THE OPEN UNIVERSITY OF SRI LANKA

Department of Civil Engineering

Bachelor of Technology - Level 5

CEX 5231 - MECHANICS OF FLUIDS

FINAL EXAMINATION - 2009/2010

Time Allowed : Three Hours

Date : 22nd March, 2010 Time : 1400 - 1700

ANSWER **ALL THREE** QUESTIONS IN PART A AND **ANY TWO** QUESTIONS IN PART B. ALL QUESTIONS CARRY EQUAL MARKS.

PART A

Answer **all three** questions in this section.

1)

A long, straight river flows in the middle of a long, straight area of deposited sediment of 2 km width - that is bounded below and on both sides by impervious rock as shown in Figure 1. The deposited sediment is homogenous and isotropic and has a permeability of 2.5 m/day . The area is recharged at an average rate of 1 mm/day due to the infiltration of rainfall. The water level in the river is maintained by a dam and the water level in the ground on both sides of the river is at a steady state. There is a well located 500 m away from the river, as shown in the figure.

On a certain day the water level in the river is 10 m above the layer of impervious rock. This level is reduced by 1 m over a short period of time by a sudden release of water from the dam. The change in the water level in the river will lead to a change in the water level in the ground over a longer time period.

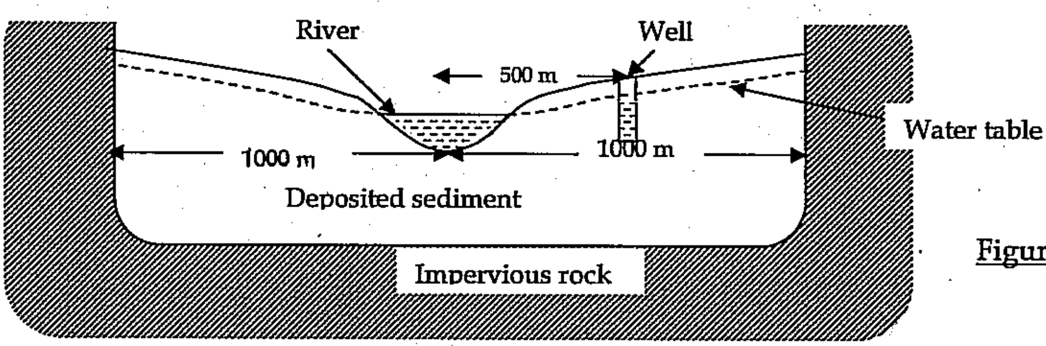

Figure 1

a) Derive, from first principles, a differential equation governing the unsteady variation of the groundwater level on both sides of the river. State all your assumptions.

b) Simplify the differential equation obtained in section a) to obtain a differential equation for the steady (equilibrium) variation of the water level with distance from the river.

c) Sketch the variation of the water level in the well with time after the sudden lowering of the water level in the river. Explain your answer.

d) By solving the equation derived in section b), obtain the final difference in the water level in the well due to the lowering of the water level in the river. Explain your answer.

2)
An open channel has a uniform rectangular cross-section that is 0.5 m in width, a constant slope of 0.005 and a Mannings' coefficient of 0.025. The channel has two gates at A and B that are 25 m apart, as shown in Figure 2. The opening of the upstream gate is 0.1 m while the opening of the downstream gate is 0.4 m. The channel carries a discharge of 1 m^3/s. There is a free hydraulic jump somewhere between the two gates.

a) Calculate the flow depth just upstream of the gate at B.

b) Does the channel have a mild slope or a steep slope for this discharge?

c) Sketch one possible free surface profile between A and B and classify the elements of the profile (from M1, M2, M3, S1, S2, S3, C1, C2, C3, etc.). Explain your answer.

d) Explain how you would estimate the location of the hydraulic jump between A and B.

e) How would the location of the jump change if the sluice gate at B was raised slightly? Explain your answer.

