The Open University of Sri Lanka
Department of Electrical and Computer Engineering
Final Examination 2009/2010
ECX6234 – Digital Signal Processing

Time: 0930 – 1230 hrs. Date: 2011-03 -12

Answer 5 questions

Select at least 1 question from section B

SECTION A

1.

(a) A linear, time invariant, discrete system was found to be causal. A student investigates the system response by applying a unit impulse $\delta[n]$ as the input.

(i) Briefly explain the meaning of underlined terms. [10 *marks*]

(ii) Sketch a typical output signal. [1 *mark*]

(b) A system is defined by the input output relation

$$y[n]=x[n]+2x[n-1]$$

Find whether the system is

(i) linear [2 *marks*]

(ii) time-invariant [2 *marks*]

(iii) causal [2 *marks*]

(c) (i) What is *BIBO* stability ? [1 *mark*]

(ii) Find whether the system described by the equation $y[n]=nx[n]+2x[n-1]$ is *BIBO* stable. [2 *marks*]

2.

(a) Define z-transform of $x[n]$.
What is *Region of Convergence* (*ROC*)? [2 *marks*]

(b) Show the following:

(i) $z\{a^n u[n]\} = \dfrac{1}{1-az^{-1}}, \quad ROC: |z|>|a|$ [3 *marks*]

(ii) $z\{x[n-n_0]\} = z^{-n_0}X(z)$ [3 *marks*]

(iii) $z\{nx[n]\} = -z\dfrac{dX(z)}{dz}$ (*Hint*: Differentiate both sides of the expression given in (a) with respect to z). [3 *marks*]

(c) Find inverse $z-transforms$ of

(i) $X(z)=\dfrac{3}{z-3}, \qquad |z|>3$ [3 *marks*]

(ii) $X(z) = \dfrac{z}{(z-1)(z-2)}, \quad |z| > 2$ [3 *marks*]

(iii) $X(z) = -\dfrac{z}{(z-1)^2}, \quad |z| > 1$ [3 *marks*]

3.

(a) The input $x[n]$ and the impulse response $h[n]$ of discrete *LTI* system is given by

$$x[n] = u[n-1]; \quad h[n] = \alpha^n u[n]$$

Find the output $y[n]$ of the system. [10 *marks*]

(b)

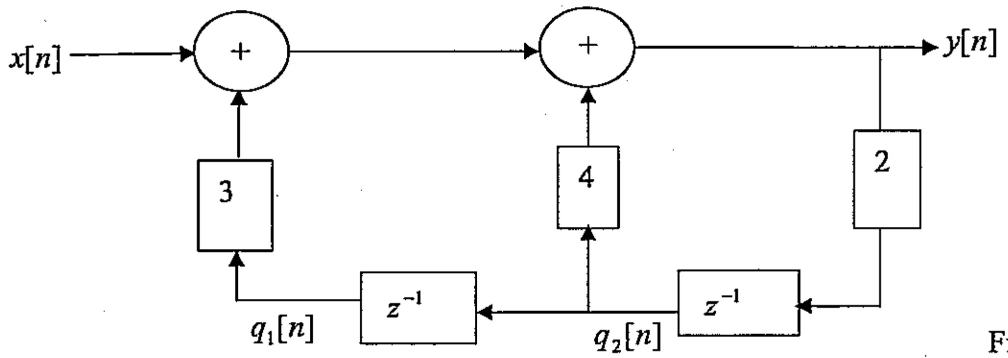


Fig.3

Find the state space representation of the system given in Fig.3.
Use $q_1[n]$ and $q_2[n]$ as state variable. [10 *marks*]

4.

