00051

The Open University of Sri Lanka
Department of Electrical and Computer Engineering
Final Examination 2011/2012
ECX5233 – Radio and Line Communication

Time: 1400 – 1700 hrs. Date: 2012-03 -16

Answer any Five Questions

1.

(a)

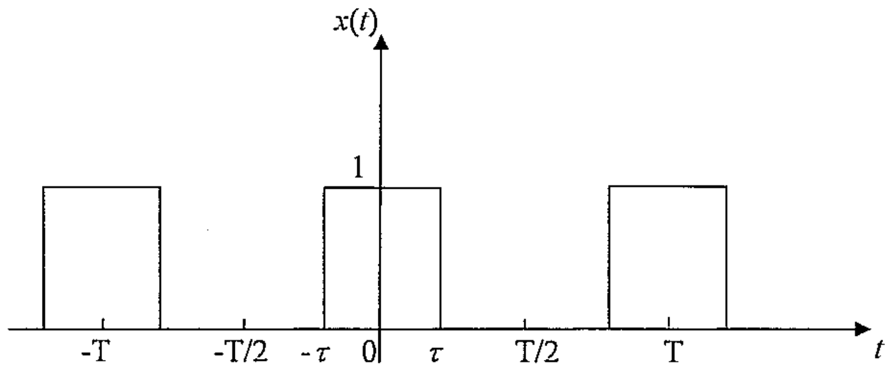


Fig.1

(i) Express $x(t)$ in the form of a Fourier series and show that it can be expressed as $x(t)=\sum_{n=-\infty}^{\infty} A_n \cos nkt$. (3 *marks*)

(ii) Find the values of k and A_n. (2 *marks*)

(iii) Sketch A_n vs. nk. What is the value of A_0? For what values of n does A_n becomes zero? (2 *marks*)

(iv) What is the distance between two adjacent spectral lines? (2 *marks*)

(v) What changes will take place in the sketch in (iii) if

(1) T is increased? (2 *marks*)

(2) τ is increased? (2 *marks*)

(vi) If $x(t)$ is fed to an ideal lowpass filter that has a cutoff frequency $f_c = \frac{4.85}{T}$, find the frequency spectrum of the output signal of the filter and sketch the approximate output waveform. (3 *marks*)

(b) A single sideband amplitude modulated signal (*SSB/AM*) can be expressed as $x_{SSB}(t) = m(t)\cos\omega_C t - \hat{m}(t)\sin\omega_c t$. $m(t)$ is the message signal. $\hat{m}(t)$ is the message signal with a 90^0 phase shift.

Show that if $x_{SSB}(t)$ is multiplied by $\cos\omega_c t$ and then lowpass filtered the message signal $m(t)$ can be recovered. (4 *marks*)

2.

(a) The Fourier Transform of $x(t)$ is $X(\omega)$.
Write an expression for the Fourier Transform of $y(t)$ in terms of $X(\omega)$ if

(i) $y(t) = x(t - t_0)$ (1 *mark*)

(ii) $y(t) = x(t)\,e^{-j\omega_0 t}$ (1 *mark*)

(iii) $y(t) = X(t)$ (1 *mark*)

(iv) $y(t) = \dfrac{d\,(x(t))}{dt}$ (1 *mark*)

(v) $y(t) = x(at)$ (1 *mark*)

[Proofs of the above expressions are NOT required].

(b)

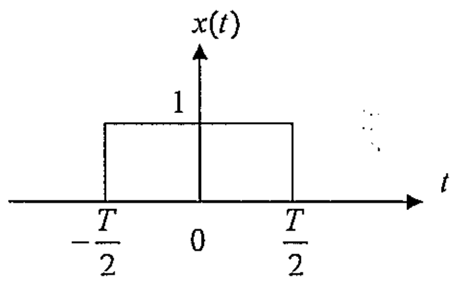


Fig.2.1

(i) Find $y(t) = \dfrac{d\,(x(t))}{dt}$ and sketch it. (2 *marks*)

(ii) Find $Y(\omega)$. (2 *marks*)

(iii) Using the value of $Y(\omega)$ find $X(\omega)$. (2 *marks*)

(c) Find $Z(\omega)$, the Fourier Transform $z(t)$ for the following:

(i) $$z(t) = \frac{\sin\left(\dfrac{\omega t}{2}\right)}{\dfrac{\omega t}{2}} = \text{sinc}\left(\frac{\omega t}{2}\right)$$ (3 *marks*)

