# The Open University of Sri Lanka
# Department of Electrical and Computer Engineering
# Final Examination 2012/2013
# ECX5233 – Communication Theory and Systems

Time: 0930 – 1230 hrs. Date: 2013-08 -16

*Answer any FIVE questions*

**1.**
(a)

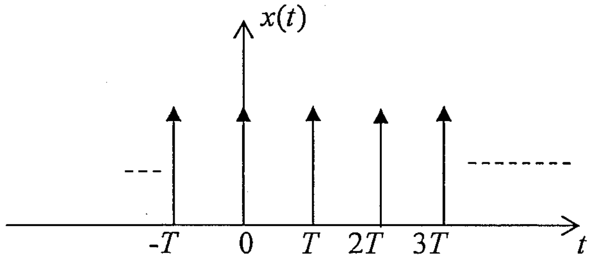


Fig.1

$x(t)$ is an impulse train extending from $-\infty$ to $\infty$.

(i) Express $x(t)$ as a Fourier series.
(ii) Sketch the frequency spectrum of $x(t)$.

(b) Above impulse train in (a) is fed to an ideal low pass filter whose lower cutoff frequency is $\omega_L$. If $\omega_L = \pi/T$ and output of the filter is $y(t)$

(i) Determine $y(t)$
(ii) Sketch $y(t)$ indicating the important values.
(iii) Calculate $Y(\omega)$ the Fourier transform of $y(t)$.
(iv) Sketch $Y(\omega)$ indicating important values.
(v) Now the repetition rate $T$ of the impulse train is reduced slowly. Describe the behavior of $Y(\omega)$.



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**2.**

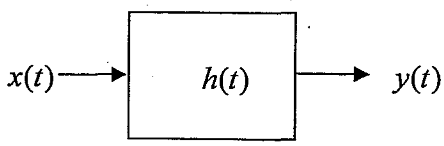
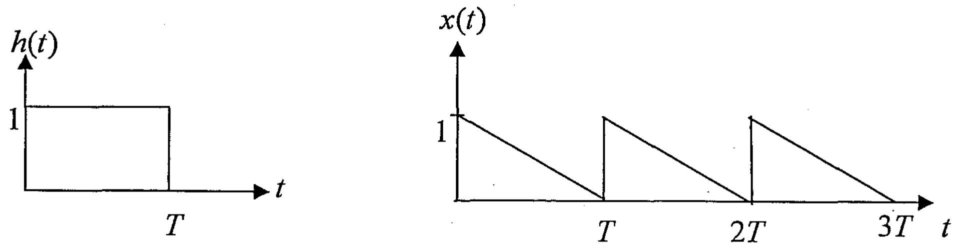



Fig.2



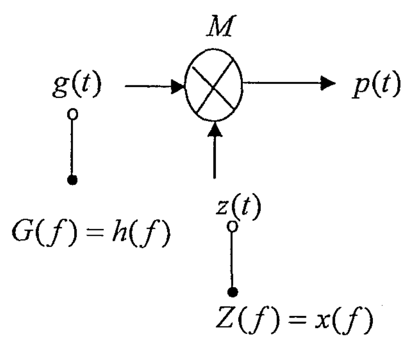

(a)

Convolution of $h(t)$ with $x(t)$ is given by $\int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$. Write an expression for $y(t)$.

(i) Evaluate $y(t)$ when $-\infty < t \leq T$.

(ii) Find $y(t)$ for $T < t \leq 2T$

(iii) Sketch $y(t)$.




(b)

Now consider the functions $g(t)$ and $z(t)$ whose Fourier transforms are $h(f)$ and $x(f)$ respectively (change $t$ to $f$ and $T$ to $f_0$ in the functions $h(t)$ and $x(t)$ ).

Two signals having time functions $g(t)$ and $z(t)$ are fed to a multiplier $M$. The output of the multiplier is $p(t)$.

(i) Describe a method to find $p(t)$.
(ii) Sketch $p(t)$.

