THE OPEN UNIVERSITY OF SRI LANKA

Department of Civil Engineering

Bachelor of Technology - Level 5

CEX5231 - MECHANICS OF FLUIDS

FINAL EXAMINATION 2011/2012

Time Allowed : Three Hours

Date : 12th March, 2012 Time : 0930 – 1230 hrs

ANSWER **ALL THREE** QUESTIONS IN PART A AND **ANY TWO** QUESTIONS IN PART B. ALL QUESTIONS CARRY EQUAL MARKS.

PART A

Answer **all three** questions in this section.

1) A wind turbine has a blade length of 10 m and is to be operated in a wind of average velocity 10 m/s. A 1:20 scale model of the turbine is tested in air with a wind velocity of 5 m/s. The density of air is 1.25 kg/m^3 and the dynamic viscosity of air is 2×10^{-5} Pa s.

The test is carried out by measuring the rotational speed of the turbine at various values of resisting torque. The results of the model test are presented in Table 1.

Resisting Torque (Nm)	3.5	2.5	1.5	0
Rotational Speed (r.p.m.)	0	40	70	100

Table 1

a) Derive a non-dimensional relationship between the power generated by the turbine and other relevant variables.

b) Use the results of the model test and the non-dimensional relationship derived in -part a) to estimate the

(i) maximum power that can be generated
(ii) rotational speed at which the maximum power will be generated

by the proto-type turbine under normal operating conditions. State all your assumptions and explain your answer.

2) The Navier-Stokes equations can be written in the form

$$\frac{\partial \underline{q}}{\partial t}+(\underline{q}.\nabla)\underline{q}=-\frac{1}{\rho}\underline{\nabla}(p^{*})+\frac{\mu}{\rho}\nabla^{2}\underline{q} \qquad (1) \qquad \text{and} \qquad \nabla.\underline{q}=0 \qquad (2)$$

where $\underline{q}=u\underline{i}+v\underline{j}+w\underline{k}$ is the velocity vector and $p^{*}=p+\rho gz$ is the piezometric pressure with z measured in the $\underline{k}$ direction.

a) List the assumptions that have been made in the derivation of equations (1) and (2).
b) Explain briefly how equation 1) has been derived.
c) Explain the physical significance of the second term in the left hand side of equation (1).

A fluid of density ρ and dynamic viscosity μ flows in a wide open channel that has a small angle of slope θ, as shown in Figure 2. The flow is steady, two-dimensional and uniform in the x direction. The depth of the flow is h. The flow is laminar.

d) Apply the principal of conservation of momentum (force momentum relation) to the elemental control volume shown in Figure 2 and obtain a relationship between the gradient of the shear stress $d\tau/dy$ and the slope of the channel. State all assumptions and explain your answer.

e) Integrate the relationship obtained in section d) and obtain expressions for the velocity profile u(y) and the discharge per unit width in the channel. State all boundary conditions and explain your answer.

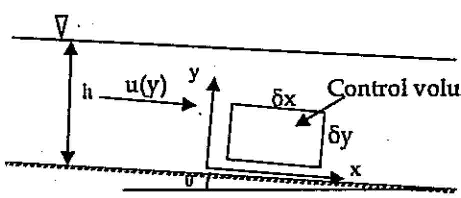

Figure 2

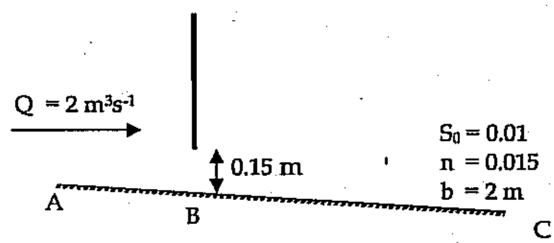

Figure 3

3) A long open channel ABC has a width of 2 m, a slope of 0.01 and a Manning's coefficient of 0.015. The channel carries a steady discharge of 2 m³/s. A gate is placed in this channel at B as shown in Figure 3. The opening of the gate is 0.15 m and uniform flow is observed far upstream and downstream of the gate.

