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The Open University of Sri Lanka
Faculty of Engineering Technology

Study Programme	:	Diploma in Technology/Bachelor of Technology (Engineering)
Name of the Examination	:	Final Examination
Course Code and Title	**:**	**MEX5233 Dynamics of Mechanical Systems**
Academic Year	:	2012/13
Date	:	28th July 2013
Time	:	0930 hrs. – 1230 hrs.
Duration	:	3 hours

General instructions

1. Read all instructions carefully before answering the questions.
2. This question paper consists of 8 questions. All questions carry equal marks.
3. Answer any 5 questions only.

QUESTION 01

A machine of mass m is mounted on the ground through an isolator consisting of a spring of stiffness k and a damper having a damping ratio β. The machine produces an excitation force of the form $P \sin \omega t$, where P is the magnitude and ω is the frequency. The displacement of vibratory motion of the machine (x) at steady state is given as

$$x = \Delta \eta \sin(\omega t - \phi), \quad \text{where} \quad \Delta = \frac{P}{k}, \quad \eta = \frac{1}{\sqrt{(1-r^2)^2 + (2\beta r)^2}}, \quad r = \frac{\omega}{\omega_n}, \quad \omega_n = \sqrt{\frac{k}{m}} \quad \text{and}$$

$$\phi = \tan^{-1}\left(\frac{2\beta r}{1-r^2}\right).$$

(a) Show that the transmissibility ratio (T) of the force transmitted to the ground at steady state is given by

$$T = \sqrt{\frac{1+(2\beta r)^2}{\left(1-r^2\right)^2 + (2\beta r)^2}}$$

(b) Plot a graph of T vs r for r = 0, 1, $\sqrt{2}$, 2 & 3 and β = 0.5. Using the graph explain the significance of $r = \sqrt{2}$.

(c) The following observations were made with respect to the performance of the above machine of mass 250 kg.

Transmissibility at resonance = 2

Transmissibility at an excitation frequency of 25 Hz = 0.5

Calculate the damping ratio of the damper and the stiffness of the spring.

QUESTION 02

Two bodies, each of mass m, and of which the lateral movements are constrained, are suspended by two springs, each having a stiffness k, from a uniform, rigid, weightless bar as shown in Fig. Q2. The bar which is free to pivot about P is connected elastically to a rigid structure by another two springs, each having a stiffness k.

(a) Determine the equations of motion and show that the frequency equation giving natural frequency (ω) can be expressed as

$$\omega^4 - \frac{3}{2}\left(\frac{k}{m}\right)\omega^2 + \frac{1}{2}\left(\frac{k}{m}\right)^2 = 0$$

(b) Determine the natural frequencies in terms of k and m and obtain the mode shapes.

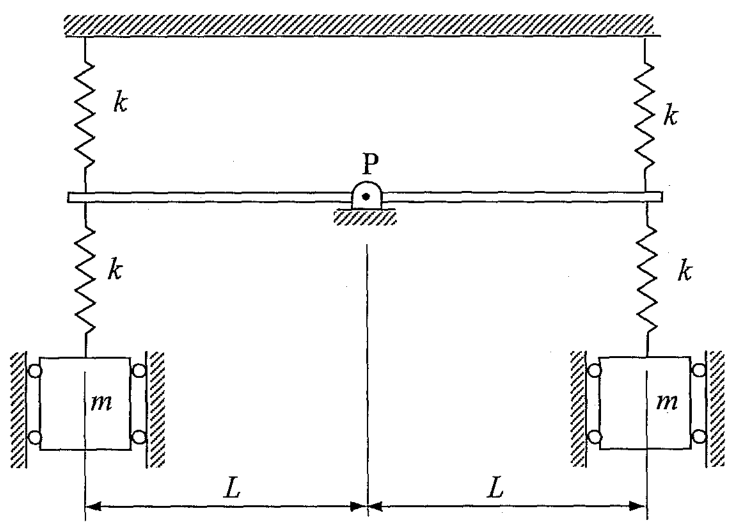

Fig. Q2

QUESTION 03

A torsional vibration system consists of four rotors, 1, 2, 3, and 4, mounted on a steel shaft of diameter 50 mm. Moments of inertia of the rotors and the lengths of the sections of the shaft are given in the Table below.

Moments of Inertia of Rotors (kgm^2)	
Rotor 1	0.6
Rotor 2	0.2
Rotor 3	0.1
Rotor 4	0.25

Lengths of Sections of Shafts between Rotors (m)	
Between rotors 1 and 2	0.50
Between rotors 2 and 3	0.25
Between rotors 3 and 4	1.0

An approximate value of the first natural frequency is given as 62.5 Hz.

(a) Check this approximate value and determine whether it is high or low.
(b) Sketch the mode of vibration at this frequency and find the position of the node.

Take rigidity modulus of steel as 80 GN/m^2.

QUESTION 04

(a) Determine the natural frequency of transverse vibration of a uniform beam with fixed ends and length L, mass per unit length m and flexural rigidity EI using Rayleigh's method. Assume that the dynamic deflection curve of the beam is of the form,

$$y = a\{x^2(L-x)^2\},$$

where y is the deflection of the beam at a section at a distance x from one end, L is the length of the beam and a is a constant.

(b) A steel pipe of inner diameter 80 mm and outer diameter 90 mm is held in clamped supports 4 m apart. The pipe is full of liquid of density 800 kg/m^3. Calculate the natural frequency of transverse vibration of the pipe. Influence of the liquid on flexural rigidity can be neglected.

