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The Open University of Sri Lanka
Department of Electrical and Computer Engineering
ECX 6241 – Field Theory
Final Examination – 2012/2013

Date: 2013-07-31 Time: 0930-1230

Answer **five questions** by selecting **two from Section A, two from Section B** and **one from Section C**.

Section A

Select two questions from this section. (15 Marks for each)

1. (a) Explain what is a "conservative vector field".

(b) Prove that the work done of moving an object from P_1 to P_2 in a conservative force field $\boldsymbol{F}$ is independent of the path joining the two points P_1 and P_2.

(c) Evaluate $\int_C \boldsymbol{A} \cdot \boldsymbol{dr}$ where $\boldsymbol{A} = 2x\,\boldsymbol{i} + 4y\,\boldsymbol{j} - 3z\,\boldsymbol{k}$ where C is a curve: $\boldsymbol{r}(t) = \cos t\,\boldsymbol{i} + \sin t\,\boldsymbol{j} + t\,\boldsymbol{k}$ is defined from $t = 0$ to π.

2. (a) V is a scalar field and $\boldsymbol{A}$ is a vector field. Verify each of the following vector identities:

(i) $\nabla \times (V\boldsymbol{A}) = \nabla V \times \boldsymbol{A} + V(\nabla \times \boldsymbol{A})$

(ii) $\nabla \times (\nabla \times \boldsymbol{A}) = \nabla(\nabla \cdot \mathrm{A}) - \nabla^2 \boldsymbol{A}$

(b) Let S be the surface of the circular cylinder $x^2 + y^2 = a^2$ in the first octant between the planes $z = 0$ and $z = h$. Evaluate $\iint_s \boldsymbol{A} \cdot \boldsymbol{n}\, ds$ where $\boldsymbol{A} = z\,\boldsymbol{i} + x\,\boldsymbol{j} - 3xy^2\,\boldsymbol{k}$.

3. (a) State the following theorems:

(i) Divergence theorem,

(ii) Stoke's theorem,

(b) Determine the curl of the following vector fields.

(i) $\boldsymbol{Q} = \rho \sin\phi\, \boldsymbol{a}_\rho + \rho^2 z\, \boldsymbol{a}_\phi + z\cos\phi\, \boldsymbol{a}_z$

(ii) $\boldsymbol{T} = \frac{1}{r^2}\cos\theta\, \boldsymbol{a}_r + r\sin\theta\cos\phi\, \boldsymbol{a}_\theta + \cos\theta\, \boldsymbol{a}_\phi$

(c) Verify the Stokes's theorem for $\boldsymbol{A} = y^2\boldsymbol{i} + xy\,\boldsymbol{j} - xz\,\boldsymbol{k}$ when s is the hemisphere $x^2 + y^2 + z^2 = a^2$ and $z = 1$.

Section B

Select **two** questions from this section. (20 Marks for each)

4. (a) State the Gauss's law.

(b) A charge distribution with spherical symmetry has density $\rho_v = \begin{cases} \frac{\rho_0 r}{R}; & 0 \le r \le R \\ 0; & r > R \end{cases}$.

Determine $\boldsymbol{E}$ and V everywhere.

(c) Plot $\boldsymbol{E}$ and V along $\boldsymbol{a}_r$ and comment on it.

5. (a) Consider length L of two coaxial conductors of inner radius a and outer radius b ($b > a$). The space between them are be filled with a homogenous dielectric with permittivity ε and the inner and outer conductors carry $+Q$ and $-Q$ respectively. Find the capacitance.
(b) Conducting spherical shells with radii a and b are maintained at a potential difference V such that $V(r = a) = V_0$ and $V(r = b) = 0$. Determine V and E in the region between the shells.

6. (a) Explain the term "Magnetic levitation".
(b) A bar magnet or a small filamentary current loop is usually referred to as a magnetic dipole. Find the magnetic vector potential in the far field.
(c) Two coaxial circular wires of radii a and b ($b > a$) are separated by distance h ($h \gg a, b$). Find the mutual inductance between wires.

Section C

Select **one** question from this section. (30 Marks)

7. (a) Write Maxwell's equations.
(b) Which of the following fields in free space are Maxwellian?

(i) $\boldsymbol{H} = 10\cos\left(10^5 t - \frac{z}{10}\right)\, \boldsymbol{a}_x$

(ii) $\boldsymbol{E} = \frac{\sin\theta}{r}\cos\left(\omega t - r\omega\sqrt{\mu_0\varepsilon_0}\right)\, \boldsymbol{a}_\theta$

(iii) $\boldsymbol{B} = (1 - \rho^2)\sin\omega t\ \boldsymbol{a}_z$

(c) In a medium characterized by $\sigma = 0$, $\mu = \mu_0$, $\varepsilon = \varepsilon_0$ and $\boldsymbol{E} = 20\sin(10^8 t - \beta z)\boldsymbol{a}_y\ V/m$. Calculate β and $\boldsymbol{H}$ by using phasors.

8. (a) Explain "Poynting's Vector".
(b) Calculate the total instantaneous power flow "W" leaving a closed surface S by using $W = \oint \boldsymbol{E} \times \boldsymbol{H} ds$. (Use the vector identity $div(\boldsymbol{E} \times \boldsymbol{H}) = \boldsymbol{H} \cdot curl\ \boldsymbol{E} - \boldsymbol{E} \cdot curl\ \boldsymbol{H}$ to show where the power leaving a close surface goes in terms of stored energy and ohmic dissipation.)
(c) A uniform plane wave propagating in a medium has $\boldsymbol{E} = 2e^{-\alpha z}\sin\left(10^8 t - \beta z\right)\boldsymbol{a}_y V/m$. If the medium is characterized by $\varepsilon_r = 1$, $\mu_r = 20$ and $\sigma = 3\ S/m$, find α, β and $\boldsymbol{H}$.

Note:

1. $\frac{1}{4\pi\varepsilon_0} \cong 9 \times 10^9\ m/F$
2. $\mu_0 = 4\pi \times 10^{-7}\ \frac{H}{m}$