# The Open University of Sri Lanka
## Department of Electrical and Computer Engineering
## Final Examination 2012/2013
## ECX6234 – Digital Signal Processing

Time: 0930 – 1230 hrs. Date: 2013- 08 - 03

***Answer any FIVE questions***

**1.**

(a) (i) Write an expression for $X(\omega)$ the Discrete Time Fourier Transform (*DTFT*) of $x[n]$.
Find $X(\omega)$ for the $x[n]$ given below:

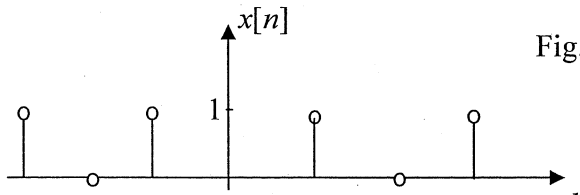


Fig.1

(ii) Define $z$-transform $X(z)$ of $x[n]$. What is the relationship between $X(z)$ and $X(\omega)$?

(b) From first principles determine the $z$-transform and the Region of convergence (*ROC*) of $2^n u[n]$.

(c) Derive an expression for the $z$-transform of $x[n-n_0]$ in terms of $X(z)$.

(d) Find the inverse $z$-transform $x[n]$ of $X(z) = \dfrac{z}{(z-1)(z-2)}$, $1 < |z| < 2$.

**2.**

(a) What is a BIBO stable system?

If the impulse response of a BIBO stable *LTI* system is $h[n]$, show that $\sum_n |h[n]|$ is always bounded.

(b) Find whether the systems with following impulse responses are BIBO stable:

(i) $h[n] = u[n]$

(ii) $h[n] = 0.5^n u[n]$

(c) What is a causal system?
Show that for a causal system, impulse response $h[n] = 0$ for $n < 0$

**3.**

(a) The input $x[n]$ and the output $y[n]$ of a causal system is related by the following difference equation:

$$y[n] - \frac{3}{8}y[n-1] + \frac{1}{8}y[n-2] = x[n]$$

(i) Find the system function $H\{z\}$.

(ii) Find the impulse response $h[n]$.

(b)

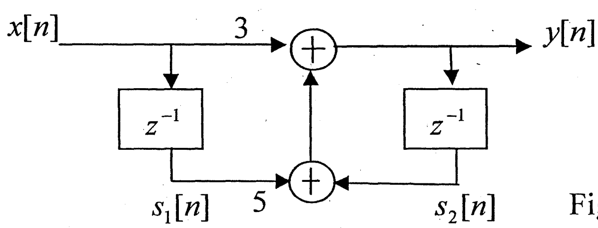


Fig.2

For the linear system shown in Fig.2, write state space equations. You may use the states $s_1[n]$ and $s_2[n]$ in your equations.

**4.**

(a) The sequence $x[n]$ is down sampled using the sampling operator $\delta_D[n]$. The resulting function $v[n] = x[n] . \delta_D[n]$.

$$\delta_D[n] = \begin{cases} 1 & \text{if } n/D \text{ is an integer} \\ 0 & \text{otherwise} \end{cases}$$

If $D = 2$ derive an expression for $\delta_D[n]$ and show that

$$V(z) = \frac{1}{2}X(z) + \frac{1}{2}X(-z)$$

(b) If $v[n] = \frac{1}{D}\sum_{k=0}^{D-1} x[n]\, e^{-j2\pi k\frac{n}{D}}$ represents the sampled signal, show that

$$V(\omega) = \frac{1}{D}\sum_{k=0}^{D-1} X(\omega - 2\pi k/D)$$

(c) A sequence $x[n]$ is up sampled by a factor $L$ ($L$-1 zeros are added between samples). If the output of the sampler is $y[n]$, write an expression for $Y(\omega)$ in terms of $X(\omega)$.
Suppose $X(\omega) = |\cos(\omega)|$, sketch $X(\omega)$ and $Y(\omega)$ for $-4\pi \leq \omega \leq 4\pi$ if $L = 4$.

**5.**
A sequence is given by $x[n] = [0.5, 1, 0.2, 0, 1, 0.7, 0, 0, 0, ....0]$, $n = 0, 1, 2, \ldots$.
This sequence is transformed in to the analogue signal $x(t)$ using the first order hold (*FOH*) function $g(t)$ shown in Fig.3.

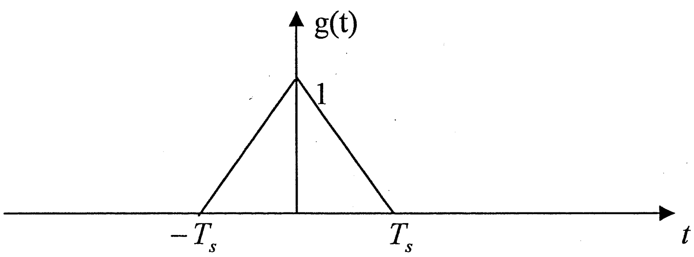


Fig.3

(a) Sketch $x[n]$.