Density of steel	-7.85×10^3 kg/m^3
Modulus of elasticity of steel	-200 GN/m^2

QUESTION 05

(a) Fig. Q5 shows the block diagram of a position control system with velocity feedback. The transient response of the closed loop system has a damping ratio of 0.6, and a frequency of damped oscillation of 4 rad/s.

(i) Determine the values of k_1 and k_2.
(ii) Determine the unit step response of the closed loop system.

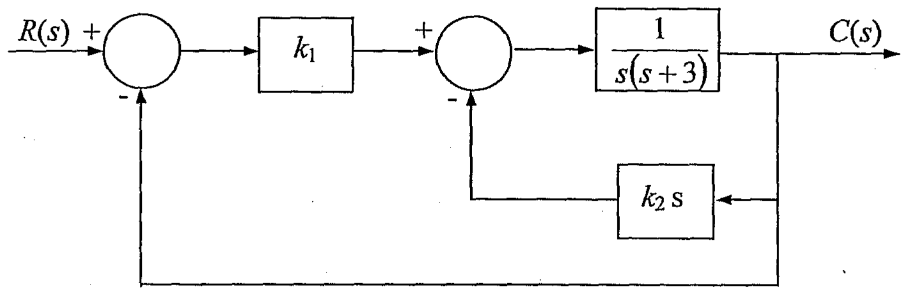

Fig. Q5

(b) A system, whose characteristic equation is,

$$s^3 + (8 + K)s^2 + 6s + (16 + 8K) = 0,$$

has been designed to give satisfactory performance with a forward path amplification gain of $K = 2$. Determine the factor by which this gain can be increased before the system becomes unstable.

QUESTION 06

The open loop transfer function of a unity feedback control system is given by

$$G(s) = \frac{K}{(s+1)(s+3)(s+5)}$$

where K is the gain of the controller.

(i) Draw the root locus as a function of K and determine the range of values of K for which the system is stable. Verify your result using Routh's criterion.

(ii) Using the root locus you have drawn show that $s = -6$ is a closed loop pole of the system for $K = 15$. Determine the other closed loop poles.

(iii) Determine the damping ratio of the closed loop system when $K = 15$.

(iv) Determine the unit impulse response of this system for $K = 15$. Assume that all initial conditions are zero.

QUESTION 07

The open loop transfer function of a unity feed back control system is given by

$$G(s) = \frac{K}{s(1+0.1s)(1+0.5s)}.$$

Plot the Bode diagram of $\frac{G(j\omega)}{K}$ and determine the values of K for a

(a) gain margin of 20 db

(b) phase margin of 45°.

QUESTION 08

Fig. Q8 shows the arrangement of a speed control system for an engine. If the engine speed increases the sleeve of the fly-ball governor moves upward and this movement acts as the input to hydraulic controller. A positive error signal (upward motion of the sleeve) causes the power piston to move downward, thus reducing the fuel-valve opening and decreasing the engine speed.

The dashpot connected to the piston in the working cylinder has a damping constant c and the stiffness of the spring attached to the dashpot is k. The oil flow in the servomechanism per unit displacement is Q and the area of cross section of the piston in the working cylinder is A. Draw a complete block diagram of the overall system and obtain the transfer function relating y and e. You may neglect fluid compressibility, fluid leakage and inertia of moving parts.

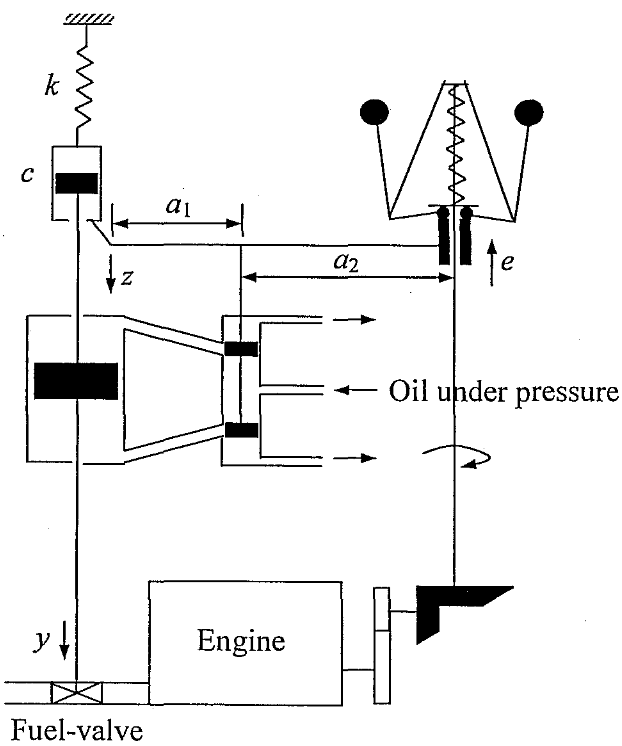

Fig. Q8

LAPLACE TRANSFORMS

TIME FUNCTION $f(t)$	**LAPLACE TRANSFORM** $F(s)$
Unit Impulse $\delta(t)$	1
Unit step	$\frac{1}{s}$
t	$\frac{1}{s^2}$
t^n	$\frac{n!}{s^{n+1}}$
$\frac{df(t)}{dt}$	$sF(s)-f(0)$
$\frac{d^n f(t)}{dt^n}$	$s^n F(s) - s^{n-1} f(0) - s^{n-2} \frac{df(0)}{dt} \ldots - \frac{d^{n-1} f(0)}{dt^{n-1}}$
e^{-at}	$\frac{1}{s+a}$
te^{-at}	$\frac{1}{(s+a)^2}$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
$e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at} \cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$