(a) A causal discrete system has a input $x[n]$ and output $y[n]$. $x[n]$ and $y[n]$ are related by the equation $y[n] - \frac{1}{4}y[n-1] + \frac{1}{5}y[n-2] = x[n]$

(i) Find the *system function* $H(z)$. [5 *marks*]

(ii) Find the *impulse response* $h[n]$. [5 *marks*]

(b) A discrete sequence $x[n]$ in general can be considered as a combination of predicted value $\hat{x}[n]$ and the error $w[n]$.

$$\hat{x}[n] = x[n-1]$$
$$x[n] = x[n-1] + w[n]$$

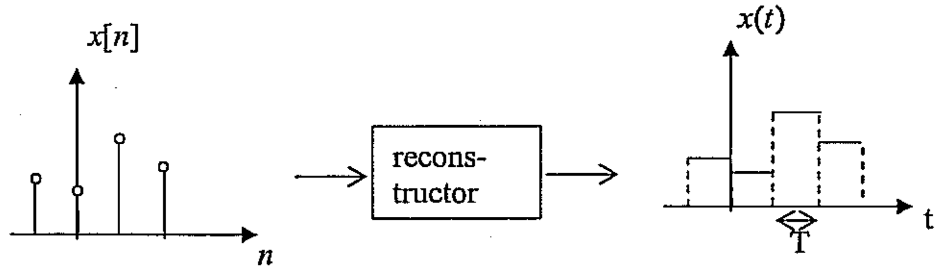
(i) Find the relationship between $\hat{X}$ and $X(z)$. [5 *marks*]

(ii) Realize system in a block diagram. [5 *marks*]

5.

(a) The function of a signal reconstructor (Zero Order Hold) is to convert a discrete signal $x[n]$ into an analogue signal $x(t)$. This is done by interpolating between the samples as given below:

Fig.5



For the interpolation between discrete values time limited function $g(t)$ is used. Interpolation is done by shifting $g(t)$ in time domain and multiplying it by corresponding $x[n]$ value.

(i) Write an expression for $x(t)$ in terms of $x[n]$ and $g(t)$. [3 *marks*].

(ii) Plot function $g(t)$. [3 *marks*]

(iii) Find $G(\omega)$ and sketch it. [3 *marks*]

(iv) Derive an expression for $X(\omega)$ the Fourier transform of $x(t)$ in terms of $G(\omega)$ and DTFT $\{x[n]\}$. [3 *marks*]

(b) (i) Write an expression for the impulse response $h[n]$ of an ideal lowpass filter. [3 *marks*]

(ii) Show that the filter in (b) (i) shows linear phase characteristics. [5 *marks*]

SECTION B

6.

(a) A signal $x[n]$ can be sampled into $v[n]$ using the operator $\delta_D[n]$ as follows:

$v[n]=\delta_D[n].x[n]$ where $\delta_D[n] = 1$ if n/D is an integer
$= 0$ otherwise

(i) Write an expression for $\delta_D[n]$ if $D=2$. [4 *marks*]

(ii) Show that $V(z)=\frac{1}{2}X(z)+\frac{1}{2}X(-z)$ for the case given in (i). [4 *marks*]

(iii) Find $V(\omega)$ the Fourier transform of $v[n]$. [4 *marks*]

(b) For values of D other than 2, general expression for $\delta_D[n]$ can be written as follows:

$$\delta_D[n] = \frac{1}{D}\sum_{k=0}^{D-1} e^{-j2\pi kn/D}$$

Derive an expression for $V(\omega)$ in terms of $X(\omega)$. [8 *marks*]

7.

(a) (i) What is difference between *a finite impulse response filter* (*FIR*) and *an infinite impulse response filter* (*IIR*)? [2 *marks*]

(ii) Sketch a typical impulse response $h(n)$ for
(1) a *FIR* filter. [2 *marks*]
(2) an *IIR* filter. [2 *marks*]

(b) If the impulse response $h(n)$ of a *FIR* filter is symmetric about the y-axis show that the frequency response $H(\omega)$ can be written in the form

$$H(\omega) = \sum_{n=1}^{M} 2h[n]\cos n\omega + h[0]$$ [7 *marks*]

(c) Explain how you would design an equiripple filter having the transfer function given in (b). [7 *marks*]

8.

When designing a *FIR* filter we first convert an ideal filter into an intermediate *FIR* filter using a function known as *a window*. Such a function has a finite length, and has a zero value for $|n| \rangle L$. Through this process infinite impulse response of an ideal filter is converted into a finite impulse response.

$$h_w[n] = h_d[n].w[n]$$

$h_d[n]$ = impulse response of the ideal filter which was approximated to the *FIR* filter.
$w[n]$ = window function.

