The Open University of Sri Lanka
Department of Electrical and Computer Engineering
Final Examination 2008/2009
ECX6234 – Digital Signal Processing

043

Time: 1400 – 1700 hrs. Date: 2009-03 -20

Answer any FIVE questions

1.

(a) Define Discrete Fourier Series (*DFS*) for a periodic sequence $x[n]$ with period N using standard expression.
Write expressions for
(i) $X[k] = DFS\{x[n]\}$
(ii) $x[n] = IDFS\{X[k]\}$

(b) A periodic signal $x[n]$ having a period $N = 12$ is defined as follows:

$$x[n] = \begin{cases} 1 & \text{if } 0 \le n \le 5 \\ 0 & \text{if } 6 \le n \le 11 \end{cases}$$

Find the values of $X[k] = DFS\{x[n]\}$

(c) For a non periodic signal $x[n]$ define

(i) Discrete Time Fourier Transform (*DTFT*)
(ii) Discrete Fourier Transform (*DFT*).

(d) (i) For the signal $x[n] = 0.8^n u[n]$ find $X(\omega) = DTFT\{x[n]\}$
(ii) Find $DFT\{x[n]\}$ if $x[n] = \{1, 1, -1, -1\}$

(e) What is understood by linearity? Show that the DFT operation is linear.

2.

(a) Define following terms with reference to *a discrete system*:

(i) Discrete linear system.
(ii) Discrete time invariant system.
(iii) Causal system.
(iv) *BIBO* stability.

(b) Determine whether the following systems are *causal* and *BIBO* stable.

(i) $y[n] = 3x^2[n] + 2x[n-1] + x[n-2]$
(ii) $y[n] = nx[n] + x[n-1]$

(c) A linear time invariant system has an impulse response $h[n] = 0.8^n u[n]$.
Determine the output sequence $y[n]$ for the following inputs:
(i) $x[n] = 1.2^n u[-n]$.
(ii) $x[n] = u[n-3]$.

(d) Calculate from the first principles, the z-transform and the region of convergence (*ROC*) for the following signals:

(i) $u[n]$.
(ii) $\delta[n-m]$.

3.

(a) Z- transform of a sequence $x[n]$ is given by $X(z) = \dfrac{z(z-5)}{(z-1)(z-3)(z-5)}$.
(i) Give all the possible regions of convergence.
(ii) For which *ROC* is the $X(z)$ the z-transform of a causal sequence?

(b) Find $x[n]$ if,

(i) $X(z) = -\dfrac{(z+1)}{(z-1)(z-2)}, \qquad 1 < |z| < 2$

(ii) $X(z) = \dfrac{z}{(z^2+1)}, \qquad |z| > 1$

(c) From the first principles prove that the z- transform of $x[n-n_0]$ is $z^{-n_0}.X(z)$.

(d) Find (i) the impulse response $h[n]$
(ii) the step response $s[n]$
of the discrete system described by the difference equation

$$y[n] + \frac{5}{6}y[n-1] - \frac{1}{6}y[n-2] = x[n]$$

4.
(a) Consider the difference equation

$$y[n] + a_1 y[n-1] + ... + a_N y[n-N] = x[n] + b_1 x[n-1] + ... + b_N x[n-N]$$

What can you say about the coefficients $a_1, a_2, ..., a_N, b_1, b_2, ... b_N$ if the above equation represents (i) a *FIR* filter?

(ii) an *IIR* filter?

(b) Suppose the desired frequency response of a filter is $H_d(e^{j\omega})$. The transfer function of the filter $H(z)$ is given by

$$H(z) = A\prod_{k=1}^{K}\frac{1+a_k z^{-1}+b_k z^{-2}}{1+c_k z^{-1}+d_k z^{-2}} = AG(z)$$

Suppose the mean square error $E = \sum_{i=1}^{M}[|H(e^{j\omega_i})| - |H_d(e^{j\omega_i})|]^2$

E can be thought of as a function of the parameters $a_1, b_1, c_1, d_1, a_2, b_2, c_2, d_2, ..., d_K, A$

The parameters for $H(z)$ can be found out by minimizing the mean square error. (When E is minimum, the partial derivatives with respect to the above parameters become zero).

(i) Find the total number of parameters in E.