(ii)

Pulse is symmetric about y-axis

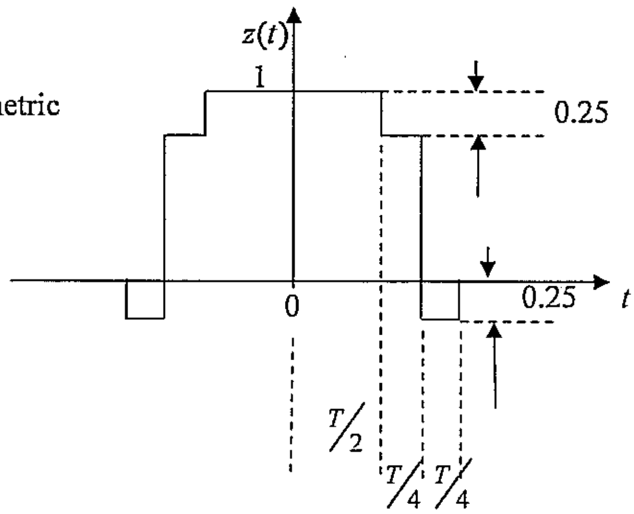


Fig.2.2

(3 *marks*)

[Hint: You may use the answers to (a) and (b)]

(iii) $z(t)$ in (ii) is passed through an ideal lowpass filter having a cutoff frequency $\omega_c = \dfrac{2\pi}{T}$. If the filter output is $z'(t)$, find $Z'(\omega)$ and sketch the approximate shape of $z'(t)$. (3 *marks*)

3.

A phase modulated carrier signal $x_c(t)$ can be written as
$x_c(t) = A\cos(\omega_c t + \beta \sin \omega_m t)$

(i) Show that $x_c(t)$ is a periodic signal and can be written as $A\ \mathrm{Re}(e^{j\omega_c t} e^{j\beta \sin \omega_m t})$ (3 *marks*)

(ii) Find the period T of the periodic signal $e^{j\beta \sin \omega_m t}$. (3 *marks*)

(iii) Express $e^{j\beta \sin \omega_m t}$ as a Fourier series and write an expression for C_n. (3 *marks*)

(iv) Show that C_n can be written as $\dfrac{1}{2\pi}\int_{-\pi}^{\pi} e^{j(\beta \sin x - nx)}dx$ (3 *marks*)

(v) The integral $\dfrac{1}{2\pi}\int_{-\pi}^{\pi} e^{j(\beta \sin x - nx)}dx$ is called a *Bessel function* of first kind of order n and denoted by $J_n(\beta)$.

Show that $x_c(t) = A\sum_{n=-\infty}^{\infty} J_n(\beta)\cos(\omega_c + n\omega_m)t$ (3 *marks*)

(vi) Using the result of (v), find the lowest value of β at which the total power of a phase modulated signal is distributed among the side lobes. Use graphs of Bessel functions of first kind (provided). (*5 marks*)

4.

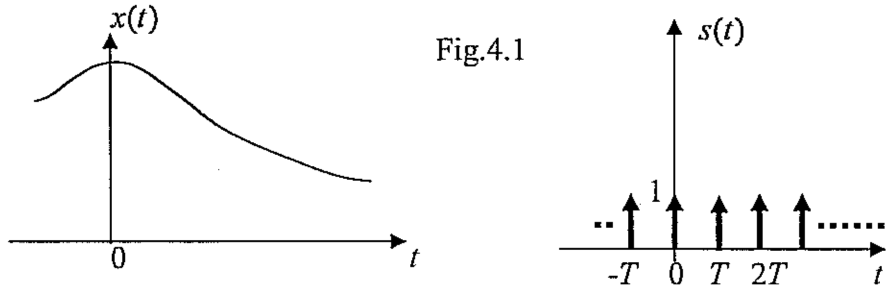


Fig.4.1

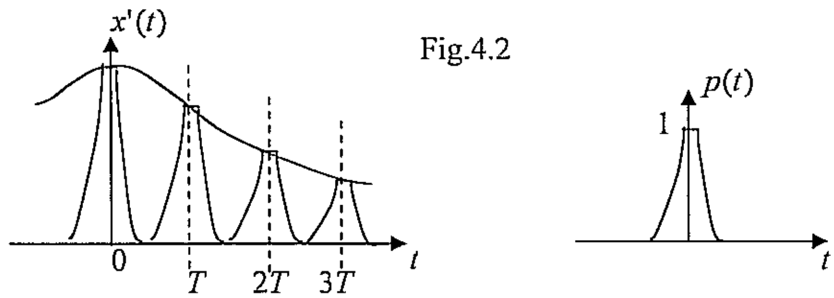


Fig.4.2

The signal $x(t)$ is to be *pulse amplitude modulated* using the pulse $p(t)$. As the first step, $x(t)$ is converted into $x_s(t)$, by multiplying $x(t)$ with the impulse train $s(t)$. Now it is necessary to convert $x_s(t)$ into $x'(t)$. The amplitude of the n^{th} pulse (in $x'(t)$) is $x(nT)$.