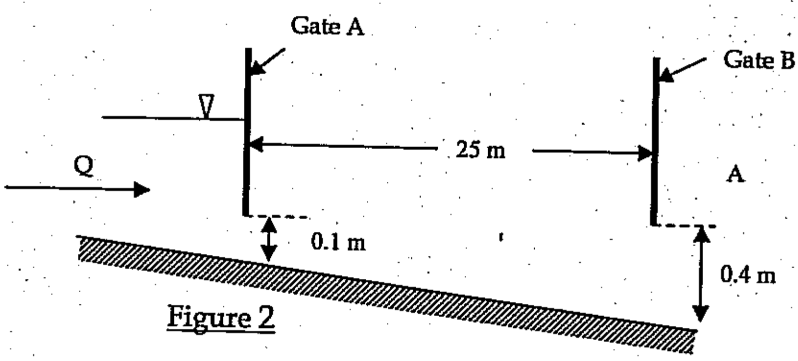

Figure 2

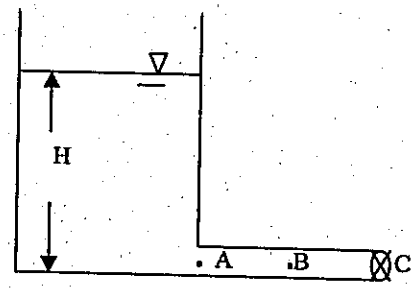

Figure 3

3)
Water flows steadily out of a large tank, X, through a horizontal pipeline ABC as shown in Figure 3. The pipeline has a length, L, a diameter d and an equivalent roughness ε. The water level in the tank is H, as shown in the figure. There is a valve at C.

It is observed that a sudden closure of the valve at C results in a rapid increase of the pressure just upstream of the valve. It is found that the maximum pressure p_m in the pipeline depends on the variables L, d, gH, ε, μ, ρ and κ. Here g is the acceleration due to gravity, μ the viscosity of water, ρ the density of water and κ the bulk modulus of water defined by $\frac{1}{\kappa} = \frac{1}{\rho}\frac{\partial \rho}{\partial p}$.

a) Explain why the sudden closure of the valve results in a rapid increase of the pressure just upstream of the valve.

b) Sketch the variation of the pressure at the locations A, B and C in the pipeline with time after the sudden closure of the valve on the same graph. Explain your answer.

c) Obtain a non-dimensional relationship between the maximum pressure p_m and the other variables.

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PART B

Answer **any two** questions in this section.

4)
A horizontal pipe of circular cross-section has a section where the radius varies linearly such that $\frac{dr}{dx}=k$ where **r(x)** is the radius of the pipe, **x** the distance along the pipe and **k** is a constant. Water, of density ρ, flows steadily along the pipe. The pipe friction factor is f.

a) Applying the principles of <u>conservation of mass</u> and <u>conservation of momentum</u> to an appropriate control volume derive differential equations governing the variation of the velocity and pressure – i.e. $\frac{dv}{dx}$ and $\frac{dp}{dx}$ - along the section where the radius of the pipe varies. Your derivation <u>should</u> include the effects of pipe friction. State any assumptions that you make.

b) Show that your result in section a) reduces to a simple form of the Bernoulli equation when the effects of pipe friction are neglected.

c) Discuss whether the result obtained in section a) will remain valid as **k** increases. Consider both positive and negative values of **k**. Explain your answer.

5)
The Navier-Stokes equations for a two-dimensional flow can be written in the form

$$\frac{\partial \underline{q}}{\partial t}+(\underline{q}.\underline{\nabla})\underline{q}=-\frac{1}{\rho}\underline{\nabla}(p)+\frac{\mu}{\rho}\nabla^2\underline{q} \quad (1) \qquad \text{and} \qquad \nabla.\underline{q}=0 \quad (2)$$

where $\underline{q}=u\underline{i}+v\underline{j}$ is the velocity vector and p is the pressure.

a) Explain the physical significance of the term $(\underline{q}.\underline{\nabla})\underline{q}$ in equation (1).

b) What is the definition of the pressure – p – in equation (1)?

c) Explain the physical significance of the term $\frac{\mu}{\rho}\nabla^2\underline{q}$ in equation (1).

