THE OPEN UNIVERSITY OF SRI LANKA

Bachelor of Technology (Civil) – Level 5

CEX 5233 - STRUCTURAL ANALYSIS 058

FINAL EXAMINATION - 2005/2006

Time Allowed : 3 hours

Date: 2006-04-05 (Wednesday) Time: 09.30 – 12.30 hrs.

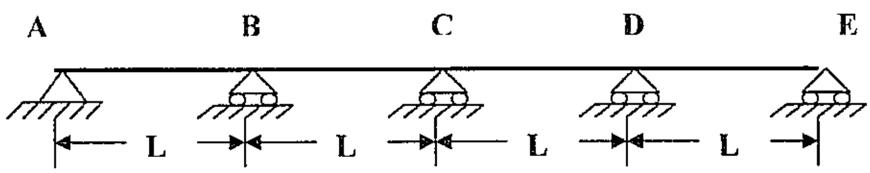
The Paper consists of Eight (8) questions. Answer **Five (5)** questions

1. Analyse the continuous beam shown below, using flexibility method, for

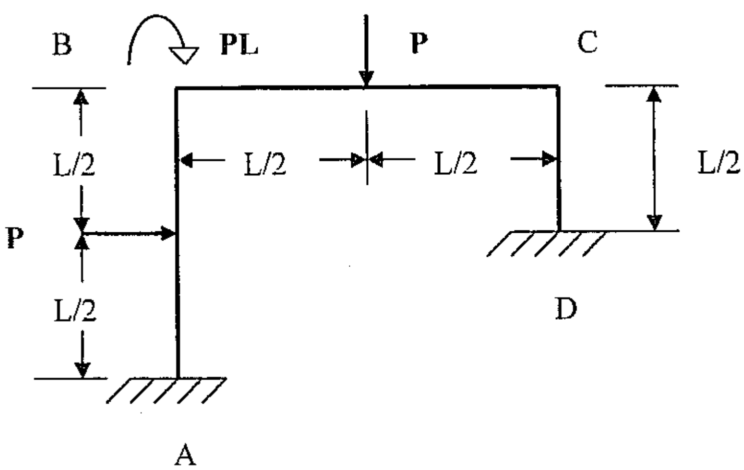
(a) a uniformly distributed load of intensity q on all spans

(b) a unit downward movement of support B

and draw the bending moment diagram.
The beam has a constant flexural rigidity EI.
(You may use Table-1 to determine flexibility coefficients)



2. The plane frame shown in consists of rigidly connected members of constant flexural rigidity EI. Analyse the frame using displacement method.
Neglect axial deformation.
(You may use Table-1 to determine stiffness coefficients)

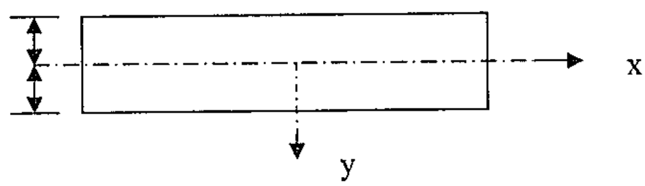


3. The components of stress at a point in a body are $\sigma_x = 0$; $\sigma_y = 300$; $\sigma_z = 100$; $\tau_{xy} = \tau_{xz} = 0$; $\tau_{yz} = 100\sqrt{3}$ N/m^2.

Determine ;
- (a) the stress invariants
- (b) the principal stresses
- (c) the direction cosines of the principal planes

4. Show that $\phi = \frac{q}{8c^3}\left[x^2\left(y^3 - 3c^2y + 2c^3\right) - \frac{1}{5}y^3\left(y^2 - 2c^2\right)\right]$ is a stress function.

Determine the stress components σ_x, σ_y and σ_{xy}.
Find what problem it solves when applied to the region included in $y = \pm c$, $x = 0$, on the side x positive.



5. A conical tank with semi-cone angle **α** and height **h** is filled with a liquid of specific weight γ. Show that the stress resultants are given by

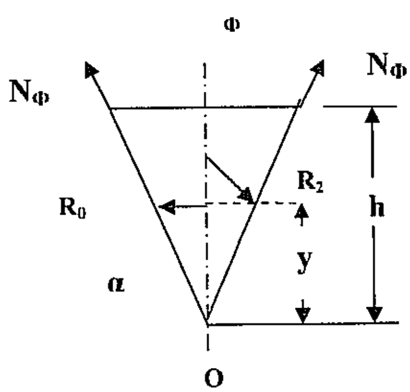
$$N_\theta = \frac{\gamma(h-y)yTan\alpha}{Cos\alpha}$$

$$N_\phi = \frac{\gamma y(h-\frac{2}{3}y)yTan\alpha}{2Cos\alpha}$$

Also show that the maximum N_θ is when **y = d/2** and maximum N_Φ is at **y = 3d/4**. Determine N_θ^{max} and N_Φ^{max}.