(a) Sketch $x_s(t)$. (2 *marks*)

(b) Express $s(t)$ as a Fourier series and show that $s(t) = \sum_{n=-\infty}^{\infty} e^{jn\omega_0 t}$.

Deduce that $S(\omega) = \frac{1}{T}\sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_0)$ (2 *marks*)

(a) Write an equation for $x_s(t)$ in terms of $x(t)$ and $s(t)$. (2 *marks*)

(b) What is the relationship between $X_s(\omega)$, $X(\omega)$ and $S(\omega)$

Find $X_s(\omega)$. (2 *marks*)

(c) Derive an expression for $X'(\omega)$ in terms of $X(\omega)$ and $P(\omega)$. (3 *marks*)

(d) Suppose $X(\omega)$ is a cosine pulse having a bandwidth B and $P(\omega)$ is a Gaussian pulse as shown in Fig.4.3.

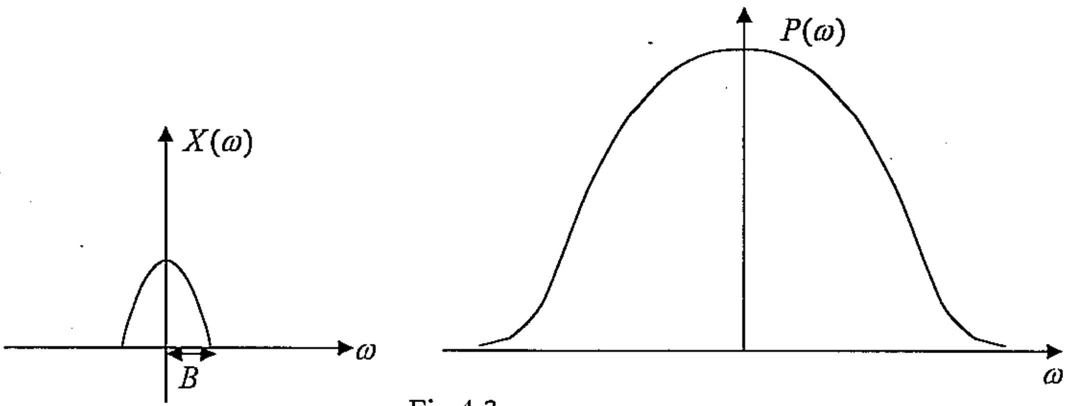


Fig.4.3

(i) Sketch $X'(\omega)$. (2 *marks*)

(ii) To extract $X(\omega)$ from $X'(\omega)$, $X'(\omega)$ is lowpass filtered. Using the sketch of (i) show that $1/T$ should not be less than B/π if the error due to overlapping of frequency components is to be avoided in the filtered output.

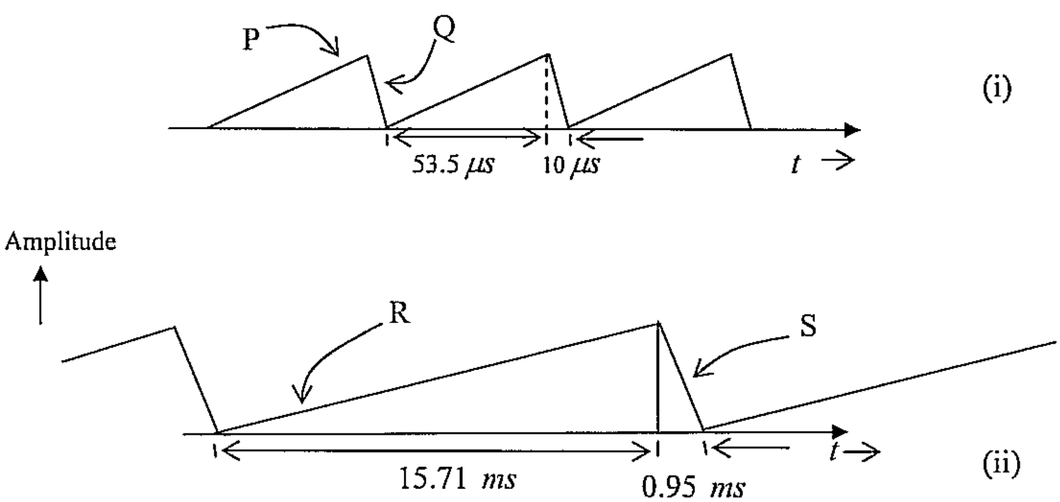
(3 *marks*)