**3.**

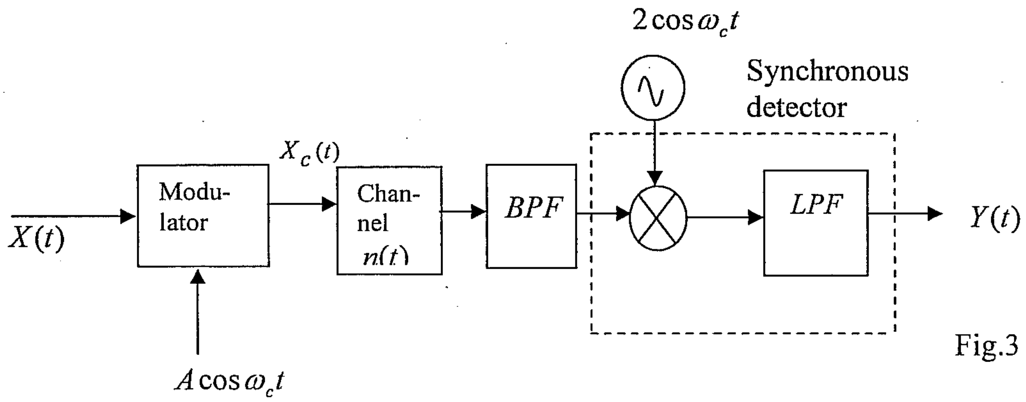


Fig.3

The carrier signal $A\cos\omega_c t$ is modulated using the information signal $X(t)$ whose bandwidth is $B$. The resulting signal is $X_c(t) = A\,X(t)\cos\omega_c t$. During the transmission noise $n(t)$ is added to the signal. The received signal is first sent through a band-pass filter and demodulated using synchronous detection.

(a) (i) What is the purpose of the band-pass filter?
(ii) What is the centre frequency of the band-pass filter?
(iii) How would you select the bandwidth of the band-pass filter?

Noise added to the signal $n(t)$ is transformed into narrowband noise due to band-pass filtering. Thus the noise $n_i(t)$ at the input to the detector can be written as
$n_i(t) = n_c(t)\cos(\omega_c t) - n_s(t)\sin(\omega_c t)$

(b) Decide a value for the lower cutoff frequency of the lowpass filter.
(c) Write an expression for the noise signal at the output of the lowpass filter.
(d) Write an expression for the information signal at the output of the lowpass filter.
(e) Spectral density of the noise signal is $\eta$. Assuming that $E[n_c^2(t)] = E[n_s^2(t)] = E[n_i^2(t)]$ find the signal to noise ratio at the lowpass filter output.

**4.**

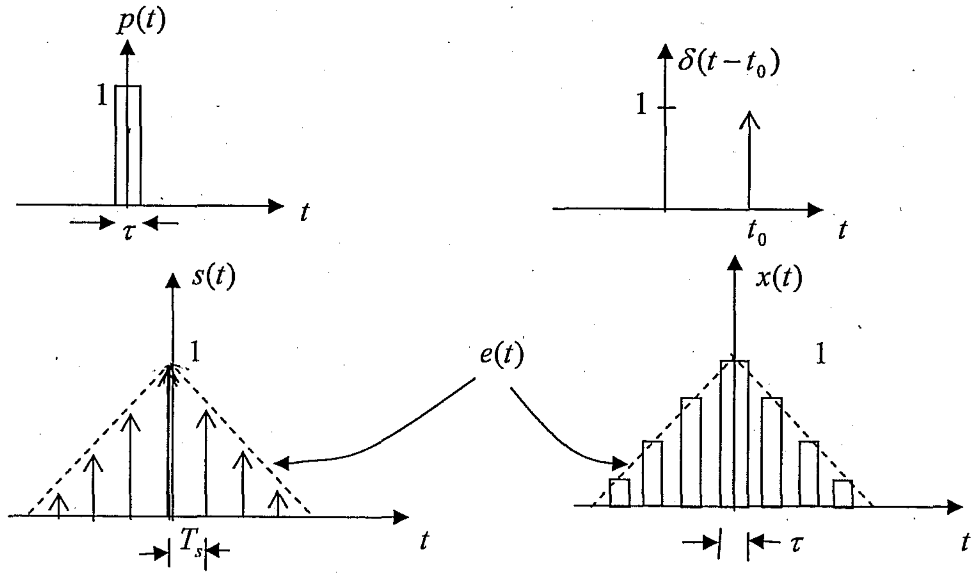



Fig.4

$s(t)$ and $x(t)$ have the same triangular envelop $e(t)$.

(a) What is the value of $p(t) * \delta(t - t_0)$ ?

(b) What is the relationship between $p(t)$, $s(t)$ and $x(t)$ ?

(c) Using (b) find $X(\omega)$ in terms of $P(\omega)$ and $S(\omega)$.

(d) A train of impulses $z(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT)$ becomes equal to $\frac{1}{T} \sum_{k=-\infty}^{\infty} e^{jk\omega_0 t}$ when expressed as a Fourier series. Show that $Z(\omega)$ is also a train of impulses.

