00112

The Open University of Sri Lanka
Department of Electrical and Computer Engineering
Final Examination 2009/2010
ECX5233 – Radio and Line Communication

Time: 1400 – 1700 hrs. Date: 2010-03 -26

Answer any FIVE questions

1.

(a) With the help of a diagram briefly explain the principle of

(i) Frequency Division Multiplexing (*FDM*)

(ii) Time Division Multiplexing (*TDM*)

(b)

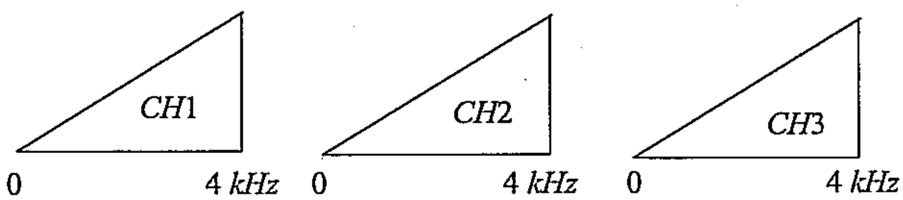

Fig.1

Three voice channels *CH*1, *CH*2 and *CH*3 are shown in Fig.1. Each channel has a bandwidth of 4 *kHz*.

(i) If the 3 channels are Frequency Division Multiplexed,

(α) sketch the *FDM* signal.
(β) give the modulation technique used in the *FDM* process.

(δ) give the values of the sub-carrier frequencies used for above modulation technique.

(γ) Now the *FDM* signal is amplitude modulated and transmitted. If the carrier frequency is 100 *kHz*, sketch the frequency spectrum of the modulated carrier. Indicate all the important frequency values on your sketch.

(ii) Suppose Time Division Multiplexing is used in the transmission of the channels.

(α) What is the lowest possible sampling frequency for each channel?
(β) If each sample is digitized into a 8-bit word, calculate the speed of data transmission in *bps*.

2.
(a)

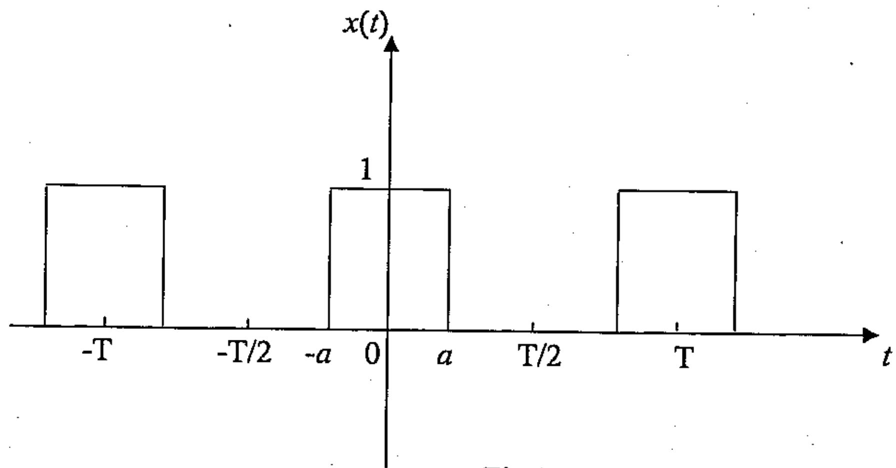

Fig.2

(i) Find the Fourier series of the signal $x(t)$ shown in Fig.2.

(ii) Sketch the frequency spectrum of $x(t)$. Indicate all the important frequency values on your diagram.

(iii) Repeat (ii) for the following values of a.

(α) $a = \frac{T}{8}$
(β) $a = \frac{T}{4}$

(iv) What is the impact of pulse width on the frequency spectrum?

(v) Consider the sketch of (ii).

(α) What is the distance between two consecutive spectral lines?
(β) What changes to the frequency spectrum will take place when T is increased?
(γ) Sketch the frequency spectrum when $T \to \infty$. Interpret this result.

(b) A repetitive waveform $x(t)$ is given by $x(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$.

(i) Sketch $x(t)$ and express $x(t)$ as a Fourier series.