Membrane solution for shells of revolution is given by;

$$\frac{N_\theta}{R_2} + \frac{N_\varphi}{R_1} = -p_r$$

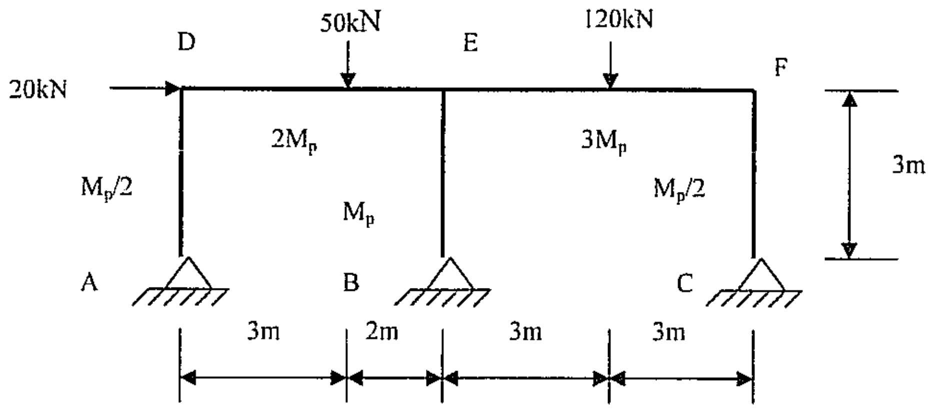


6. A circular plate of radius **a** is carrying a concentrated load **P** at the centre. Assuming that the edge of the plate is built-in, show that the deflection of the plate is given by;

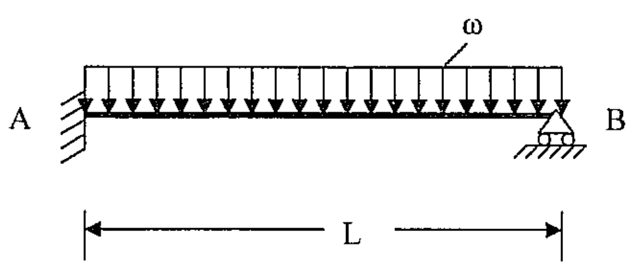
$$\omega = \frac{Pr^2}{8\pi D} Log \frac{r}{a} + \frac{P}{16\pi D}\left(a^2 - r^2\right)$$

Hence determine the maximum deflection of the plate.
Also find the moment M_r at built-in edge.

7. Analyse the rigid frame shown below using plastic method of analysis and determine the Plastic Moment M_p, for a load factor of 1.8.



8. (a) Determine the collapse load for the fixed beam shown in the figure below, if the Plastic Moment Capacity of the beam is M_P.



(b) Determine the ultimate load for the propped cantilever shown in the figure below, if the Plastic Moment Capacity of the beam is M_P.

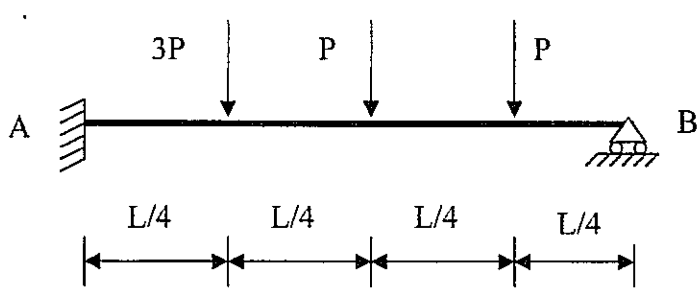
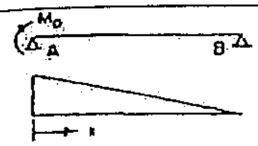
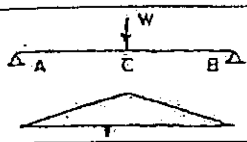
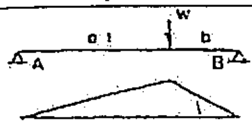
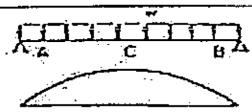
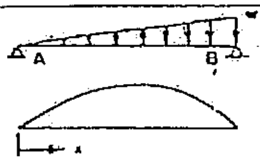
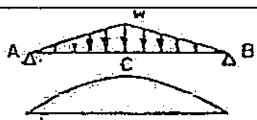
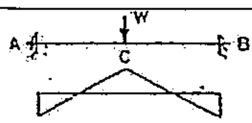
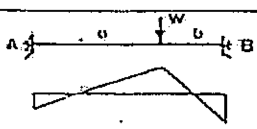
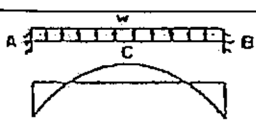
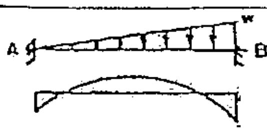
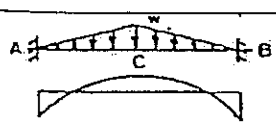