A two-dimensional ideal fluid flow has a velocity potential given by the function

$w(z)=Uz+ik\ln(z)$ where $z=x+iy$.

d) Obtain an expression for the horizontal and vertical velocity components of this flow along the y axis (i.e. the line $x=0$).

e) What are the stagnation points of the flow?

f) Sketch the streamlines of this flow.

6)

An irrigation channel has a rectangular cross-section of width 2 m. The channel carries a discharge of $1\ \text{m}^3/\text{s}$ at a depth of 0.6 m. Between locations A and B the channel bed is raised smoothly by 0.2 m over a channel length of 20 m, as shown in Figure 6.

It is necessary to maintain the elevation of the free surface of the flow in the channel at a constant elevation <u>above a horizontal datum</u>. This is to be done by changing the width of the channel smoothly between A and B while keeping the rectangular cross-section.

a) Obtain a relationship between y/y_c and E/y_c for an open channel of rectangular cross-section. Here y is the flow depth, y_c is the critical depth and E is the specific energy of the flow.

b) Calculate the required channel width at B. Explain your answer.

c) Indicate the flow conditions at A and B on a graph of y/y_c against E/y_c.

d) Calculate the magnitude and direction of the force on the channel section AB due to the flow.

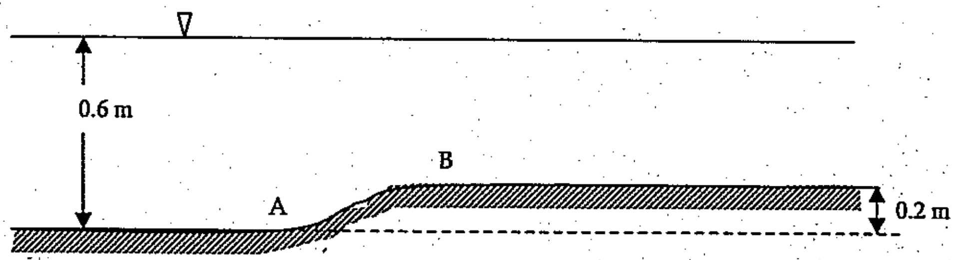

Figure 6

7)
A turbine is tested in a wind of constant velocity, U, as shown in Figure 7. The test is carried out by measuring the rotational speed, N, at different values of the torque T, applied against the direction of rotation of the shaft.

The blades of the turbine are made of rectangular, two-dimensional (flat) pieces of metal.

a) Sketch the variation of the Torque, T, as a function of the rotational speed, N, that you would expect to obtain from such a test. Explain this variation.

b) Sketch the variation of the shaft power, P, as a function of the rotational speed, N, that you would expect to obtain from such a test. Explain this variation.

c) Sketch the variation of the maximum power obtained as a function of the blade angle, θ that you would expect to obtain from such a test. Explain this variation.

d) Explain how you would obtain a theoretical expression relating the torque, T, to the wind speed, U, the rotational speed, N, the blade angle, θ and the geometry of the turbine.

Note : You do NOT have to derive an expression here. You have only to state the fundamental principle and explain how it is used.

e) Explain how you would use the results of this test to estimate the maximum power that could be extracted by the same turbine for a different wind velocity.

