THE OPEN UNIVERSITY OF SRI LANKA

BACHELOR OF TECHNOLOGY – LEVEL 05

FINAL EXAMINATION – 2012/2013

MPJ 5134 – THE NATURE OF MATHEMATICS I

DURARTION: THREE HOURS (3.00)

Date: 28th July 2013	Time: 1330hrs – 1630hrs

Instructions:

- **Answer five (05) questions**
- **Number of pages in the paper – 04.**
- **All symbols are in standard notation.**

01. (a) Let $a, b \in \mathbb{R}$. Using field and order axioms prove that:

(i) $a.0 = a,$

(ii) $-(-a) = a,$

(iii) $a.(-b) = -a.b,$

(iv) Deduced that $(-b)^2 = b^2$. Hence, show that $(a-b)^2 = a^2 - 2ab + b^2$. [50%]

(b) (i) Let $x, a, b \in \mathbb{R}$, show that $|x - a| < b$ if and only if $a - b < x < a + b$.

(ii) Solve $\left|\frac{1}{x-1} - 2\right| < 1$ for all possible solutions and draw the related graph. [30%]

(c) Prove by mathematical induction that $n^5 - n$ is divisible by 5 for all natural numbers n.[20%]

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02. (a) Let p, q and r be propositions.

(i) Show that $p \Rightarrow q \equiv \sim p \vee q$. Hence, show that the proposition $\left(q \wedge (p \Rightarrow \sim q)\right) \Rightarrow \sim p$ is tautology. [30%]

(b) Use truth tables to discover whether the following "equivalences" are valid or not.

(i) $\sim (p \vee \sim q) \vee (\sim p \wedge \sim q) \equiv p$.

(ii) $(p \Rightarrow q) \vee (p \Rightarrow r) \equiv p \Rightarrow (q \vee r)$. [40%]

(c) Determine the validity of the arguments:

(i) $(p \Rightarrow q) \wedge p \vdash q$.

(ii) $\left((p \Rightarrow q) \wedge \sim q\right) \vdash \sim p$. [30%]

03. (a) Give a direct proof for :

(i) If a is an odd integer and b is an even integer, then ab is an odd integer.

(ii) Let x be a real number. Then $\left|x + \frac{1}{x}\right| > 1$. [30%]

(b) Show using proof by contradiction that:

(i) If x is an irrational number, then $x + 7$ is irrational.

(ii) Suppose a is an integer. If $a^2 - 2a + 7$ is even, then a is odd. [40%]

(c) (i) Show that if $|x^2 - 4x + 6| \neq 2$, then $x \neq 2$.

(ii) Suppose a and b are positive real numbers such that $a^{\sqrt{b}} \neq b^{\sqrt{a}}$. Show that $a \neq b$. [30%]

04. (a) Determine whether or not each set is the null set:

(i) $A = \{x |\ x \in \mathbb{R} \ and \ x^2 + x + 1 = 0 \ or \ x = 1\}$.

(ii) $B = \{x | x \in \mathbb{R} \ \ and \ |x - 8| + 4 = 0 \ \}$.

(iii) $C = \left\{x \middle|\ x \in \mathbb{R} \ \ and \ \ \{x\} \subseteq \phi\right\}$. [30%]

(b) Suppose A, B are sets and $A = A \cap B$

(i) What is $A \cup B$ equal to?

(ii) What is $B \setminus A$ equal to?

(iii) What is $(B \setminus A) \cup A$ equal to? [30%]

(c) Suppose A, B, C are sets.

(i) Show by the Venn diagram method that $A \cup B \setminus C = (A \setminus C) \cup (B \setminus C)$.

(ii) Give an example to show that $A \cap (B \cup C) \neq (A \cap B) \cup C$. [40%]

05. (a) Suppose E is a universal set. Now consider the following three results: For any subsets A, B, C of E,

C(1): $(A^c)^c = A$

C(2): $(A \cup B)^c = A^c \cap B^c$ } - De Morgan's Laws

C(3): $(A \cap B)^c = A^c \cup B^c$

C(4): $A \cap A^c = \phi$

C(5): $A \cup A^c = E$

C(6): $A \subseteq B \Rightarrow B^c \subseteq A^c$

U(1): $A \cup A = A$. - Idempotency

U(2): $A \cup B = B \cup A$. - Commutativity

U(3): $A \cup \phi = A$.

U(4): $A \cup (B \cup C) = (A \cup B) \cup C$. - Associativity

U(5): $A \cup B = B \Leftrightarrow A \subseteq B$.

Prove that: (i) $\phi^c = E$, (ii). $E^c = \phi$, (iii). $A \subseteq B \Leftrightarrow B^c \subseteq A^c$. [30%]

(b) Show by the logic method that :

(i) $\{x \mid x \in \mathbb{R},\ |x-1| < 1\} = \{x \mid x \in \mathbb{R},\ x < 1\} \cap \{x \mid x \in \mathbb{R},\ x > 0\}$.

(ii) $\{x \mid x \in \mathbb{R},\ x^2 - 3x + 5 > 3\} = \{x \mid x \in \mathbb{R},\ x < 1\} \cup \{x \mid x \in \mathbb{R},\ x > 2\}$.
[30%]

(c) Suppose A, B, C are subsets of universal set E. Prove by the logic method that:

(i) If $A \subseteq B$ and $C \subseteq D$, then $A \cap C \subseteq B \cap D$.

(ii) $A\setminus(B\cap C)=(A\setminus B)\cup(A\setminus C)$. [40%]

06. (a) Let $A=\{1,2,3,4\}$, $B=\{2,\ 4,6\}$ ñ is the relation from $A\times B$ where for any $x\in A$, $y\in B$ xñ$y \Leftrightarrow x+1<y$.

Let $S=\{(x,y)|\ x\in A, y\in B$ and xñ$y\}$, $T=\{(x,y)|\ x\in A, y\in B$ and xñ$^{-1}y\}$ and

$U=\{(x,y)|\ x\in B, y\in A$ and x ñ$^{-1}y\}$. Write down all members of S, T and U. [40%]

(b) Consider the relation ñ on $\mathbb{N}$ where $\forall x,y\in\mathbb{N}$, xñ$y \Leftrightarrow 7|x-y$.

Show that ñ is an equivalence relation on $\mathbb{N}$. Find all disjoint equivalence classes

[2],[5] and [13]. [60%]

07. (a) Let a, b and c be integers.

(i) Show that if $d|a$ and $d|b$, then $d|a+b$.

(ii) If $a|c$ and $b|d$, then $ab|cd$, where $a\neq 0, b\neq 0$.

(iii) Let $\gcd(a,b)=g$. Show that $\gcd(a,a+b)=g$. [50%]

(b) Using well-ordering principle of $\mathbb{N}$,

(i) show that for all $n\in\mathbb{N}$, $1+3+5+\cdots+(2n-1)=n^2$.

(ii) show that for all $n\in\mathbb{N}$, $1+2+2^2+2^3+\cdots+2^n=2^{n+1}-1$. [50%]

END