Table 1 Formulas for Beams

Structure	Shear	Moment	Slope	Deflection
Simply supported Beam				
	$S_A = -\frac{M_o}{L}$	M_o	$\theta_A = \frac{M_o L}{3EI}$ $\theta_B = -\frac{M_o L}{6EI}$	$Y_{max} = 0.062\frac{M_o L^2}{EI}$ $at\ x = 0.422L$
	$S_A = \frac{W}{2}$	$M_o = \frac{WL}{4}$	$\theta_A = -\theta_B = \frac{WL^2}{16EI}$	$Y_c = \frac{WL^3}{48EI}$
	$S_A = \frac{Wb}{L}$ $S_B = \frac{Wa}{L}$	$M_o = \frac{Wab}{L}$	$\theta_A = \frac{Wab}{6EIL}(L+b)$ $\theta_B = -\frac{Wab}{6EIL}(L+a)$	$Y_o = \frac{Wa^2b^2}{3EIL}$
	$S_A = \frac{WL}{2}$	$M_c = \frac{WL^2}{8}$	$\theta_A = -\theta_B = \frac{W\ L^3}{24EI}$	$Y_c = \frac{5WL^4}{384EI}$
	$S_A = \frac{WL}{6}$ $S_B = \frac{WL}{3}$	$M_{max} = 0.064Wl^2$ $at\ x = 0.577L$	$\theta_A = \frac{7WL^3}{360EI}$ $\theta_B = -\frac{8WL^3}{360EI}$	$Y_{max} = 0.00652\frac{WL^4}{EI}$ $at\ x = 0.519L$
	$S_A = \frac{WL}{4}$	$M_c = \frac{WL^2}{12}$	$\theta_A = -\theta_B = \frac{5W\ L^3}{192EI}$	$Y_c = \frac{WL^4}{120EI}$
Fixed Beams				
	$S_A = \frac{W}{2}$	$M_c = \frac{WL}{8}$	$\theta_A = \theta_B = 0$	$Y_c = \frac{WL^3}{192EI}$
	$S_A = \frac{Wb^2}{L^3}(3a+b)$ $S_B = \frac{Wa^2}{L^3}(3b+a)$	$M_A = -\frac{Wab^2}{L^2}$ $M_B = -\frac{Wba^2}{L^2}$	$\theta_A = \theta_B = 0$	$Y_o = \frac{Wa^3b^3}{3EIL^3}$
	$S_A = \frac{WL}{2}$	$M_A = M_B = -\frac{WL^2}{12}$	$\theta_A = \theta_B = 0$	$Y_c = \frac{WL^4}{384EI}$
	$S_A = \frac{3WL}{20}$ $S_B = -\frac{7WL}{20}$	$M_A = -\frac{WL^2}{30}$ $M_B = -\frac{WL^2}{20}$	$\theta_A = \theta_B = 0$	$Y_{max} = 0.00131\frac{WL^4}{EI}$ $at\ x = 0.525L$
	$S_A = \frac{WL}{4}$	$M_A = M_B = -\frac{5WL^2}{96}$	$\theta_A = \theta_B = 0$	$Y_c = \frac{0.7WL^4}{384EI}$