a) Calculate the critical depth for the given channel and discharge.
b) Calculate the uniform depth for the given channel and discharge.
c) Calculate the depth of flow just upstream of the gate at B.
d) Show that there is a free hydraulic jump somewhere near the gate.
e) Sketch the profile of the flow (variation of flow depth) from the uniform flow far upstream of the gate to the uniform flow far downstream of the gate. Identify the flow profile type (from M1, M2, M3, S1, S2, S3) of each section and indicate the location of any hydraulic jumps.
f) Estimate the location of the hydraulic jump. Explain your answer.

Note : The equations $\frac{dy}{dx}=\frac{S_0-S_f}{1-F_r^2}$ and $\frac{y_1}{y_2}=\frac{1}{2}\left(\sqrt{1+8F_{r2}^2}-1\right)$ may be used.

PART B

Answer **any two** questions in this section.

4) A rectangular open channel has a width of 2 m and carries a discharge of $3\ m^3/s$ at a depth of 1.2 m. The channel undergoes a smooth transition between points A and B where the bed level of the channel is raised by Δ m while the width of the channel is reduced by Δ m at the same time. These changes are shown in Figures 4a and 4b.

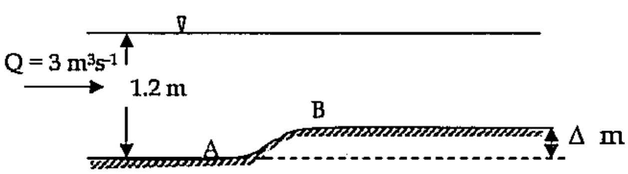

Figure 4a: Longitudinal view of channel

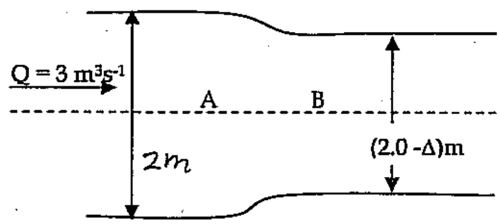

Figure 4b: Plan view of channel

a) Derive a relationship between the non-dimensional specific energy (E/y_c) and the non-dimensional depth (y/y_c) for a channel with a rectangular cross-section.

b) Calculate the flow depth at B when $\Delta = 0.1$ m. Explain your answer using a graphical version of the relationship derived in part a).

It is observed that the flow depth at A changes when Δ is increased beyond a certain critical value.

c) Explain this observation using the graphical version of the relationship derived in part a).

d) Calculate the critical value of Δ beyond which the flow depth at A changes.

5) The Manning's coefficient for a uniform gravel bed is estimated by the equation $n = 0.039 d^{1/6}$ where d is the grain size in meters. The Shields' parameter is defined by the equation $\Pi_2 = \frac{\rho u_{*cr}^2}{\gamma_{s*} d}$.

a) Identify and define the variables in the above definition of the Shields' parameter.

b) Sketch the Shields curve. Identify and define the variables on the two axes.

c) Explain briefly how the Shields' curve has been obtained.

A long uniform open channel of rectangular cross-section has a width of 2 m and a slope of 0.0005. The channel bed is lined with a loose gravel of uniform diameter 6 mm. When the discharge in the channel is increased gradually it is found that the gravel begins to move when the discharges reaches a value of $3\ m^3/s$. The density of water is $1000\ kg/m^3$ and the density of gravel is 2650 kg/m^3.

d) Calculate the critical value of the Shields parameter for the initiation of motion. State all your assumptions and explain your answer.

e) Calculate the minimum diameter of gravel that should be used to maintain a stable bed if the channel is to carry a discharge of $5\ m^3/s$. State all your assumptions and explain your answer.