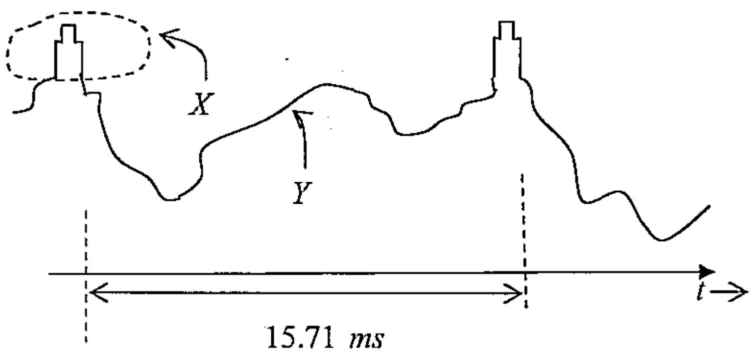
(iii) Frequency distortion is introduced during filtering due to $P(\omega)$. How does this happen? (2 *marks*)

(iv) Ideally what should be the shape of $p(t)$ so that no error is introduced due to $p(t)$? Justify your answer. (2 *marks*)

5.

(a) Three signals shown in Fig.5 are used in a TV receiver.





(iii)

Fig.5

Each waveform is divided into two different parts P,Q; R,S and X,Y

(i) Explain the role of each waveform. (3 *marks*)
(ii) Explain the functions of P, Q, R, S and Y. (3 *marks*)
(iii) X consists of a top part and a bottom part. Explain the function of each of these parts. (3 *marks*)
(iv) A certain black and white picture consists of a pure black dot D_B and a pure white dot D_W. Select the correct waveform from the above and redraw it, marking D_B and D_W on it. (3 *marks*)

(v) Calculate the number of lines per frame. Deduce the number of lines per picture. (4 *marks*)

(b) What are the advantages of digital TV system over analog TV system? (4 *marks*)

6.

$x(t)$ is a random process with an autocorrelation function $\Re_{xx}(\tau)$ and a power spectral density function $P_{xx}(\omega)$. $x(t)$ is passed through a lowpass filter whose time constant $\tau = RC = k$ shown in Fig.6 (a).

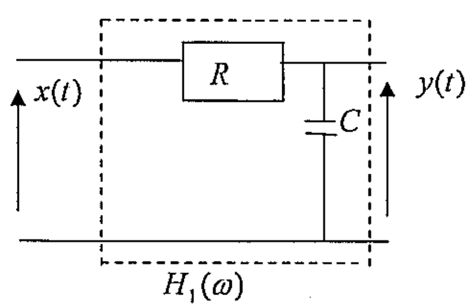


Fig.6(a)

(a) Find the transfer function ($H_1(\omega)$) of the filter. (3 *marks*)

Now $y(t)$ is differentiated and fed to a summer circuit together with $y(t)$ as shown in Fig.6(b). (Two systems given in Fig.6(a) and Fig.6(b) are cascaded)

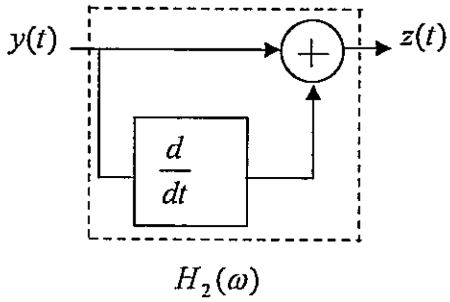


Fig.6(b)

(b) Find the transfer function $H_2(\omega)$. (4 *marks*)

The Power spectral density function of $x(t)$ is given by $P_{xx}(\omega) = \cos\omega T$, where T is a constant.

(c) Find the power spectral density function $P_{zz}(\omega)$ of the random process $z(t)$. (6*marks*)

(d) Find the auto correlation function $\Re_{zz}(\tau)$ of $z(t)$ if $k = 1$. (7 *marks*)

7.

