THE OPEN UNIVERSITY OF SRI LANKA

Bachelor of Technology (Civil) – Level 5

073

CEX 5233 - STRUCTURAL ANALYSIS

FINAL EXAMINATION - 2006/2007

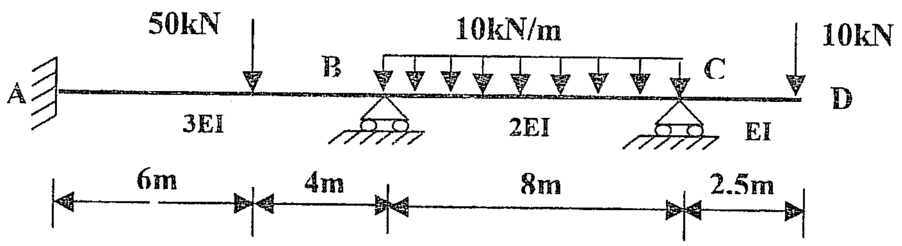
Time Allowed : 3 hours

Date: 2007-04-03 (Tuesday) Time: 09.30 – 12.30 hrs.

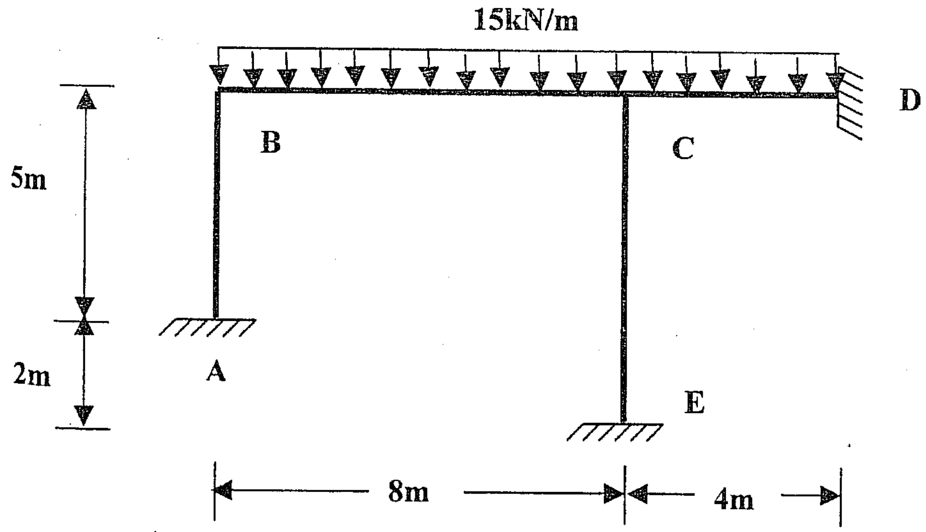
The Paper consists of Eight (8) questions. Answer **Five (5)** questions

1. Analyse the continuous beam with an overhang shown in the figure below, using Flexibility method and draw the bending moment diagram.

(You may use Table-1 to determine flexibility coefficients)



2. Analyse the frame shown in the figure below using Stiffness method. Neglect axial deformation. (EI = Constant).
(You may use Table-1 to determine stiffness coefficients)



3. The stress tensor at a point with reference to axes **x,y,z** is given by

$$\sigma_{ij} = \begin{bmatrix} 4 & 1 & 2 \\ 1 & 6 & 0 \\ 2 & 0 & 8 \end{bmatrix} \text{ MPa.}$$

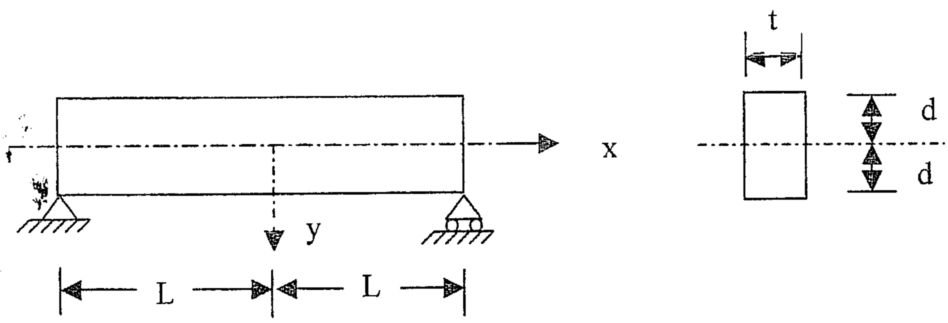
Show that by transformation of the axes by 45° about the **z** axis, the new stress components are given by $\sigma'_{ij} = \begin{bmatrix} 6 & 1 & \sqrt{2} \\ 1 & 4 & -\sqrt{2} \\ \sqrt{2} & -\sqrt{2} & 8 \end{bmatrix}$ MPa

Show that after the above transformation of axes, the stress invariants remain unchanged.