(ii) Show that the Fourier Transform $X(\omega)$ of $x(t)$ is $\frac{1}{T}\sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_0)$,

where $\omega_0 = \frac{2\pi}{T}$.

Interpret the above result.

3.

(a) Define $x(t) * s(t)$, the convolution of the function $x(t)$ with the function $s(t)$.

Functions $x(t)$ and $s(t)$ are defined as follows:

$x(t) = m.t$ and $s(t) = 2m.t$, for $0 < t < T$

$x(t) = y(t) = 0$, otherwise

(i) Draw $x(\tau)$ and $s(t-\tau)$ for $T > t > 0$ in the same diagram.

(ii) Find the value of $x(\tau)\, s(t-\tau)$.

(iii) Calculate the value of $y(t) = x(t) * s(t)$, for $T > t > 0$.

(iv) Following steps similar to (i), (ii) and (iii), calculate $y(t) = x(t) * s(t)$ for $2T > t > T$.

(v) Find $y(t) = x(t) * s(t)$ and sketch it.

(b) A non-sinusoidal carrier signal $s(t)$ is amplitude modulated (*DSB*) using a base-band signal $m(t)$.

(i) Write an expression for the amplitude modulated carrier signal $y(t)$ in terms of $m(t)$ and $s(t)$.

(ii) The Fourier transforms $M(\omega)$ and $S(\omega)$ of $m(t)$ and $s(t)$ respectively are given below in Fig.3

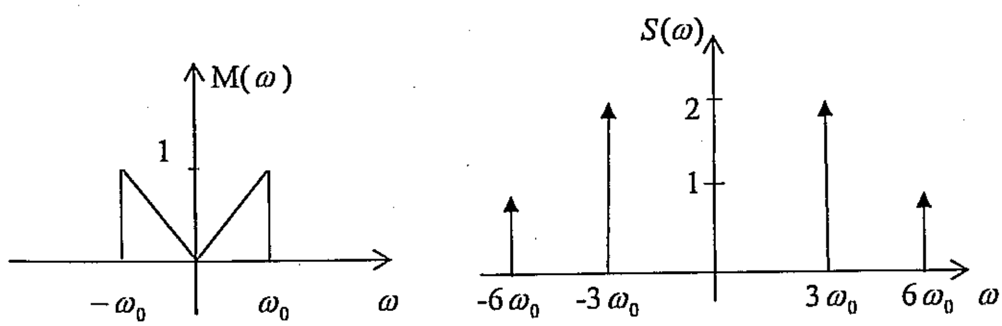

Fig.3

(α) Find $Y(\omega)$, the Fourier transform of $y(t)$.

(β) Sketch $Y(\omega)$.

(γ) Now $y(t)$ is transmitted with the carrier suppressed. Assuming that the transmission channel to be ideal, explain how would you recover $m(t)$ at the receiver.

(η) Write an expression for the received signal $y_R(t)$ in terms of $m(t)$.

(κ) If the transmission channel exhibits ideal low-pass characteristics (lower cutoff frequency $f_L = \frac{4.3}{2\pi}\omega_0$), explain how would you recover $m(t)$. Justify your answer.

4.

(a) A sinusoidal signal $x(t) = \sin \omega_0 t$ is sampled using an impulse train $s(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$, where $T_s = \frac{2\pi}{8\omega_0}$ and $\omega_0 = \frac{2\pi}{T}$.

(i) Sketch the sampled signal $x_s(t)$.

(ii) Find the Fourier transform $X_s(\omega)$ of $x_s(t)$.

(iii) Sketch $X_s(\omega)$ and explain whether $x(t)$ can be recovered without distortion.

(b) Consider the signal $v(t) = \sum_{n=-\infty}^{\infty} p(t - nT_s)$,

where $p(t) = 1$ for $|t| \le \frac{T}{16}$.
$= 0$ otherwise.

$v(t)$ is Pulse Amplitude Modulated (*PAM*) using $x(t)$.

(i) Sketch *PAM* signal $v_x(t)$.

(ii) Write an expression for $v_x(t)$.

(iii) Find $V_x(\omega)$ the Fourier transform of $v_x(t)$.

