the load Z_L is

$$Z_i = Z_0\left[\frac{Z_l\cos(\beta l) + jZ_0\sin(\beta l)}{Z_0\cos(\beta l) + jZ_l\sin(\beta l)}\right]$$ (Assume attenuation constant $\alpha = 0$)

c. Use the result above to prove that the characteristic impedance Z_0 of a transmission line is given by the equation, $Z_0 = \sqrt{Z_{io}Z_{is}}$

Where Z_{io} is the open circuited input impedance

Z_{is} is the short circuited input impedance

Q6)

a. What do you mean by the term "cut-off frequency" in the waveguide theory?

b. Prove that the cut-off frequency f_c of an air-filled ***mn*** mode rectangular waveguide is given by

$$f_c = f\sqrt{1 - \frac{v_g}{v_p}},$$

Where f is the operating frequency of the waveguide

v_g is the Group velocity

v_p is the Phase velocity

c. The field equations for the TE_{mn} mode are given by

$$H_z = A_{mn}\cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)e^{-\gamma z}$$

$$H_x = \frac{\gamma m\pi}{k^2 a}A_{mn}\sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)e^{-\gamma z}$$

$$H_y = \frac{\gamma n\pi}{k^2 b}A_{mn}\cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)e^{-\gamma z}$$

$$E_x = \frac{j\omega\mu n\pi}{k^2 b} A_{mn} \cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)e^{-\gamma z}$$

$$E_y = \frac{-j\omega\mu m\pi}{k^2 a} A_{mn} \sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)e^{-\gamma z}$$

Prove that the time average power transmitted by the wave through the transverse (x-y) plane (z direction) in the TE_{10} mode is given by $P_{av} = \frac{b}{a}\frac{\omega\mu\beta}{4}\left(\frac{\pi A_{10}}{k^2}\right)^2$

Assume attenuation constant α is negligible.

Q7)

a. Define the term "directivity" of an antenna.
b. What is the pattern solid angle (Ω_p) and the directivity (D) for an isotropic antenna? Find the pattern solid angle (Ω_p) and the directivity (D) for a semi-isotropic antenna that radiated equally in all directions above $\theta = \pi/2$, but is zero otherwise.
c. Suppose in the far-field for a particular antenna at the origin, the electric field is

$E_{os} = \eta_0 I_0 \frac{e^{-j\beta r}}{\pi r} \sin\theta \underline{u_\theta}$, What is the radiation resistance of this antenna?

d. Determine the pattern solid angle and directivity for the following normalized radiation intensities:
   i. $P_n(\theta,\phi) = \sin\theta$
   ii. $P_n(\theta,\phi) = \sin^2\theta$

Q8)

"Microwave Link Design is a methodical, systematic and sometimes lengthy process"

a. Write down the steps that have to be followed during the Microwave Link Design process.
b. Define the term "Fersnel Zone", write down the equation for fersnel zone calculation and factors considered during the fersnel zone selection of the link design process.
c. Derive that the minimum coverage distance D from the antenna with h_t m height is given by the equation $D = \sqrt{2h_t R}$, where R is the Curvature of the earth.
d. Assume that a microwave link is designed in the metropolitan area that has huge buildings. Suggest a method to increase the receiving signal strength.