(a) A binary transmitter transmits either a '1' or a '0'. Thus the transmitter output signal $s(t)$ can take one of the following values:

'1' – a positive d.c. signal having an amplitude $+5 \pm 0.5$ *Volts*
'0' – a negative d.c. signal having an amplitude -5 ± 0.5 *Volts*

The probability density function of $s(t)$ is given below in Fig.7.1.

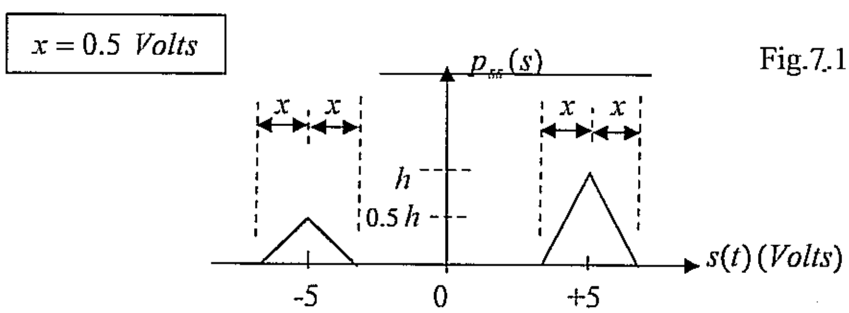


Fig.7.1

(i) Find the value of h. What is unit of h? (2 *marks*)

(ii) Find the average power of the signal. (4 *marks*)

(b) Uncorrelated channel noise is added to $s(t)$ during the transmission. The probability density function of the noise signal is given in Fig.7.2

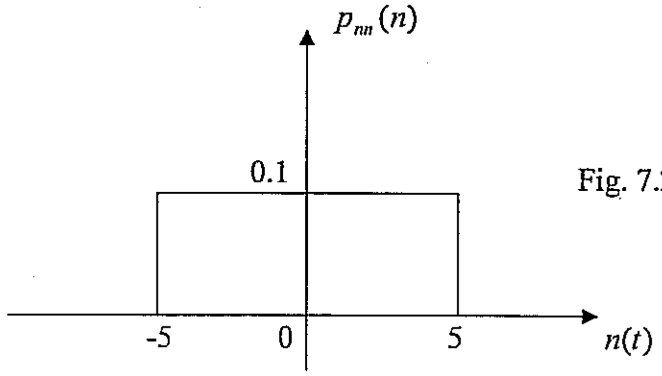


Fig. 7.2

(i) Find the joint probability density function $p_{sn}(s(t), n(t))$.
(3 *marks*)

(ii) Find the average value of channel noise. (3 *marks*)

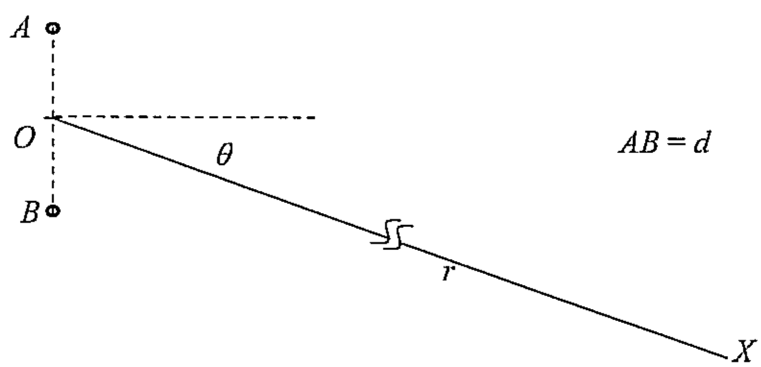
(iii) If the received signal (at the receiver input) is $r(t)$, find the power of $r(t)$. Note that $r(t)$ consists of data and noise. (4 *marks*)

(c) If $p_{nn}(n) = k\delta(n(t))$, recalculate the power of the received signal.
(4 *marks*)

8.

(a) What is an isotropic antenna? Draw the radiation pattern of it.
(3 *marks*)

(b)



When an isotropic antenna is placed at O (the mid-point of AB) the field strength at a distant point X (at a distance $r(>> d)$) is E_0.
Now the above antenna is moved to A and another identical antenna is placed at B.