(e) What is the relationship between $e(t)$, $s(t)$ and $z(t)$ ?

(f) Using (e) show that $S(\omega) = \sum_{k=-\infty}^{\infty} E(\omega - k\omega_0)$.

(g) $P(\omega)$ and $E(\omega)$ are sinc( ) and sinc$^2$( ) functions respectively. Sketch the approximate shape of $X(\omega)$.

**5.**

(a) Why is
  (i) horizontal blanking
  (ii) vertical blanking

  is necessary in a *TV* receiver?

(b) In a *TV* receiver the length of time between two consecutive horizontal blanking pulses is $t_0$. A *TV* picture consists of 2 frames and each frame consists of $N$ lines.

The aspect ratio of the *TV* screen is $n:1$. Assuming that the horizontal blanking time is negligible compared to the horizontal scanning time derive an expression for the *TV* picture bandwidth.

(c) The field strength of a certain transmitting antenna can be measured at a point $P$ using an antenna $X$ and a field strength meter. Using this setup explain how would you

(i) plot the radiation pattern of $X$.
(ii) find the *beamwidth* of $X$.

**6.**

A data source transmits a discrete signal $x(t)$. During the transmission noise $n(t)$ is added to the signal. At the receiver the received signal $y(t) = x(t) + n(t)$ is sent through a device $D$ which decides the value $x(t)$ of the transmitted signal. The output $z(t)$ of $D$ is a reasonable estimate for $x(t)$.

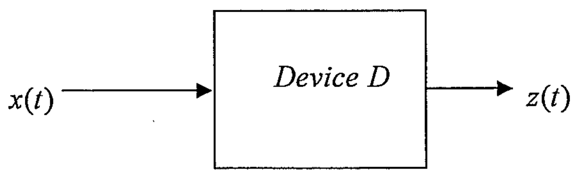



Fig.6

$x(t)$ can have one of the 4 normalized amplitude values -2, -1, +1 and +2 with the following probabilities:

$$P(-2) = P(+2) = \tfrac{1}{8}; \quad P(-1) = P(+1) = \tfrac{3}{8}$$

Noise signal $n(t)$ is statistically independent of the data signal $x(t)$.
The probability density function $p_{n(t)}$ of $n(t)$ is given by

$$p_{n(t)} = \begin{cases} a & -1 < n(t) < +1 \\ 0 & \text{otherwise} \end{cases}$$

(a) Write an expression for the power of the received signal $y(t)$.

(b) Calculate the power of $y(t)$..

Depending on the value of $y(t)$, device $D$ decides a value for $x(t)$ according to the following criterion:

$$x(t)=\begin{cases}+2 & \text{if} \quad y(t)>2\\ +1 & \text{if} \quad 0<y(t)\leq +2\\ -1 & \text{if} \quad -2<y(t)\leq 0\\ -2 & \text{if} \quad y(t)\leq -2\end{cases}$$

(c) What is the probability that $x(t) = +2$ is detected as +1?

(d) What is the probability that $x(t) = +1$ is detected with an error?

7.

(a) For a certain noise signal you are given the probability density function and the auto correlation function.
Describe the practical use of these two functions.

(b) (i) What is a wide sense stationary random process?

A random process is given by $X(t) = A\cos\omega t$. $A$ is a random variable uniformly distributed in the range $-a$ to $+a$ and $\omega$ is a constant.

(ii) Find the autocorrelation function $\Re_{XX}(t_1,t_2)$.

(iii) Find whether the process is wide sense stationary.

**8.**

(a) (i) What is entropy of an information signal?

(ii) Find entropy of $x(t)$ given in question 6.

(b) (i) Write an expression for the transfer function $H(\omega)$ of a quadrature filter.

(ii) Show that the impulse response $h(t)$ of a quadrature filter is given by $\dfrac{1}{\pi t}$.

[Use the fact that the Fourier transform of sgn($t$) is $\dfrac{2}{j\omega}$.]

(iii) Find the Hilbert transform of $\cos\omega_0 t$.

## Appendix

$$\cos x = \frac{1}{2}\left[e^{jx} + e^{-jx}\right] \qquad \cos x \cos y = \frac{1}{2}\left[\cos(x-y) + \cos(x+y)\right]$$

$$\sin x = \frac{1}{2j}\left[e^{jx} - e^{-jx}\right] \qquad \sin x \sin y = \frac{1}{2}\left[\cos(x-y) - \cos(x+y)\right]$$