(ii) How many equations will result in the mean square method? Write these equations in the general form (actual calculations are not required).

(iii) Derive an expression for $\frac{\partial E}{\partial |A|}$.

(iv) Show that $|A| = \frac{\sum_{i=1}^{M}|G(e^{j\omega_i})| - |H_d(e^{j\omega_i})|}{\sum_{i=1}^{M}|G(e^{j\omega_i})|^2}$

5.

In the least square design of *IIR* filters following criteria are used:

(i) The output of the inverse of $H(z)$ must approximate a unit sample when the input is $h_d(n)$, where $h_d(n)$ is the desired impulse response and $H(z)$ is the transfer function of the filter.

(ii) Coefficients of $H(z)$ are chosen so as to minimize the Error $E = \sum_{n=1}^{\infty}(v(n))^2$ where $v(n)$ denotes the output of the inverse system with the transfer function $1/H(z)$.

Thus we can write the relation $V(z) = \dfrac{H_d(z)}{H(z)}$.

Assume the filter transfer function $H(z)$ to be of the form

$$H(z) = \frac{b_0}{1 - \sum_{k=1}^{N} a_k z^{-k}}.$$

(a) Write the difference equation relating $v(n)$ and $h_d(n)$.

(b) Show that $E = \dfrac{1}{b_0^2} \sum_{n=1}^{\infty} (h_d(n))^2 - 2\sum_{n=1}^{\infty} h_d(n) \sum_{k=1}^{N} a_k h_d(n-k) + \sum_{n=1}^{\infty} \left[\sum_{k=1}^{N} a_k h_d(n-k) \right]^2$.

(c) Find $\dfrac{\partial E}{\partial a_i}$.

(d) Show that $\sum_{k=1}^{N} a_k \sum_{n=1}^{\infty} h_d(n-k)(n-i) = \sum_{n=1}^{\infty} h_d(n)(n-i)$ for E to become a minimum with respect to a_i.

6.

(a) Sketch the impulse response of an ideal low pass filter $h_d(n)$.

(b) In order to realize the above filter as a *FIR* filter, it is necessary to truncate the desired impulse response $h_d(n)$.

Therefore the impulse response of the *FIR* filter can be written as

$$h(n) = h_d(n).w(n)$$

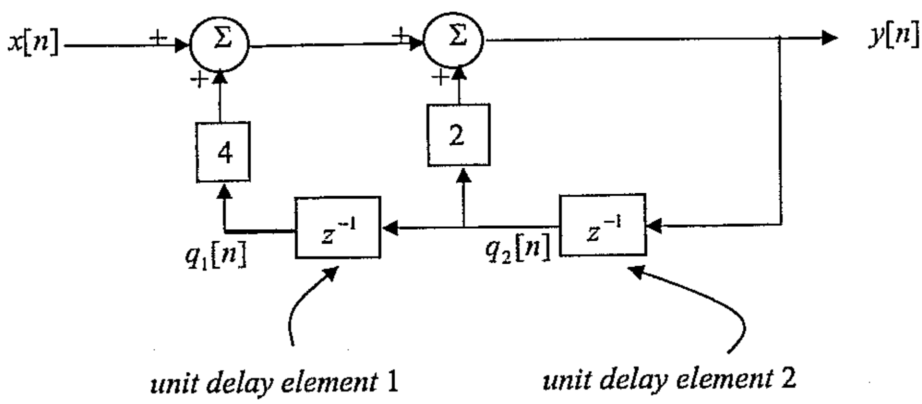
(i) What is $w(n)$?
(ii) If $w(n) = 1, \quad 0 \le n \le L$
(α) sketch $W(\omega)$.
(β) write the relationship between $H(\omega), H_d(\omega)$ and $W(\omega)$.

(c) Design a digital low pass filter using window techniques, to meet the following requirements:

Pass band = 3 kHz
Stop band = 6 kHz, with at least 30 dB attenuation.
Sample frequency = 30 kHz.

(d) How does the length of a window affect the transition width $\Delta\omega$ of the filter?

7.
(a) Consider the discrete, linear time invariant system given below:



(i) Explain different functional elements of the system.

(ii) By selecting the outputs of the unit delay elements 1 and 2 as state variables (as shown in the figure), find the state space representation of the system.

