THE OPEN UNIVERSITY OF SRI LANKA
BACHELOR OF TECHNOLOGY/ BACHELOR OF SOFTWARE ENGINEERING – LEVEL 05
FINAL EXAMINATION – 2011/2012
MPZ5140/ MPZ5160 – DISCRETE MATHEMATICS II
DURATION : THREE (03) HOURS.

Date : 20th February 2012 **Time . 930-1230 hrs.**

Instructions:

- **Answer only six questions.**
- **Please answer a total of six questions choosing at least one from each single section.**
- **State any assumption that you made.**
- **All symbols are in standard notation.**

SECTION – A

01. i. Define a abelian group in usual notation.

(20 marks)

ii. Determine the commutativity and the associativity of each of the following birary operations.

a) * Defined on Q (rational numbers) by a * b = ab +1.

b) * defined on Z (positive integers) by a * b = a – b.

(40 marks)

iii. Let $= \{1, -1, i, -i\}$ and * denotes complex multiplication. Show that (G , *) is a group. (note: $i^2 = -1$)

(40 marks)

02. i. Let G be a group and prove that the set G of all matrices.

$A^{\alpha} = \begin{bmatrix} Cos\alpha & -Sin\alpha \\ Sin\alpha & Cos\alpha \end{bmatrix}, \alpha \in R$ together with matrix multiplication is an abelian group.

(25 marks)

ii. Let $R = \{0,1,2,3,4\}$ be a group under the operation $\oplus_5$. The operation $\oplus_5$ is defined by $a \oplus_5 b = r$, $0 \leq r < 5$, where r is the non-negative remainder when ordinary addition a + b is divided by 5.

a. Determine the identity element of R.

b. Determine the inverse of each element $a \in R$.

(45 marks)

iii. Let $A = \left\{ \begin{pmatrix} a & b \\ c & o \end{pmatrix}, a,b,c \in Z, bc \neq 0 \right\}$. Does A form a group under matrix multiplication? Justify your answer.

(30 marks)

03. i. Define a semi-group in usual notation. Let * be the operation on Q (rational numbers) defined by

a) $a * b = 2(a+b)$ b) $a * b = \frac{ab}{3}$

Is $(Q,*)$ a semi group?

(50 marks)

ii. Define a Homomorphism and Isomorphism for group in usual notation.

If $G = \left\{ \begin{pmatrix} r & o \\ o & r \end{pmatrix} | r \in H \right\}$, $f : G \to H$ defined by $f(r) = \begin{pmatrix} r & o \\ o & r \end{pmatrix}$

for all $\begin{pmatrix} r & o \\ o & r \end{pmatrix} \in G$.

Show that f is an Isomorphism. Where (G, +) and (H, +) are group is usual notation.

(50 marks)

SECTION – B

04. i. Why study graph theory.

(10 marks)

ii. Draw each of the following groups and indicate which are simple or not.

a) Define a graph G such that

$V(G) = \{2,3,4,5,11,12,13,14\}$ and two vertices 'm' and 'n' are adjacent if and only if $gcd\ (m,n) = 1$.

b) $G_1 = \{V_1, E_1\}$, *where* $V_1 = \{1,2,3,4,5,6,7,8\}$ and

$E_1 = \{\{1,2\},\{1,7\},\{2,3\},\{2,4\},\{4,5\},\{3,4\},\{5,6\}\}$

c) $G_2 = \{V_2 E_2\}$ *where* $V_2 = \{1,2,3,4,5,6\}$

$E = \{\{x,y\} \mid 3x + y$ is even and $x \leq y\}$

(50 marks)

iii. Briefly explain connected and disconnected graphs.

(10 marks)

iv. What is the largest possible number of vertices in a graph with 432 edges if all the vertices have degree at least three?

(15 marks)

v. Is there a graph with degree 2, 3, 3, 5, 7 , 8 on six vertices? Justify your answer.

(15 marks)

05. i. Let G be a graph of 8 vertices and 15 edges in which each vertex is of degree 3 or 5. How many vertices of degree 5 does G have? Construct one such graph G. (20 marks)

ii. Construct a multigraph of six vertices and seven edges in which every vertex is odd.

(10 marks)

iii. Four teams of three specialist students each (Electrical, Electronics and Computer) are to be study into engineering degree. However, some of the students cannot work well with some other subjects. The following table shows the students number, their specializations and who they do not like subjects.