(iv) Sketch $V_x(\omega)$.

(v) Can $x(t)$ be recovered without any distortion? Justify your answer.

5.

(a) A sinusoidal signal having amplitude A is pulse code modulated. If the number of quantization levels is L,

(i) what is the maximum possible quantization error ($e_{\max}$)?

(ii) how can $e_{\max}$ be minimized?

(iii) calculate the average quantization noise power (N_{ave}).

(iv) show that signal to noise ratio (expressed in decibels) is given by

$$\frac{S}{N} = 1.76 + 20 \log L$$

(Assume that the quantization error is equally distributed)

(b) An audio signal has a frequency distribution as shown in Fig.5.

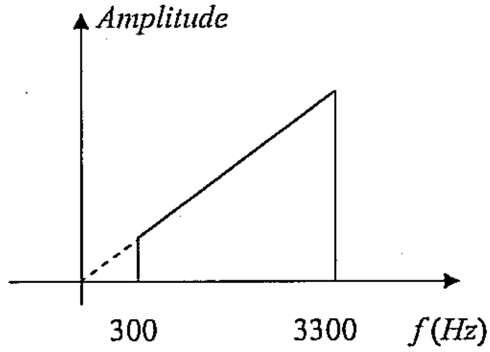

Fig. 5

This signal is sampled at a rate of 7000 *samples /second*. Each sample is then converted into a digital signal. If the required S/N ratio for the *PCM* process is 28 *dB*, calculate

(i) the minimum no. of quantization levels.
(ii) the minimum system bandwidth required.

(c) Sometimes signal compression is done in *PCM* using $\mu - law$.
(i) Why is compression done in *PCM*?
(ii) Briefly explain the principle of $\mu - law$ compression.

6.
(a) A Quadrature Amplitude Modulated (*QAM*) signal is given by

$$x_{QAM}(t) = s_1(t)\cos\omega_0 t + s_2(t)\sin\omega_0 t$$

where $s_1(t)$ and $s_2(t)$ are 2 base-band signals to be transmitted simultaneously using the same carrier signal.

(i) Find the Fourier transform of $x_{QAM}(t)$.

(ii) Find the bandwidth of $x_{QAM}(t)$.

(iii) If it is necessary to recover $s_1(t)$ and $s_2(t)$ from $x_{QAM}(t)$, explain the demodulation process of $x_{QAM}(t)$. Using necessary calculations show how $s_1(t)$ and $s_2(t)$ are safely recovered.

(b) A transmitter sends a random binary signal $s(t)$ having two amplitude levels -3V and +2V. The probabilities of transmitting -3V and +2V are 0.4 and 0.6 respectively. During the transmission channel noise $n(t)$ is added to the signal. $n(t)$ is random in nature and has the probability density function given in Fig.6.

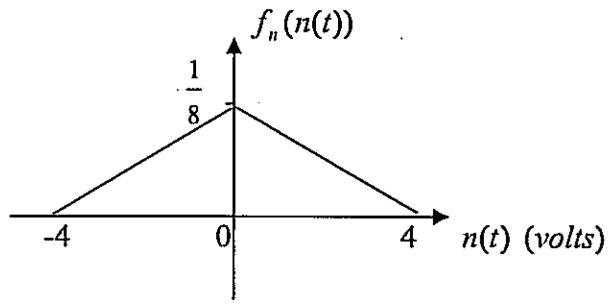

Fig.6

(i) What is the mean signal level $\overline{s(t)}$ at the transmitter?

(ii) Find the power of the total signal at the receiver input. (Assume that $s(t)$ and $n(t)$ are uncorrelated)

7.

(a) Define following terms:

(i) Stationary random process

(ii) Wide-sense stationary random process.

(iii) Ergodic random process.

(b) (i) Define *Autocorrelation function* $\Re_x$ of a random process $x(t)$.