(iii) What is understood by *a unit delay element*?

(b) Briefly explain TWO digital signal processing (*DSP*) techniques used in restoration of an old image.

(c) (i) What is a Kalman filter? Describe the function of a Kalman filter.
(ii) How does the filter calculate the best value for the input estimator $\hat{x}_k$?

8.

(a) A sampling sequence $\delta_D[n]$ can be written as

$$\delta_D[n] = \begin{cases} 1 \text{ if } n/D \text{ is an integer} \\ 0 \text{ otherwise} \end{cases}$$

If the sampling- and sampled sequences are $x[n]$ and $v[n]$ respectively

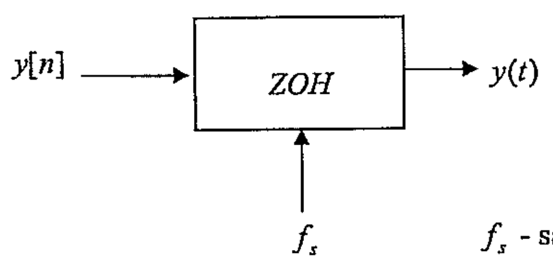
(i) write the relationship between $\delta_D[n]$, $x[n]$ and $v[n]$.

(ii) show that $V(z) = \frac{1}{D}\sum_{k=0}^{n-1} X(e^{j\frac{2\pi k}{D}}.z)$.

(b) Consider a sinusoidal signal $x[n] = 2\cos(0.1\pi n)$

Determine the frequency spectrum of the signal $v[n] = \delta_3[n].x[n]$

(c) Zero order hold (*ZOH*) is used as a reconstructor. It takes a numerical sequence $y[n]$ and outputs a continuous time signal $y(t)$.



f_s - sampling frequency
T_s - sampling period

ZOH is linear and uses the interpolating function $g(t)$ (to provide interpolation between the samples). To find $y(t)$ first we weight each $y[n]$ using $g(t - nT_s)$. Then all the weighted $y[n]$'s are added together to form $y(t)$.

(i) write the relationship between $y(t)$, $g(t)$ *and* $y[n]$.

(ii) Show that $Y(\omega) = G(\omega)(\sum_{n=-\infty}^{\infty} e^{-j\omega nT_s} y[n])$.

$Y(\omega)$ and $G(\omega)$ are the Fourier transforms of $y(t)$ and $g(t)$ respectively.

<u>**Supplementary material**</u>

(a) Window functions

Window type	$w(n)$	$\Delta\omega$	Attenuation
Rectangular	1	$\frac{4\pi}{N}$	-13dB
Bartlett	$\frac{2}{N-1}\left(\frac{N-1}{2}-\left\lvert n-\frac{N-1}{2}\right\rvert\right)$	$\frac{8\pi}{N}$	-27dB
Hanning	$0.5+0.5\cos\left(\frac{2\pi n}{N-1}\right)$	$\frac{8\pi}{N}$	-32dB
Hamming	$0.54+0.46\cos\left(\frac{2\pi n}{N-1}\right)$	$\frac{8\pi}{N}$	-43dB
Blackman	$0.42+0.5\cos\left(\frac{2\pi n}{N-1}\right)+0.08\cos\left(\frac{2\pi n}{N-1}\right)0{\cdot}42+$	$\frac{12\pi}{N}$	-53dB

(b) Some important Z-transforms

Function	z-transform	*ROC*
$\delta[n]$	1	All z
$u[n]$	$\frac{z}{z-1}$	$\lvert z\rvert > 1$
$a^n u[n]$	$\frac{z}{z-a}$	$\lvert z\rvert > a$
$-a^n u[-n-1]$	$\frac{z}{z-a}$	$\lvert z\rvert < a$
$nx[n]$	$X\left(\frac{1}{z}\right)$	$R'=\frac{1}{R}$
$x[-n]$	$-z\frac{dX(z)}{dz}$	$R'=R$
$z_0^n x[n]$	$X\left(\frac{z}{z_0}\right)$	$R'=\lvert z_0\rvert R$
$e^{j\Omega_0 n}x[n]$	$X(e^{j\Omega_0 n}z)$	R'=R

$X(z)$ is the z-transform of $x[n]$. R is the *ROC* of $X(z)$