The Open University of Sri Lanka
Department of Electrical and Computer Engineering
ECX6233– Microwave Communication Systems
Final Examination 2008/2009

072

Time : 0930 - 1230 hrs. Date: 2009- 03 - 18

Answer any FIVE questions.

1.
Consider a time varying electromagnetic field in an isotropic medium.

(a) Write an expression for $\nabla \times \vec{H}$.

(b) Using the vector identity $\nabla.(\nabla \times \vec{A}) = 0$, show that $\rho\frac{\sigma}{\varepsilon} + \frac{\partial \rho}{\partial t} = 0$, where ρ is the electric charge density and σ is conductivity of the medium respectively.

(c) Show that the solution for ρ is given by $\rho = \rho_0 e^{-\left(\frac{\sigma}{\varepsilon}\right)t}$.

(d) For copper $\sigma = 5.8 \times 10^7 \ \Omega^{-1}m^{-1}$ and $\varepsilon = \varepsilon_0$.
Show that ρ disappears within an extremely short interval compared to the period of a microwave.

(e) Prove that

$$\nabla^2 \vec{E} = \mu\sigma\frac{\partial \vec{E}}{\partial t} + \mu\varepsilon\frac{\partial^2 \vec{E}}{\partial t^2}$$

(f) If field components vary according to $e^{j\omega t}$, show that the propagation constant γ is given by $\gamma^2 = j\omega\mu(\sigma + j\omega\varepsilon)$.

(g) Show that $\gamma = \sqrt{\omega\mu\sigma}\ \angle 45^0$ for a good conductor.

2.
(a) In order to calculate the field components within a rectangular waveguide, we solve Maxwell's equations for the inner space of the waveguide. What boundary conditions are to be considered here?

(b) Field components inside a rectangular waveguide for *Transverse Electric* mode is given below:

$$\vec{E}_x = j\omega\mu\left(\frac{n\pi}{b}\right)\vec{A}\cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)e^{-j\beta z}$$

$$\vec{E}_y = -j\omega\mu\left(\frac{m\pi}{a}\right)\vec{A}\sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)e^{-j\beta z}$$

$$\vec{H}_x = -j\beta\left(\frac{m\pi}{a}\right)\vec{A}\sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)e^{-j\beta z}$$

$$\vec{H}_y = j\beta\left(\frac{n\pi}{b}\right)\vec{A}\cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)e^{-j\beta z}$$

$$\vec{H}_z = \beta_c^2\left(\frac{n\pi}{b}\right)\vec{A}\cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)e^{-j\beta z}$$

where $\beta_c^2 = \sqrt{\left(\frac{m\pi x}{a}\right)^2 + \left(\frac{n\pi y}{b}\right)^2}$, $\vec{A}$ - constant

(i) Write an equation for $\vec{H}_z$.

(ii) Calculate and sketch the E-field pattern across the waveguide for TE_{20} mode.

For TE_{11} mode,

(iii) find the direction of lines of force of the E-field across the waveguide when
(α) $x = 0, y = b/2$.
(β) $x = a/2, y = 0$.
(γ) $x = a/2, y = b$.
(δ) $x = a, y = b/2$.

(iv) find the value of E-field strength when $x = a/2, y = b/2$.

(v) Sketch the results of (iii) and (iv).

(vi) Sketch the total E-field across the waveguide.

3.

Propagation constant γ_{mn} (for mn^{th} mode) in a rectangular waveguide is given by

$$\gamma_{mn} = j\beta_{mn} = j\sqrt{(k^2 - k_{c,mn}^2)}$$

(i) If the guide dimensions are a and b, write the values of k and k_{cmn}. Take the frequency and the wavelength of the signal as λ and f respectively.

(ii) Derive an expression for the cutoff frequency $f_{c,mn}$.

Suppose $a{:}b = 2:1$

(iii) What is the dominant TE mode in a rectangular waveguide?
Derive an expression for the cutoff wave length for the dominant TE mode.

(iv) If the cutoff frequency of the dominant mode is f_0, derive an expression for $f_{c,mn}$ (the cutoff frequency for TE_{mn} mode) in terms of f_0.
Find the values of $f_{c,mn}$ for TE_{21} and TE_{32} modes.

(v) If the cutoff frequency for dominant TM mode is f_0', find f_0'/f_0.

(vi) A waveguide having a breadth $a = 6$ cm is to be used as a cutoff attenuator in TE_{10} mode.
How would you select the frequency of the input signal?
Find the attenuation per meter when $f = 2$ GHz.

4.

(a) A Bethe-hole coupler has following secondary magnetic field components for TE_{10} mode:

$$B_1 = -\frac{j\omega\varepsilon_0}{abY_\omega}\frac{2}{3}\gamma_0^3 A\sin^2\left(\frac{\pi d}{a}\right)$$