(ii) Autocorrelation functions $\Re_x$ and $\Re_y$ of two independent random processes $x(t)$ and $y(t)$ respectively are given below:

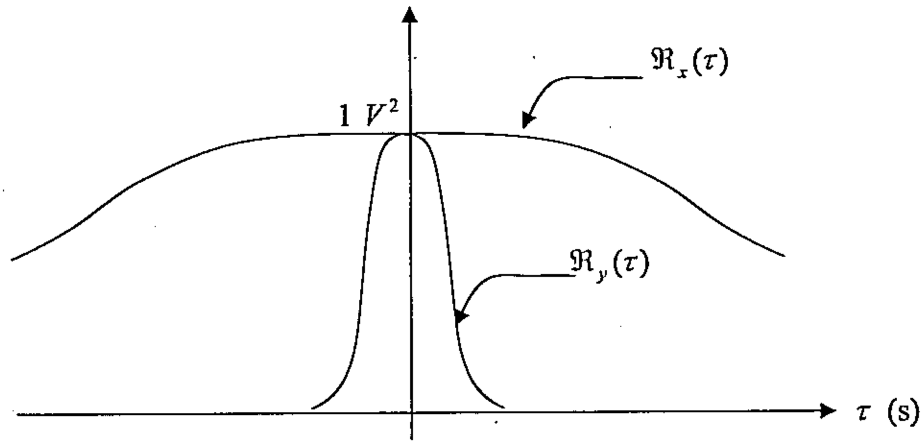

Fig.7 (a)

(α) Compare the nature of two processes.
(β) Find the average power of each process.
(δ) A third random process $z(t)$ is defined by the equation

$$z(t) = x(t) + y(t)$$

Derive an expression for $\Re_z$, the autocorrelation function of $z(t)$.

(c) A random process is given by $x(t) = E\cos(\omega t + \alpha)$, where E is a random variable equally distributed between $-\dfrac{E_0}{2}$ and $+\dfrac{E_0}{2}$ (Fig.7(b)). ω and α are constants.

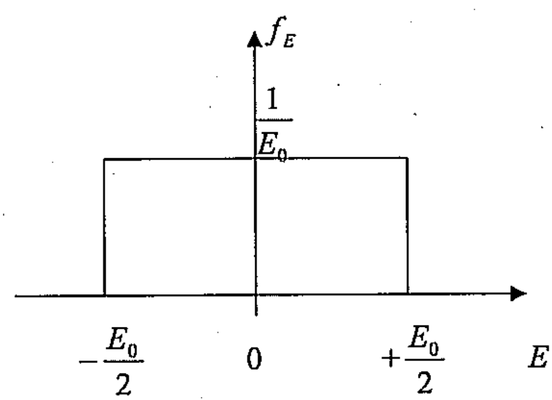

Fig.7(b)

(i) Find the autocorrelation function $\Re_x$.
(ii) Is $x(t)$ wide sense stationary? Justify your answer.

8.

(a) Consider a memoriless source m emitting messages $m_1, m_2, \ldots, m_i. . m_n$ with probabilities $P_1, P_2, \ldots P_i \ldots, P_n$ respectively.

(i) Write an expression for the average information content (entropy).

(ii) A data source emits one of the 4 digital signals -1, 0, +1 or +2. The probabilities of emission of 0, +1 and +2 are $\frac{1}{4}$, $\frac{1}{4}$ and $\frac{1}{4}$ respectively.

Find the entropy of the source.

(iii) Two binary channels are cascaded as shown in the Fig.8. Transition probabilities for various values of x_i and y_j are indicated in the diagram.

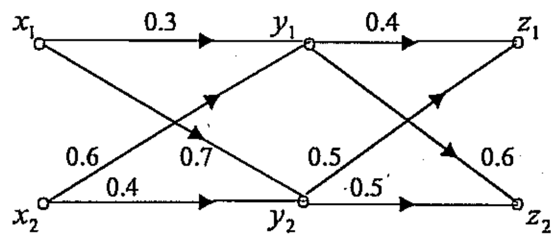

Fig.8

(α) Find the overall channel matrix of the resultant channel and draw the resultant equivalent channel diagram.

(β) If the probabilities $P(x_1) = 0.7$ and $P(x_2) = 0.3$, find $P(z_1)$ and $P(z_2)$.

(b) Briefly explain the following:

(i) *Schwarz* inequality.
(ii) *Matched* filter.