6) The velocity vector, U in a two-dimensional flow can be written as $U = u\underline{i} + v\underline{j}$ where i and j are the unit vectors in the x and y directions and u and v are the velocity components in the x and y directions. The stream function ψ and the potential function ϕ are defined by the equations

$u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$ and $u = \frac{\partial \phi}{\partial x}$ and $v = \frac{\partial \phi}{\partial y}$, respectively.

a) Show that relationship between the velocity components and the stream function satisfies the equation for the conservation of mass in a steady, two-dimensional flow..

b) Show that the potential function satisfies the Laplace equation (i.e. $\nabla^2 \phi = 0$) when the flow is steady.

The complex potential of a two-dimensional, ideal fluid flow is given by $W = Uz + ik\ln(z)$

c) What is the relationship between the stream function, potential function and complex potential of a two-dimensional, ideal fluid flow?

d) What is the definition of a streamline in a two-dimensional, ideal fluid flow?

e) Obtain expressions for the velocity components of this flow in the x and y directions.

f) Identify any stagnation points in the flow.

g) Sketch the streamlines of this flow. Explain your answer.

7) A large well of diameter 5 m is made in a horizontal layer of permeable material, as shown in Figure 7. The permeable material has a permeability of 2.5 m/day and lies on top of a horizontal layer of impervious rock, as shown in the figure. The well is used to supply water to a distant town and the average rate of pumping is 100,000 litres/day. The average rate of recharge of the aquifer is 1.5 mm/day. The height of the free surface of the aquifer far from the well is 10 m above the level of the impervious rock.

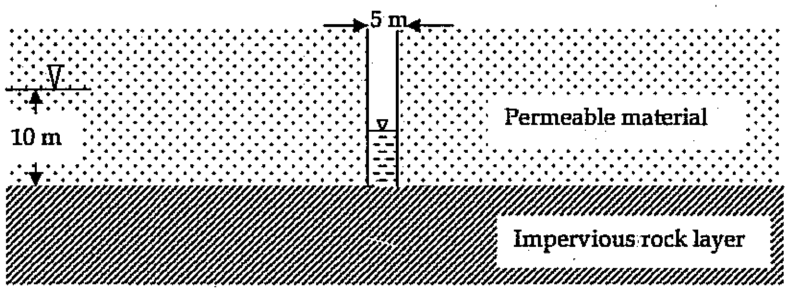

Figure 7

a) Estimate the radius of influence of the well. The radius of influence is the radius at which the groundwater level is not affected by the well.

b) Consider an appropriate control volume at a radius r from the centre of the well and derive a differential equation governing the variation of the water level of the aquifer with the radius r. State all your assumptions.

c) Solve the differential equation derived in part b) using appropriate boundary conditions and calculate the depth of water in the well.

8) Water is supplied from a Tank X to taps P, Q and R through the pipes AB, BP, BC, CQ and CR as shown in Figure 8. All the pipes have a diameter of 25 mm and a friction factor of 0.01. The lengths of the pipes and the elevations of the taps and the free surface of the tank are given in the figure.

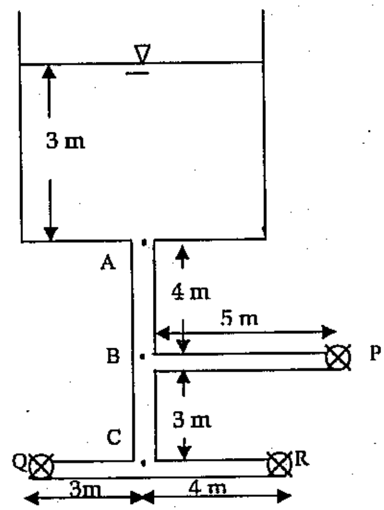

Figure 8

a) Calculate the discharge through taps P and Q when taps P and Q are open but tap R is closed. Explain your method. Assume reasonable values for any quantities that are not given.

b) Explain how you would calculate the discharge through taps P, Q and R when all three taps are open.