(i) Find the resultant field strength at X due to the antenna array in terms of E_0, θ, d and r. (4 *marks*)

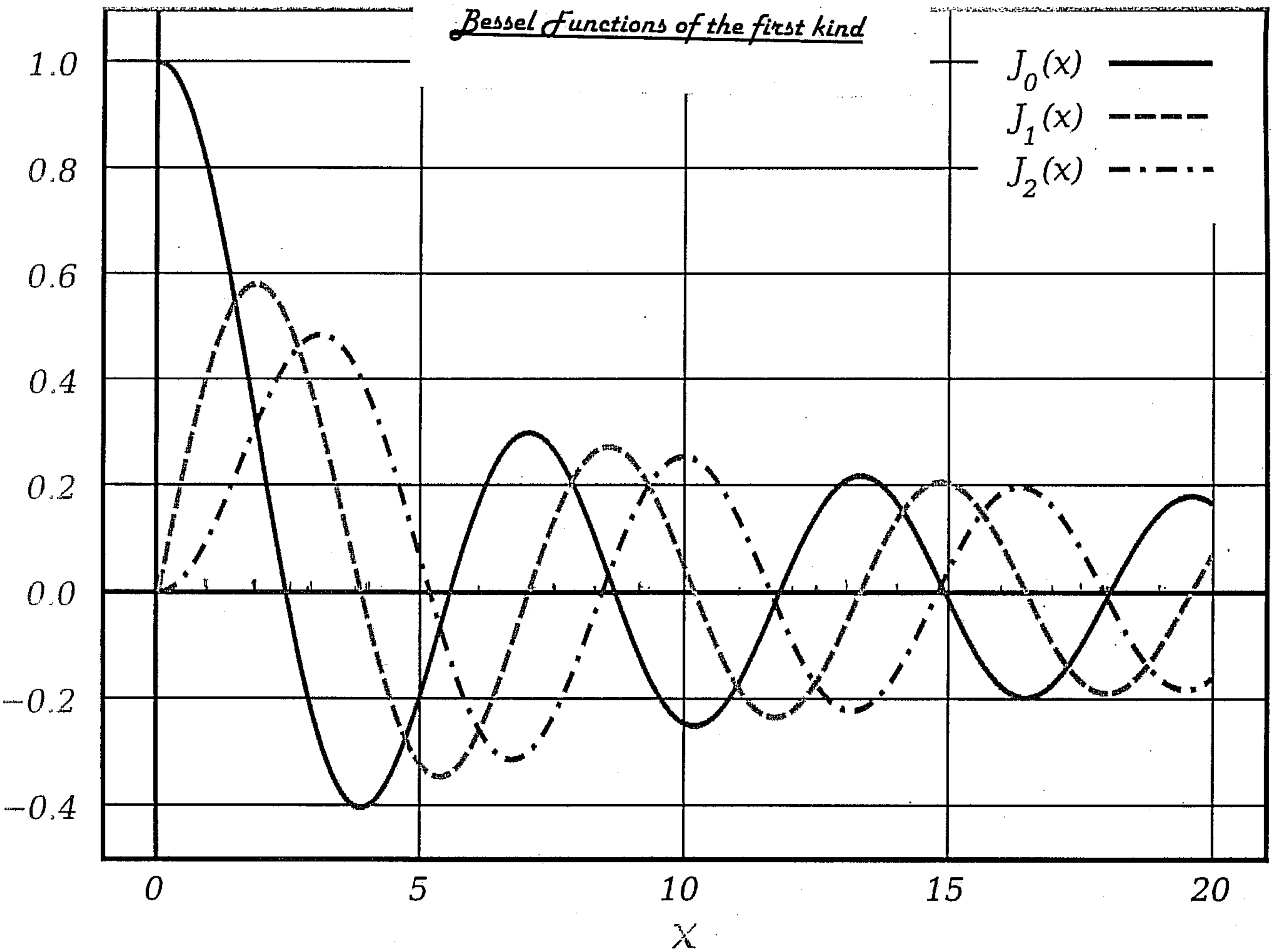
(ii) Plot the radiation pattern and calculate the beam-width of the antenna if $d = \lambda/4$. (4 *marks*)

(c) Briefly explain the following:

(i) Principle of pattern multiplication . (3 *marks*)

(ii) Entropy of a memoryless source. (3 *marks*)

(iii) Quadrature component and in-phase component of narrowband noise. (3 *marks*)


Bessel Functions of the first kind
$J_0(x)$
$J_1(x)$
$J_2(x)$
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
−0.4
0
5
10
15
20
X