(b) If $x(t) = \sum_{n=0}^{m} x[n]\, g(t - nT_s)$, sketch $x(t)$ for $m = 1$.

(c) Decide a suitable value for $m$ and sketch the analogue signal $x(t)$.

(d) Derive an expression for $X(F)$ (Fourier Transform of $x(t)$) in terms of $G(F)$ and $X(\omega)$, where $X(\omega)$ is the Discrete Time Fourier Transform of $x[n]$.

(e) Normally $x[n]$ is sent through an antialiasing filter before the FOH. Why is that?

**6.**

(a) A filter has a difference equation $y[n] = a\, y[n-1] + x[n] + x[n-1]$, where $a$ is constant. Find what kind of filter does the equation represent (a FIR filter or a IIR filter) if (i) $|a| < 1$
(ii) $|a| \geq 1$

(b) The impulse response of a certain causal filter can be represented by the sequence $h[n]$ which has finite energy.
(i) Can the filter be recursive (IIR filter)? Is it necessary that the filter be a finite impulse response filter?

(ii) Sketch $h[n]$ for a typical case.

(c) Paley-Wiener theorem states that if $h[n]$ is a causal sequence with finite energy then $\int_{-\pi}^{\pi} | \log | H(\omega) | | \, d\omega < +\infty$, where $H(\omega) = DTFT\{h[n]\}$

Using this result show that an ideal lowpass filter cannot be causal.

7.

(a) Sketch the frequency response of a non ideal lowpass filter and mark important regions. Compare critical parameters of a non ideal filter with an ideal filter. In the design of a lowpass filter explain how these values can be utilized.

(b) In the design of a window based filter the transfer function of the final filter can be written in the form $H(\omega) = e^{-j\omega L} H_w(\omega)$.

What is $L$? How is $H_w(\omega)$ related to the window function $w[n]$?

Derive the above equation for $H(\omega)$ using first principles.

(c) Design a digital lowpass filter having a pass band of 4.2 *kHz*, a stop band of 5 *kHz* with at least 30 *dB* attenuation and the sampling frequency of 22 *kHz*.

**8.**

Briefly explain the following:

(a) DFT Filter banks and transmultiplexers
(b) Multistage implementation of digital filter.
(c) Fast Fourier Transform.
(d) Estimator model of a Kalman filter

**<u>Supplementary material</u>**

(a) Window functions

| Window type | $w(n)$ | Δω | Attenuation |
|---|---|---|---|
| Rectangular | 1 | $\frac{4\pi}{N}$ | -13dB |
| Bartlett | $\frac{2}{N-1}\left(\frac{N-1}{2} - \left\vert n - \frac{N-1}{2} \right\vert\right)$ | $\frac{8\pi}{N}$ | -27dB |

| | | | |
|---|---|---|---|
| Hanning | $0.5+0.5\cos\left(\frac{2\pi\, n}{N-1}\right)$ | $\frac{8\pi}{N}$ | -32dB |
| Hamming | $0.54+0.46\cos\left(\frac{2\pi\, n}{N-1}\right)$ | $\frac{8\pi}{N}$ | -43dB |
| Blackman | $0.42+0.5\cos\left(\frac{2\pi\, n}{N-1}\right)+0.08\cos\left(\frac{2\pi\, n}{N-1}\right)0{\cdot}42+$ | $\frac{12\pi}{N}$ | -53dB |

(b) Some important Z-transforms

| Function | z-transform | *ROC* |
|---|---|---|
| $\delta[n]$ | 1 | All z |
| $u[n]$ | $\frac{z}{z-1}$ | $\lvert z\rvert>1$ |
| $a^n u[n]$ | $\frac{z}{z-a}$ | $\lvert z\rvert>a$ |
| $-a^n u[-n-1]$ | $\frac{z}{z-a}$ | $\lvert z\rvert<a$ |
| $nx[n]$ | $X\left(\frac{1}{z}\right)$ | $R'=\frac{1}{R}$ |
| $x[-n]$ | $-z\frac{dX(z)}{dz}$ | $R'=R$ |
| $z_0^n x[n]$ | $X\left(\frac{z}{z_0}\right)$ | $R'=\lvert z_0\rvert R$ |
| $e^{j\Omega_0 n}x[n]$ | $X(e^{j\Omega_0 n}z)$ | R'=R |

$X(z)$ is the z-transform of $x[n]$. $R$ is the *ROC* of $X(z)$