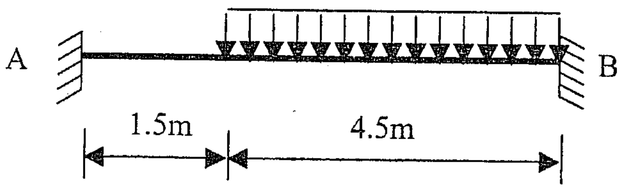
4. Show that $\phi = \frac{A}{6}\left[L^2y^3 - x^2y^3 + \frac{y^5}{5}\right] + Bx^2 + Cx^2y + Dy^3$ is a possible stress function.

Find expressions for σ_x, σ_y and σ_{xy}.

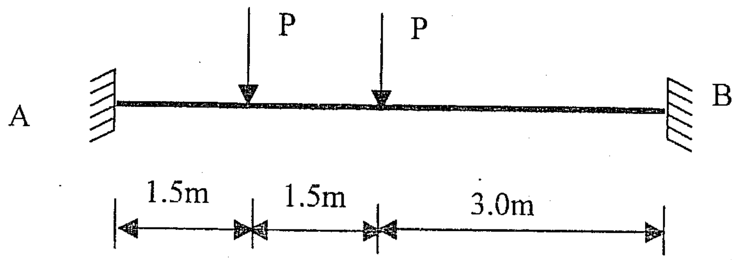
If this stress function gives solution to the problem of a beam, simply supported at the ends and loaded with a udl of intensity **q** per unit area, determine the constants **A, B, C** and **D.**



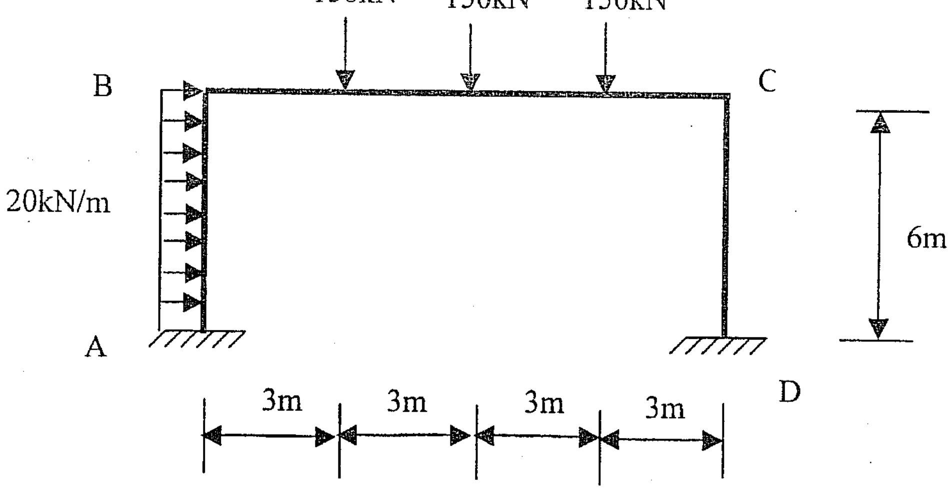
5. **(a)** A fixed beam of span 6.0m carries a udl of 60 kN/m on the right hand 4.5m as shown in the figure below. If the load factor is 2.0, locate the positions of the plastic hinges and determine the plastic moment M_p.



(b) A fixed beam of constant section carries two point loads as shown in the figure below. Calculate the collapse load for the beam, if the plastic moment is M_p.



6. The portal frame shown in the figure below is of uniform section throughout. The loads shown are ultimate loads. Using Plastic Analysis, determine the Plastic Moment Capacity of the frame M_P.