This alone is not sufficient since the impulse response is still non-causal. To make it causal h_w[n] can be shifted by a length L so that the filter becomes causal. Finally we get the impulse response of the *FIR* filter h[n] which is practically realizable.

$h[n] = h_w[n - L]$ (refer to Fig.8)

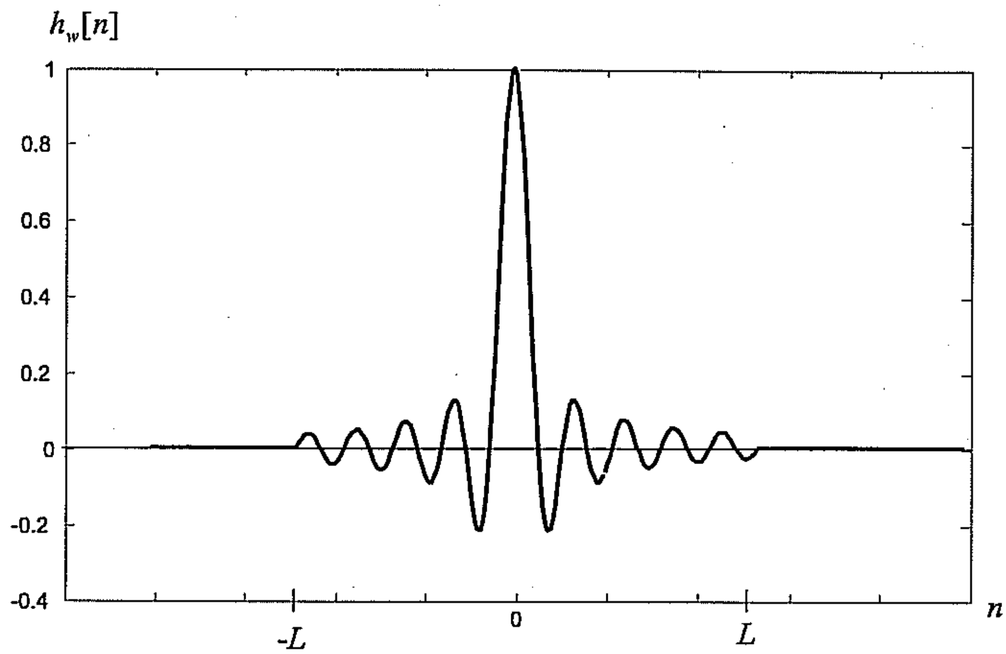


Fig.8

Fig.8 shows the intermediate *FIR* filter:

(a) If $h_d[n]$ corresponds to the impulse response of an ideal low-pass filter and $w[n]$ corresponds to a *Hamming* window sketch $h_d[n]$ and $w[n]$. [3 *marks*]

(b) What is the filter length N of the filter? [3 *marks*]

(c) Suppose we want to design a digital low-pass filter with pass band 4 kHz and stopband 5 kHz (with at least -35 dB attenuation). The sampling frequency is 20 kHz.

(i) Convert pass band and the stop band frequencies into digital frequencies. [3 *marks*]

(ii) Design a low-pass filter using the specification given above. Write the value of $h[n]$ and sketch it. [11 *marks*]

Window type	$w(n)$	Δw	Attenuation
Rectangular	1	$\frac{4\pi}{N}$	-13dB
Bartlett	$\frac{2}{N-1}\left(\frac{N-1}{2}-\left\|n-\frac{N-1}{2}\right\|\right)$	$\frac{8\pi}{N}$	-27dB
Hanning	$0.5+0.5\cos\left(\frac{2\pi n}{N-1}\right)$	$\frac{8\pi}{N}$	-32dB
Hamming	$0.54+0.46\cos\left(\frac{2\pi n}{N-1}\right)$	$\frac{8\pi}{N}$	-43dB
Blackman	$0.42+0.5\cos\left(\frac{2\pi n}{N-1}\right)+0.08\cos\left(\frac{4\pi n}{N-1}\right)$	$\frac{12\pi}{N}$	-53dB