00128

The Open University of Sri Lanka
Department of Electrical and Computer Engineering
ECX 6241 – Field Theory
Final Examination – 2011/2012

Date: 2012-02-29 Time: 0930-1230

Answer **five questions** by selecting **two from Section A, two from Section B** and **one from Section C.**

Section A

Select two questions from this section. (15 Marks for each)

Q1.

(a) Given $W = x^2y^2 + xyz$, compute ∇W and the directional derivative dW/dl in the direction $3\underline{\boldsymbol{a}_x} + 3\underline{\boldsymbol{a}_y} + 12\underline{\boldsymbol{a}_z}$ at $(2, -1, 0)$.

(b) If the integral $\int_A^B F.dl$ is regarded as the work done in moving a particle from A to B, find the work done by force filed $\boldsymbol{F} = 2xy\,\underline{\boldsymbol{a}_x} + (x^2 - z^2)\underline{\boldsymbol{a}_y} - 3xz^2\underline{\boldsymbol{a}_z}$ on a particle that travels from $A(0,0,0)$to $B(2,1,3)$ along

(i) The segment (0,0,0) → (0,1,0) → (2,1,0) → (2,1,3)

(ii) The straight line (0,0,0) → (2,1,3)

Q2.

(a) State the Divergence theorem.

(b) If $\boldsymbol{G}(r) = 10e^{-2z}(\rho\underline{\boldsymbol{a}_\rho} + \underline{\boldsymbol{a}_z})$, determine the flux of $\boldsymbol{G}$ out of the entire surface of the cylinder $\rho = 1$, $0 \le z \le 1$. Confirm the result by using the divergence theorem.

Q3.

(a) For a vector field $\boldsymbol{A}$, show explicitly that $\nabla \cdot \nabla \times \boldsymbol{A} = 0$; that is, the divergence of the curl of any vector field is zero.

(b) If $\boldsymbol{F} = 2\rho z\underline{\boldsymbol{a}_\rho} + 3z\sin\phi\,\underline{\boldsymbol{a}_\phi} - 4\rho\cos\phi\,\underline{\boldsymbol{a}_z}$, verify Stokes's theorem for the open surface defined by $z = 1$, $0 < \rho < 2$, $0 < \phi < {}^{\pi}/_{4}$.

Section B

Select two questions from this section. (20 Marks for each)

Q4.

(a) State Coulomb's law for electrostatic charges.

(b) A practical application of electrostatics in electrostatic separation of solids. For example, Florida phosphate ore, consisting of small particles of quartz (negatively charged) and phosphate (positively charged) rock, can be separated into its components by applying a uniform electric field. Assuming zero initial velocity and displacement, determine the separation between the particles after falling 80 cm. Take $E = 500\ kV/m$ and $Q/m = 9\ \mu C/kg$ for both positively and negatively charged particles.

Q5.

(a) Derive the Poisson equation for the electric field.

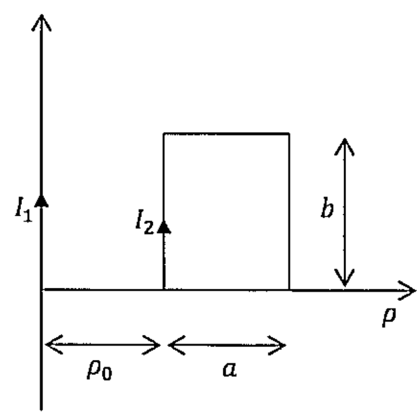
(b) Semi infinite conducting planes at $\phi = 0$ and $\phi = \pi/6$ are separated by an infinitesimal insulating gap. If $V(\phi = 0) = 0$ and $V(\phi = \pi/6) = 100\ V$, calculate V and E in the region between the planes.

Q6.

(a) State Ampere's circuit law.

(b) A rectangular loop carrying current I_2 is placed parallel to an infinitely long filamentary wire carrying current I_1 as shown in the figure. Show that the force experienced by the loop is given by

$$\boldsymbol{F} = -\frac{\mu_0 I_1 I_2 b}{2\pi}\left[\frac{1}{\rho_0} - \frac{1}{\rho_0 + a}\right]\underline{a_\rho N}$$



Section C

Select one question from this section. (30 Marks)

Q7.

(a) Show that the fields $\boldsymbol{E} = E_m \sin x \sin t\, \underline{a_y}$ and $\boldsymbol{H} = \frac{E_m}{\mu_0} \cos x \cos t\, \underline{a_z}$ in the free space are not valid solutions of Maxwell's equations.

(b) The electric field and magnetic field in free space are given by $\boldsymbol{E} = \frac{50}{\rho} \cos(10^6 t + \beta z)\underline{a_\phi}\, V/m$ $\boldsymbol{H} = \frac{H_0}{\rho} \cos(10^6 t + \beta z)\underline{a_\rho}\, A/m$. Express these in phasor form and determine the constants H_0 and β such that the fields satisfy Maxwell's equations.

Q8.

(a) Define the Poynting vector and state the Poynting theorem.

(b) In a nonmagnetic medium $\boldsymbol{E} = 4\sin(2\pi \times 10^7 t - 0.8x)\, \underline{a_z}\, V/m$. Find

- (i) ε_r, η
- (ii) The time-average power carried by the wave
- (iii) The total power crossing $100\ cm^2$ of plane $2x + y = 5$

Note: Useful formulae in cylindrical coordinates

1. $\nabla . A = \frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{1}{r}\frac{\partial A_\varphi}{\partial \varphi} + \frac{\partial A_z}{\partial z}$
2. $\nabla \times A = \frac{1}{r}\begin{vmatrix} \underline{a_r} & r\underline{a_\varphi} & \underline{a_z} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \varphi} & \frac{\partial}{\partial z} \\ A_r & rA_\varphi & A_z \end{vmatrix}$