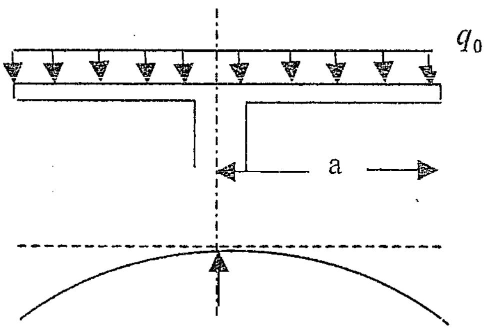
7. A vertical cylindrical tank of height **h**, radius **r** and thickness **t** is fixed at the bottom. If the tank is filled to the top with a liquid of unit weight **ρ**, using Membrane Theory show that the stress resultants of the cylindrical shell are given by;

$$N_\phi = r\rho g(h - x)$$
$$N_x = 0$$
$$N_{\phi x} = 0$$

Governing equations for an axisymmetrical cylindrical shell are given by;

$$\frac{\partial N_x}{\partial x} + \frac{1}{r}\frac{\partial N_{\phi x}}{\partial \phi} + X = 0$$
$$\frac{1}{r}\frac{\partial N_\phi}{\partial \phi} + \frac{\partial N_{x\phi}}{\partial x} + Y = 0$$
$$\frac{1}{r}N_\phi + Z = 0$$
$$N_{\phi x} = N_{x\phi}$$

8. An isotropic circular plate with radius **a** is carrying a udl q_0 and is supported on a column at its center as shown in the figure below.



Deflected configuration

Show that the deflection of the plate is given by;

$$\omega = \frac{q_0 a^4}{64D}\left[\frac{7+3\nu}{1+\nu}\left(1-\rho^2\right)+\rho^2\left(1-\rho^2+8\log\rho\right)\right]$$

where $\rho = \dfrac{r}{a}$

Hint: Consider the given problem as a superposition of two separate problems as shown in the figures below.

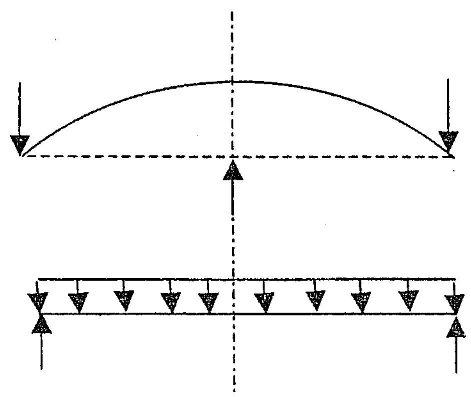
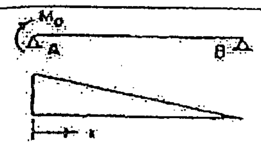
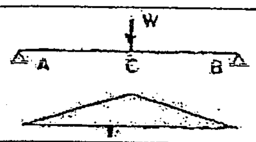
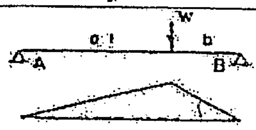
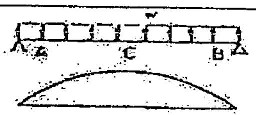
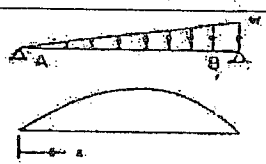
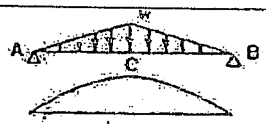
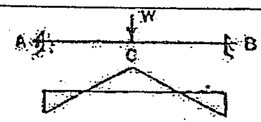
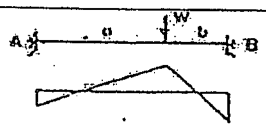
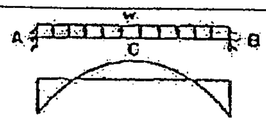
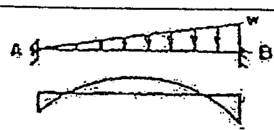
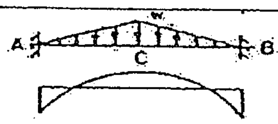


