00175

The Open University of Sri Lanka
Department of Electrical and Computer Engineering
ECX 6241 – Field Theory
Final Examination – 2010/2011

Date: 2011-03-09 Time: 0930-1230

Answer **five questions** by selecting **two from Section A, two from Section B** and **one from Section C.**

Section A

Select two questions from this section.

Q1.

(a) Let $\underline{E} = (2xy + z^3)\underline{u}_x + x^2\underline{u}_y + 3z^2x\,\underline{u}_z$.

i) Show that E is a conservative vector field.
ii) Find its scalar potential.
iii) Find the work done in moving a charge from $(1,-2,1)$ to $(3,1,4)$.

(b) If $\underline{A} = xy\,\underline{u}_x + y^2\underline{u}_y + 2xz\,\underline{u}_z$, show that the line integration of $\underline{A}$ along the closed path of a triangle with its vertices at (0,0,0), (0,2,0) and (0,0,3) results in zero.

(c)

i) What is an irrotational vector field?
ii) Given a vector function $\underline{F} = (2y - c_1z)\underline{u}_x + (c_2x - 2z)\underline{u}_y - (c_3y + z)\underline{u}_z$. Determine the constants c_1, c_2 and c_3 if $\underline{F}$ is irrotational.

Q2.

(a) State Divergence theorem.

(b) Let $\underline{B} = r^2\,\underline{u}_r + 2z\,\underline{u}_z$

i) Determine the net flux of the vector field $\underline{B}$ leaving a closed surface defined by $r = R, z = 0$, and $z = h$.
ii) Verify your result using divergence theorem.

$$(\nabla . A = \frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{1}{r}\frac{\partial A_\varphi}{\partial \varphi} + \frac{\partial A_z}{\partial z})$$

(c) If

$$\nabla.E = 0,$$
$$\nabla.H = 0,$$
$$\nabla \times E = -\frac{\partial H}{\partial t}$$
$$\nabla \times H = \frac{\partial E}{\partial t}.$$

Show that E and H satisfy the wave equation

$$\nabla^2 u = \frac{\partial^2 u}{\partial t^2}.$$

Q3.

(a) Verify the following vector identities in Cartesian coordinates,

i) $\nabla.(A \times B) = B.\nabla \times \mathrm{A} - \mathrm{A}.\nabla \times \mathrm{B}$

ii) $\nabla \times (\nabla \times A) = \nabla(\nabla.A) - \nabla^2 A$

iii) $\nabla.\phi A = A.\nabla\phi + \phi\nabla.A$

(b) For two continuous functions f and g Green's theorem is written as

$$\iiint (f\nabla^2 g - g\nabla^2 f).dR = \iint (f\nabla g - g\nabla f).dS$$

Evaluate

$$\int_C e^{-x}(\sin y\, dx + \cos y\, dy),$$

where C is the rectangle with vertices (0,0), $(\pi, 0)$, $(\pi, \frac{\pi}{2})$ and $(0, \frac{\pi}{2})$.

(c) An electric field is given by $\underline{E} = \sin y\, \underline{u}_x + x(1 + \cos y)\underline{u}_y$. Evaluate the line integral over the circular path $x^2 + y^2 = a^2, z = 0$.

Section B

Select two questions from this section.

Q4.

(a) State Coulomb's law for electrostatic charges.

(b) Using Coulomb's law, determine the fundamental units of the permittivity of free space.

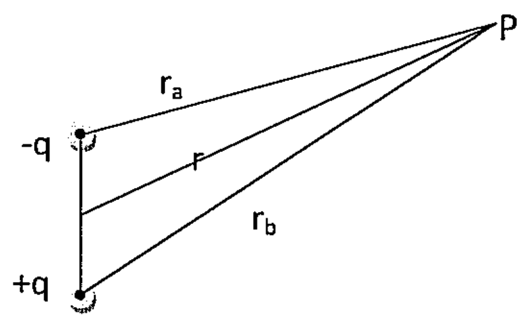
(c) Show that the electric potential caused at a distance $r = R$ by a long, thin transmission line of radius $a < R$, with a reference zero potential at $r = R_0$ and carrying a charge q C/m, is

$$\phi = -\frac{q}{2\pi\varepsilon}\ln\frac{R}{R_0}$$

Hint: $\int \frac{1}{\sqrt{a^2+x^2}}dx = ln\left[sec\left(\tan^{-1}\frac{x}{a}\right) + \frac{x}{a}\right] + conatant$

Q5.

(a) Derive the Poisson equation for the electric field.
(b) Show that the electric potential at the point P in the figure is given by $V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r_a} - \frac{1}{r_b}\right]$.



(c) If r is much greater than the separation distance show that the potential at P may be approximated by $= \frac{q\cos\theta}{4\pi\varepsilon_0 r^2}$.
(d) Draw the potential distribution with the distance.

Q6.

(a) The center conductor of a coaxial cable carries a current I_0 in the $+\underline{u}_z$ direction (out of the page) and this current return in the outer conductor. Calculate the magnetic flux density both within the coaxial cable and in the region external to the outer conductor.

(b) Calculate the capacitance of a coaxial cable whose length is Δz and it consists of a cylindrical metallic rod whose radius is a and it is surrounded concentrically with a metallic sleeve whose radius is b. There is a dielectric material separating the two conducting surfaces and it has a relative dielectric constant ε_r.

Section C

Select one question from this section.

Q7.

(a) Write Maxwell's equations.

(b) Derive the continuity equation for charge density ρ from Maxwell's equations.

(c) Find a charge density ρ that could produce an electric field $\underline{E} = E_o \cos x \cos t\, \underline{u}_x$ in vacuum

(d) Show that the two "divergence" equations are implied by the two "curl" equations and the equation of continuity.

Q8.

(a) Define Poynting vector. What physical quantity does it represent? What is the direction of Poynting vector?

(b) The electric and magnetic fields in free space in the spherical coordinate system are given by

$$\underline{E} = \frac{4}{r} sin\theta \cos\left(\omega t - \frac{2\pi r}{3}\right) \underline{u}_\theta \text{ V/m}$$

$$\underline{H} = \frac{4}{120\pi r} sin\theta \cos\left(\omega t - \frac{2\pi r}{3}\right) \underline{u}_\phi \text{ A/m}$$

i) Determine the Poynting vector.
ii) What is the direction of power flow?
iii) Determine the total average power leaving spherically closed regions of radius 100 and 1000m.