The Open University of Sri Lanka
Department of Electrical and Computer Engineering
Final Examination 2011/2012
ECX6234 – Digital Signal Processing

Time: 0930 – 1230 hrs. Date: 2012- 03 - 03

Answer any FIVE questions

1.

(a) A sinusoidal signal $x(t) = \sin(\omega_0 t)$ is converted into a discrete sequence $x[n]$ by sampling. The frequency of the sampling signal is f_s.

(i) Write an expression for $x[n]$.

(ii) What is the digital frequency of $x[n]$?

(iii) If $x(t) = \sin(50t) + 2\sin(100t)$ write an expression for $x[n]$ if the lowest possible value for f_s is used.

(b) (i) What is a linear time invariant (LTI) system?

(ii) Find whether the system defined by the input- output relation $y[n] = 2x[n] + x[n-1]$ is a LTI system.

(c) The input – output relationship of a discrete system is given by

$$y[n] = 2x[n] + x[n-1].$$

(i) Is the system BIBO stable?

(ii) Is the system causal?

Justify your answers.

2.

(a) Define the z-transform of a discrete sequence $x[n]$. What is region of convergence (ROC)?

(b) If the z-transform of $x[n]$ is $X(z)$, write an expression for the z-transform of $y[n]$ in terms $X(z)$ for the following:

(i) $y[n] = x[n-n_0]$ (ii) $y[n] = a^n x[n]$ (iii) $y[n] = x[-n]$ (iv) $y[n] = nx(n)$

(c) Find z-transform of $u[n]$ from first principles.

(d) Find the z-transform of the following sequences:

(i) $y[n] = a^n u[n]$., $|z| > a$
(ii) $y[n] = 0, 0, 0, 0, -1.2, -2.2^2, -3.2^3, -4.2^4, -5.2^5, \ldots$
(iii) $y[n] = 0, 0, 0, 0, -1, -4, -12, -32, -90, \ldots$

(e) Find the inverse z-transform of $X(z) = \dfrac{z}{(z-1)(z-2)}$, $|z| > 2$

3.
(a) Define Discrete Time Fourier Transform (DTFT) for the sequence $x[n]$.

(b) (i) If the DTFT of a signal $x[n]$ is known explain how to find $x[n]$.
(ii) Show that DTFT of $e^{j\omega_0 n}$ is $2\pi\delta(\omega - \omega_0)$
[Hint: You may use the inverse transform]

(c) $x[n] = u[n] - u[n-M]$
(i) Plot $x[n]$.
(ii) Find $X(\omega) = \text{DTFT}(x[n])$ and specify the range of ω.
(iii) Plot $|X(\omega)|$ and important values on your diagram.

(d) The output signal of a digital to analog converter is given by

y(t) = $\sum_{n=-\infty}^{\infty} g(t - nT_s) y[n]$ where $y[n]$ is the digital signal and $g(t)$ is the interpolating function. The Fourier Transforms of y(t) and $g(t)$ are Y(F) and G(F) respectively. If the DTFT of $y(t)$ is $Y(\omega)$ show that $Y(F) = G(F).Y(\omega)$, where $\omega = 2\pi F/F_s = 2\pi FT_s$ and $Y(\omega)$ is the DTFT of $y[n]$.

4.
(a) $h[n] = (u[n+6] - u[n-7]) \cos n\alpha$

(i) Express $h[n]$ in the form $\Sigma C[n]\,\delta[n-k]$ and find the value of $C[n]$. Plot $x[n]$ if $\alpha = \pi/6$.

(ii) $h[n]$ is the impulse response of a discrete system. If the input and the output of the system are $x[n]$ and $y[n]$ respectively, find the difference equation of the system.

(iii) Find the step response of the system given in (ii).

(b) The difference equation below represents the input-output relationship of a system.

$$y[n] - \frac{5}{6} y[n-1] + \frac{1}{6} y[n-2] = x[n]$$

(i) Is the system causal?
(ii) Find the system function $H(z)$.
(iii) Find the impulse response $h[n]$.

5.

(a) The impulse response of a FIR filter is h[n].

(i) If the system is causal, write the difference equation to represent the input / output relationship.

(ii) What is the length of the filter?

(iii) Implement the filter in a block diagram.

(b) The state-space realization of an IIR filter is given by

$$s[n+1] = As[n] + Bx[n]$$
$$y[n] = Cs[n] + Dx[n]$$

Show that the transfer function $H(z)$ of the system is given by

$$H(z) = C(zI-A)^{-1} B + D$$

(c) Write state- space equations for the system shown below:

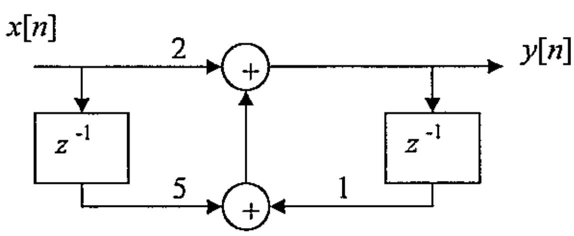

Fig.5

Explain the method of determining the poles of the system using the above state space equations. (Write the necessary equation(s) only)

6.

(a) Parseval's theorem says that $\sum_n |x[n]|^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} |X(\omega)|^2 \, d\omega.$

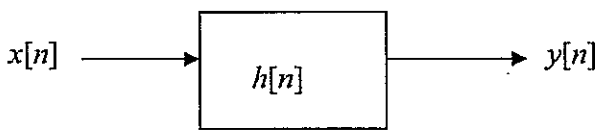

Fig.6

Show that for the system given in Fig.6, $\sum_n |y[n]|^2 \leq H^2_{max}(\omega) \sum_n |x[n]|^2$

(b) Discrete Fourier Transform (DFT) of a sequence $x[n]$ can be written as

$$X_N[k] = \sum_{n=0}^{N-1} x[n]\, w_N^{kn}, \; k = 0, \ldots, N-1, \text{ where } w_N = e^{-j2\pi/N}$$

Show that

(i) $X_N[k] = X[k]^{even}_{N/2} + w_N^k\, X[k]^{odd}_{N/2}$, $k = 0, \ldots, N\text{-}1$

(ii) $X_N[k + N/2] = X[k]^{even}_{N/2} - w_N^k\, X[k]^{odd}_{N/2}$, $k = 0, \ldots, N\text{-}1$

$X^{even}_{N/2}$ - summation of even values of n of the above summation

$X^{odd}_{N/2}$ - summation of odd values of n of the above summation

7.

(a) The signal $x[n]$ is sampled using the sampling sequence $\delta_D[n]$. The resulting signal is v[n].

$$v[n] = \delta_D[n]\,.x[n] \quad \text{if } n/D \text{ is an integer}$$
$$= 0 \qquad \text{otherwise}$$

(i) Show that $\delta_D[n] = \frac{1}{D}\sum_{k=0}^{D-1} e^{-j\frac{2\pi k n}{D}}$ satisfies the requirement for $\delta_D[n]$.

(ii) Derive an expression for V(z) in terms of X(z) .

(b) It is necessary to design a digital lowpass filter with the desired impulse response $h_d[n]$.
Explain how you would design a FIR filter using a window function $w[n]$.
What are the parameters that decide the length of the filter?

8.

(a) Give a block diagram of *Kalman* filter and briefly explain the principle of operation.

(b) Briefly explain the following:

(i) Determination of maxima and minima for a FIR filter, in equiripple approximation method.
(ii) Prediction based sampling method.
(iii) Multistage implementation of digital filters