Table 1 Formulas for Beams

Structure	Shear	Moment	Slope	Deflection ↓
Simply supported Beam				
	$S_A = -\frac{M_o}{L}$	M_o	$\theta_A = \frac{M_o L}{3EI}$ $\theta_B = -\frac{M_o L}{6EI}$	$Y_{max} = 0.062\frac{M_o L^2}{EI}$ $at\ x = 0.422L$
	$S_A = \frac{W}{2}$	$M_o = \frac{WL}{4}$	$\theta_A = -\theta_B = \frac{WL^2}{16EI}$	$Y_c = \frac{WL^3}{48EI}$
	$S_A = \frac{Wb}{L}$ $S_B = \frac{Wa}{L}$	$M_o = \frac{Wab}{L}$	$\theta_A = \frac{Wab}{6EIL}(L+b)$ $\theta_B = -\frac{Wab}{6EIL}(L+a)$	$Y_o = \frac{Wa^2b^2}{3EIL}$
	$S_A = \frac{WL}{2}$	$M_c = \frac{WL^2}{8}$	$\theta_A = -\theta_B = \frac{W\ L^3}{24EI}$	$Y_c = \frac{5WL^4}{384EI}$
	$S_A = \frac{WL}{6}$ $S_B = \frac{WL}{3}$	$M_{max} = 0.064Wl^2$ $at\ x = 0.577L$	$\theta_A = \frac{7WL^3}{360EI}$ $\theta_B = -\frac{8WL^3}{360EI}$	$Y_{max} = 0.00652\frac{WL^4}{EI}$ $at\ x = 0.519L$
	$S_A = \frac{WL}{4}$	$M_c = \frac{WL^2}{12}$	$\theta_A = -\theta_B = \frac{5W\ L^3}{192EI}$	$Y_c = \frac{WL^4}{120EI}$
Fixed Beams				
	$S_A = \frac{W}{2}$	$M_c = \frac{WL}{8}$	$\theta_A = \theta_B = 0$	$Y_c = \frac{WL^3}{192EI}$
	$S_A = \frac{Wb^2}{L^3}(3a+b)$ $S_B = \frac{Wa^2}{L^3}(3b+a)$	$M_A = -\frac{Wab^2}{L^2}$ $M_B = -\frac{Wba^2}{L^2}$	$\theta_A = \theta_B = 0$	$Y_o = \frac{Wa^3b^3}{3EIL^3}$
	$S_A = \frac{WL}{2}$	$M_A = M_B = -\frac{WL^2}{12}$	$\theta_A = \theta_B = 0$	$Y_c = \frac{WL^4}{384EI}$
	$S_A = \frac{3WL}{20}$ $S_B = -\frac{7WL}{20}$	$M_A = -\frac{WL^2}{30}$ $M_B = -\frac{WL^2}{20}$	$\theta_A = \theta_B = 0$	$Y_{max} = 0.00131\frac{WL^4}{EI}$ $at\ x = 0.525L$
	$S_A = \frac{WL}{4}$	$M_A = M_B = -\frac{5WL^2}{96}$	$\theta_A = \theta_B = 0$	$Y_c = \frac{0.7WL^4}{384EI}$