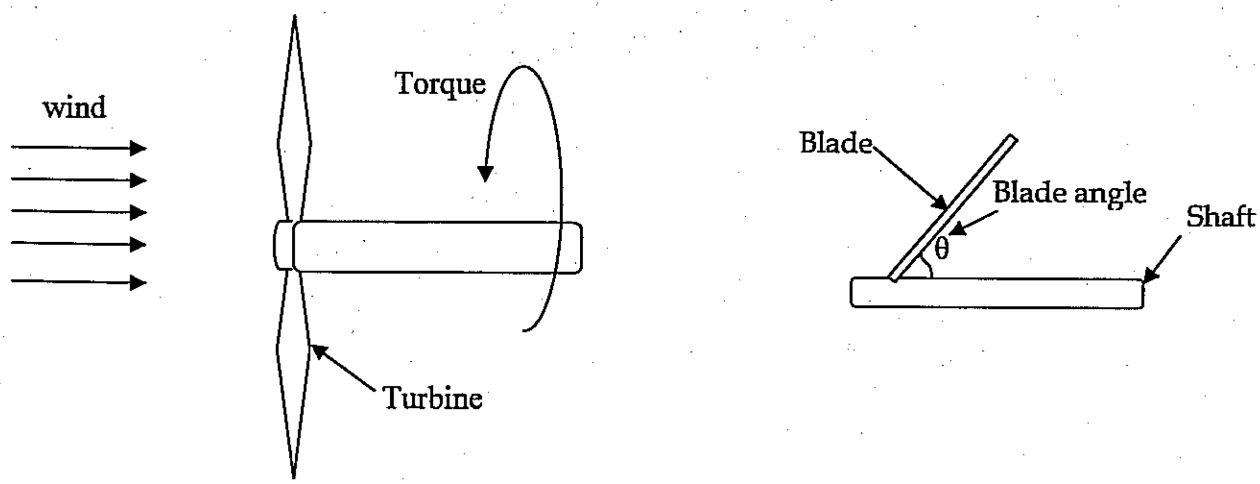

Figure 7

8)

A student sets up a pedestal fan a short distance from a flat surface, as shown in Figure 8, to study the movement of particles on a flat surface. The flow from the fan is horizontal and the flat surface is also horizontal and has a smooth surface.

The student uses the four types of particles listed in Table 8 below.

Particle Type	Material	Size and shape
A	Paper	1 cm square
B	Paper	1 cm square crumpled into 3 mm spheres
C	Polystyrene	3 mm spheres
D	Quartz	3 mm grains

Table 8

The student places each of these types of particles on the table and records the minimum distance from the edge of the table to the location where the particles remain stationary in the flow.

a) Explain, using neat diagrams, how the velocity profile (variation of velocity with distance from the surface of the table) would vary with the distance from the edge of the table. Assume that the velocity profile at the edge of the table is uniform.

b) Sketch the variation of the shear stress on the table with distance from the edge of the table and explain this variation with reference to the velocity profiles you discussed in section a).

c) Sketch, on the same graph, the variation of the shear stress on the table with distance from the edge of the table for a smooth surface and a rough surface. Explain the differences.

d) The student finds that particles of type A moved further along the table than particles of type B – even though both types of particles have the same weight. Explain this observation.

e) The size and shape of particle types B, C and D are the same. What differences would you expect to find between the stable locations of these particles from the edge of the table? Explain your answer.

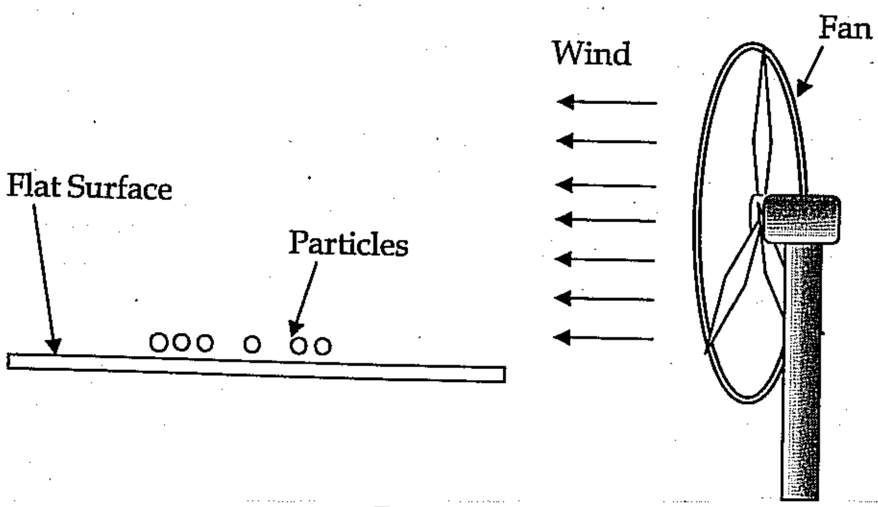

Figure 8

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