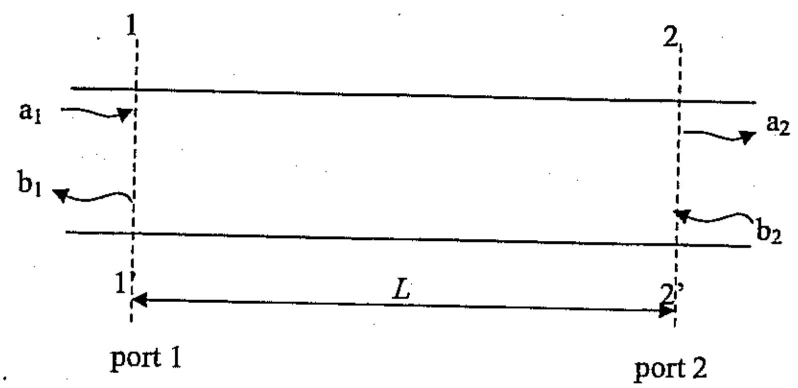
$$B_2 = \frac{j\omega\mu_0 Y_\omega}{ab}\frac{4}{3}\gamma_0^3 A\left[\sin^2\left(\frac{\pi d}{a}\right) + \frac{\pi^2}{\beta^2 a^2}\cos^2\left(\frac{\pi d}{a}\right)\right]$$

$$B_3 = \frac{j\omega\mu_0 Y_\omega}{ab}\frac{4}{3}\gamma_0^3 A\left[-\sin^2\left(\frac{\pi d}{a}\right) + \frac{\pi^2}{\beta^2 a^2}\cos^2\left(\frac{\pi d}{a}\right)\right]$$

d – distance of the aperture from the side.
γ_0 - radius of the aperture.
a, b – wave guide dimensions.
Y_ω – wave admittance.
β – phase constant.
B_1 – magnetic field radiated in both the directions by an electric dipole.
B_2 – magnetic field radiated by the magnetic dipole in the direction of port 4.
B_3 – magnetic field radiated by the magnetic dipole in the direction of port 3.

(i) Draw a sketch of a Bethe-hole coupler. Show that $\tan\left(\frac{\pi d}{a}\right) = \frac{\lambda}{\sqrt{6}a}$ if the port 3 is decoupled, where λ is the wave length.
(ii) Mention an application of the above coupler.

(b) Given below is a section of a uniform transmission line of length L.



Incident- and the reflected waves are represented by a_n and b_n respectively (n = 1, 2 ; m = 1, 2). Assuming that the line is lossless and the phase constant to be β, write the relationship between

(i) a_1 and b_2 (ii) a_2 and b_1.

Write the s- matrix for the section.

(c) In a lossless E-plane Tee, the auxiliary arm is port3. A generator is connected to the auxiliary arm.
Sketch the Tee junction and draw the E-field distribution in ports 1, 2 and 3.
If a cross section is taken at port 3, how would you see the H-field distribution?

(d) Evaluate the s- matrix for a lossless H-plane Tee, if port 3 is matched.

5.

(a) Why is the angle of twist of a Faraday rotation isolator 45^0? What happens if this angle is changed to 50^0?

(b) How can the depth of modulation of a ferrite modulator be controlled?

(c) Find the s-matrix of a lossless, matched, non-reciprocal 3-port microwave junction. Describe the principle of operation of such a device.

(d) (i) With the help of a diagram describe the principle of operation of a directional coupler.

(ii) State the necessary and sufficient conditions for a device to be a directional coupler.

(iii) An antenna is to be connected to a reflex klystron (for test purposes) and to a transmitter simultaneously.
With the help of a diagram explain how this is done using a directional coupler.

6.

(a) Write a short note on cylindrical waveguide and show the E-field distribution of the TE_{11} mode. What is the cutoff wavelength of this mode?

(b) Electrical field component in the z-direction inside a cylindrical waveguide for *TM* waves is given by $E_z = B_n J_n(\rho k_c)\cos n\varphi$, where B_n is a constant and $J_n(\rho k_c)$ is a Bessel function of the first kind and n^{th} order. Radius of the waveguide is a.

(i) Show that $J_n(k_c a) = 0$.

(ii) If the m^{th} root of the above equation is ρ_{nm}, write an expression for $\beta_{g,nm}$ the propagation constant for the nm^{th} mode.

(iii) Using the expression for $\beta_{g,nm}$, derive an expression for the cutoff frequency for TM_{mn} mode.

(iv) If the cutoff frequency of the waveguide for TM_{21} mode is 7 GHz, find the radius of the waveguide.

Some roots of $J_n(k_c a) = 0$ are given below:

Value of n	ρ_{nm} $(= k_c a)$		
	ρ_{n1}	ρ_{n2}	ρ_{n3}
0	2.405	5.520	8.654
1	3.832	7.016	10.174
2	5.135	8.417	11.602

(v) What are the modes that can propagate at 10 GHz in the above waveguide?

7.

(a) The frequency of a rectangular cavity resonator can be changed using a movable short-circuiting plunger in the cavity. The plunger changes the length of the cavity. Cross-sectional dimensions of the cavity are $a = 30$ mm and $b = 15$ mm. For the entire range of oscillations in the TE_{102} resonance mode, the variation of the cavity length d was observed to be between 4 cm and 5 cm.

Find the output frequency range of the resonator.

(b) A cylindrical resonator has a diameter of 6 cm and a length of 8 cm. Find the resonance frequency of the resonator in the TM_{211} mode.

(c) (i) With the help of a diagram describe how the output of a cylindrical cavity resonator can be coupled to a coaxial cable using *inductive coupling*.

(ii) If the above coaxial cable is matched terminated what can you say about the reflection coefficient at the termination?

(d) What are the types of matched terminations used in microwave applications? Briefly describe their principle of operation.

8.

(a) Discuss design aspects of a parabolic reflector used in satellite communication.

(b) Draw a block diagram of a satellite communication receiver. In your diagram include all the devices up to the de-multiplexer.

(c) Describe the principle of operation of (i) Travelling Wave Tube (*TWT*)
(ii) Klystron

What is the main advantage of TWT over Klystron?

(d) What are the problems associated with the alignment of a microwave link?