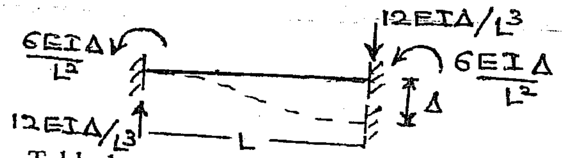
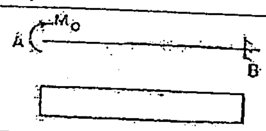
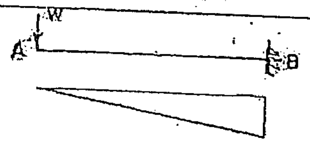
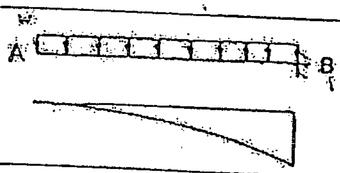
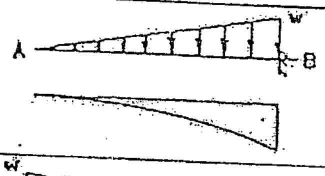
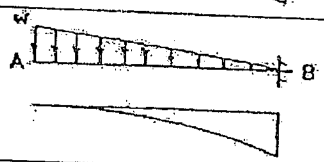
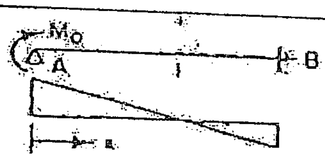
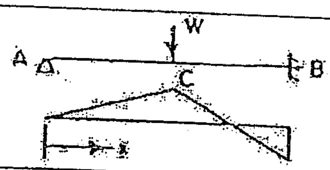
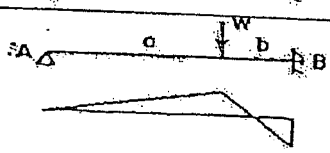
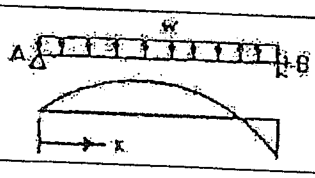
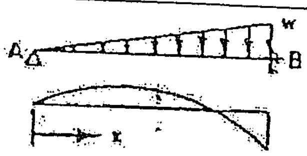
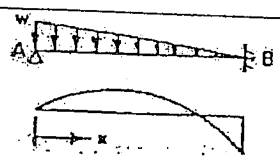


Table 1

Formulas for Beams

Structure	Shear	Moment	Slope	Deflection
Cantilever Beam				
	0	M_o	$\theta_A = \frac{M_o L}{EI}$	$Y_A = \frac{M_o L}{2EI}$
	W	$M_B = -WL$	$\theta_A = -\frac{WL^2}{2EI}$	$Y_A = \frac{WL^3}{3EI}$
	$S_B = -WL$	$M_B = -\frac{WL^2}{2}$	$\theta_A = -\frac{WL^3}{6EI}$	$Y_A = \frac{WL^4}{8EI}$
	$S_B = -\frac{WL}{2}$	$M_B = -\frac{WL^2}{6}$	$\theta_A = -\frac{WL^3}{24EI}$	$Y_A = \frac{WL^4}{8EI}$
	$S_B = -\frac{WL}{2}$	$M_B = -\frac{WL^2}{2}$	$\theta_A = -\frac{WL^3}{8EI}$	$Y_A = \frac{11WL^4}{120EI}$
Propped Cantilever				
	$S_A = -\frac{3M_o}{2L}$	$M_B = -\frac{M_o}{2}$	$\theta_A = -\frac{M_o L}{4EI}$	$Y_{max} = \frac{M_o L^2}{27EI}$ $at\ x = \frac{L}{3}$
	$S_A = -\frac{3M_o}{2L}$	$M_B = -\frac{3WL}{16}$ $M_c = \frac{5WL}{32}$	$\theta_A = \frac{WL^2}{32EI}$	$Y_{max} = 0.00962\frac{WL^3}{ELI}$ $at\ x = 0.4471$
	$S_A = \frac{Wb^2}{2L^3}(a+2L)$ $S_B = -\frac{Wa}{2L^3}(3L^2 - a^2)$	$M_B = -\frac{Wab}{L^2}(a+\frac{b}{2})$	$\theta_A = \frac{Wab^2}{4EIL}$	$Yo = \frac{Wa^2b^2}{12EIL^3}(3L+$
	$S_A = +\frac{3WL}{8}$	$M_B = -\frac{WL^2}{8}$	$\theta_A = \frac{WL^3}{48EI}$	$Y_{max} = 0.0054\frac{WL^4}{ELI}$ $at\ x = 0.422l$
	$S_A = +\frac{WL}{10}$	$M_{max} = 0.03WL^2$ $at\ x = 0.447L$ $M_B = -\frac{WL^2}{15}$	$\theta_A = \frac{WL^3}{120EI}$	$Y_{max} = 0.00239\frac{WL^4}{EI}$ $at\ x = 0.447L$
	$S_A = +\frac{11WL}{40}$	$M_{max} = 0.0423WL^2$ $at\ x = 0.329L$ $M_B = -\frac{7WL^2}{120}$	$\theta_A = \frac{WL^3}{80EI}$	$Y_{max} = 0.00305\frac{WL^4}{EI}$ $at\ x = 0.402L$