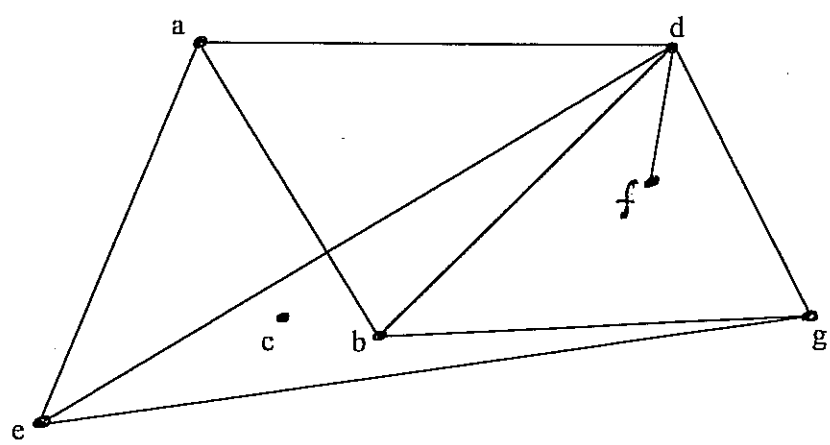
Draw a multigraph to model the situation so that we may see how to form 3 man teams such that each specialization is represented and every member of the team can study with every other. State clearly what the vertices represent and under what condition(s) two vertices are joined an edge and draw the graph.

Student No.	Specialization	Cannot cooperate with
1	Electrical	5, 7, 10
2	Electrical	-----
3	Electrical	5,6,8,9,11
4	Electrical	8, 12
5	Electronics	1,3,9
6	Electronics	3,10,11
7	Electronics	1,9,12
8	Electronics	3,4,9,10
9	Computer	3,5,7,8
10	Computer	1,6,8
11	Computer	3,6
12	Computer	4,7

(50 marks)

iv. Find the adjacency matrix of the following graph G.

(20 marks)



06. i. G is the graph whose adjacency matrix A is given by

$$A=\begin{bmatrix}0 & 1 & 1 & 0\\ 1 & 0 & 0 & 1\\ 1 & 0 & 0 & 1\\ 0 & 1 & 1 & 0\end{bmatrix}$$

a) If $V(G)=\{a_1,a_2,a_3,a_4\}$ find the number of paths of length four (4) joining vertices a_2 *and* a_3. What are these paths, if any?

(45 marks)

b) Without drawing a diagram of G, determine whether G is connected or not.

(20 marks)

ii. Prove that if a tree has at least 2 vertices, then it has at least 2 vertices of degree 1. If a tree has n nodes, what is the maximal number of vertices of degree 1?

(20 marks)

iii. Draw a tree with 20 vertices at least half of which have degree 1.

(15 marks)

SECTION – C

07. i. Iterate the relationship $x_{n+1}=\lambda x_n(1-x_n)$ for $\lambda=2.8$ and indicate initial condition $x_0=0.7$ (at least 6 iteration step are necessary).

(20 marks)

ii. Draw the diagram $x_{x+1}=\lambda x_n(1-x_n)$ for $\lambda=2.8$, when n tends to ∞, x tends to some x^* value, taking $x_0=0.7$. Hence find x^*.

(20 marks)

iii. Draw the Bifurcation diagram and comments on the diagram when λ varies from 0 to 3.5 ($x_0=0.5$).

(60 marks)

08. A three dimensional systems governed by the following three differential equations.

$$\frac{dx}{dt}=5x+2y-2z$$
$$\frac{dy}{dt}=2x+5y-2z$$
$$\frac{dz}{dt}=-2x-2y+5z$$

At $t=0,(x,y,z)=(1,1,0)$ Find the phase space value (x_n,y_n,z_n) for n = 1,2.

(100 marks)

09. i. What is a grammar? (20 marks)

ii. Show that the string $(-n*n)+(n-n)$ is a sentence generated by the grammar G, where,
$G=\{\{E,K\},\{+,*,-,(,),n\},P,E\}$ and P is the set of productions.

E $\longrightarrow$ E K E
E $\longrightarrow$ n
E $\longrightarrow$ (E)
E $\longrightarrow$ E * E
E $\longrightarrow$ - E
K $\longrightarrow$ +
K $\longrightarrow$ –

(20 marks)

iii. Let $L=\{011,\ 010,\ 001,\ 111\}$ then find L^2.

(20 marks)

iv. How to present a DFA (Deterministic finite automation)? Draw the directed graph the describes the DFA with the following state transition table.

States	Input		
	a	b	c
S_0	S_1	S_0	S_0
S_1	S_1	S_2	S_0
S_2	S_1	S_0	S_3
S_3	S_3	S_3	S_3

Initial state S_0 and accepting state S_3.

(40 marks)