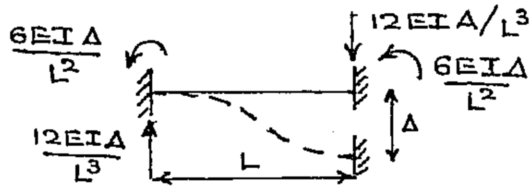
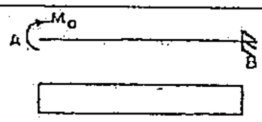
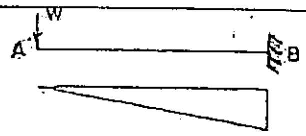
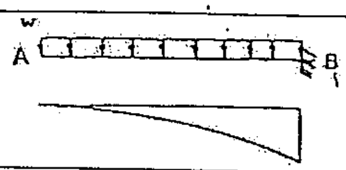
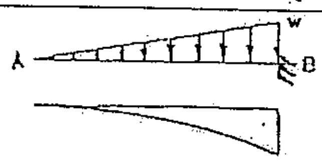
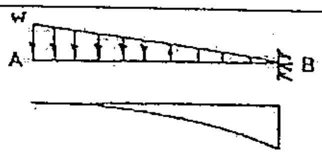
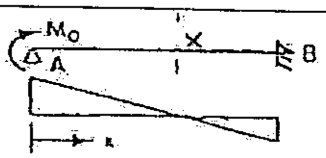
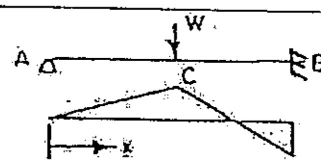
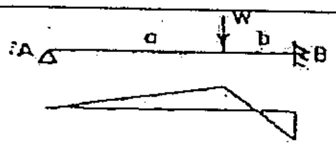
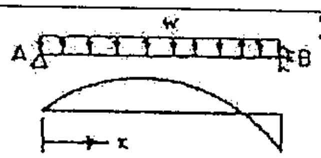
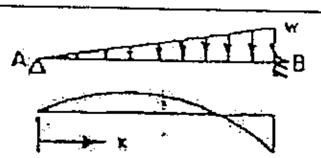
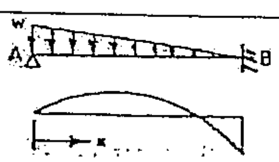


Table 1 Formulas for Beams

Structure	Shear	Moment	Slope	Deflection
Cantilever Beam				
	0	M_o	$\theta_A = \frac{M_o L}{EI}$	$Y_A = \frac{M_o L^2}{2EI}$
	W	$M_B = -WL$	$\theta_A = -\frac{WL^2}{2EI}$	$Y_A = \frac{WL^3}{3EI}$
	$S_B = -WL$	$M_B = -\frac{WL^2}{2}$	$\theta_A = -\frac{WL^3}{6EI}$	$Y_A = \frac{WL^4}{8EI}$
	$S_B = -\frac{WL}{2}$	$M_B = -\frac{WL^2}{6}$	$\theta_A = -\frac{WL^3}{24EI}$	$Y_A = \frac{WL^4}{8EI}$
	$S_B = -\frac{WL}{2}$	$M_B = -\frac{WL^2}{2}$	$\theta_A = -\frac{WL^3}{8EI}$	$Y_A = \frac{11WL^4}{120EI}$
Propped Cantilever				
	$S_A = -\frac{3M_o}{2L}$	$M_B = -\frac{M_o}{2}$	$\theta_A = -\frac{M_o L}{4EI}$	$Y_{max} = \frac{M_o L^2}{27EI}$ at $x = \frac{L}{3}$
	$S_A = -\frac{3M_o}{2L}$	$M_B = -\frac{3WL}{16}$ $M_c = \frac{5WL}{32}$	$\theta_A = \frac{WL^2}{32EI}$	$Y_{max} = 0.00962\frac{WL^3}{ELI}$ at $x = 0.4471$
	$S_A = \frac{Wb^2}{2L^3}(a+2L)$ $S_B = -\frac{Wa}{2L^3}(3L^2 - a^2)$	$M_B = -\frac{Wab}{L^2}(a+\frac{b}{2})$	$\theta_A = \frac{Wab^2}{4EIL}$	$Yo = \frac{Wa^2b^2}{12EIL^3}(3L+$
	$S_A = +\frac{3WL}{8}$	$M_B = -\frac{WL^2}{8}$	$\theta_A = \frac{WL^3}{48EI}$	$Y_{max} = 0.0054\frac{WL^4}{ELI}$ at $x = 0.422$
	$S_A = +\frac{WL}{10}$	$M_{max} = 0.03WL^2$ at $x = 0.447L$ $M_B = -\frac{WL^2}{15}$	$\theta_A = \frac{WL^3}{120EI}$	$Y_{max} = 0.00239\frac{WL^4}{EI}$ at $x = 0.447$
	$S_A = \frac{11WL}{40}$	$M_{max} = 0.0423WL^2$ at $x = 0.329L$ $M_B = -\frac{7WL^2}{120}$	$\theta_A = \frac{WL^3}{80EI}$	$Y_{max} = 0.00305\frac{WL^4}{EI}$ at $x = 0.402$