THE OPEN UNIVERSITY OF SRI LANKA

Bachelor of Technology (Civil) – Level 5

CEX 5233 - STRUCTURAL ANALYSIS

FINAL EXAMINATION - 2008/2009

Time Allowed : 3 hours

092

Date: 2009-03-19 (Thursday) Time: 09.30 – 12.30 hrs.

The Paper consists of Eight (8) questions. Answer **Five (5)** questions

1(a) Set up the Stiffness matrix for the frame shown in Fig.Q-1(a). (Neglect axial deformation). (6 marks)

(b) Analyse the structure shown in Fig.Q-1(b) using Flexibility method of structural analysis and draw the bending moment diagram. (14 marks)

(You may use Table-1 to determine stiffness and flexibility coefficients)

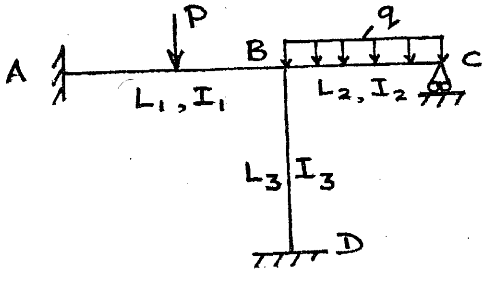


Fig.Q-1(a)

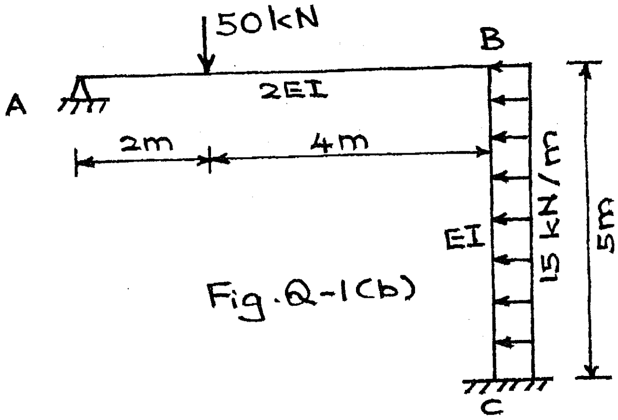


Fig.Q-1(b)

2. Analyse the continuous beam shown in figure Q-2 using Stiffness method of structural analysis and draw the bending moment diagram.
(You may use Table-1 to determine stiffness coefficients) (20 marks)

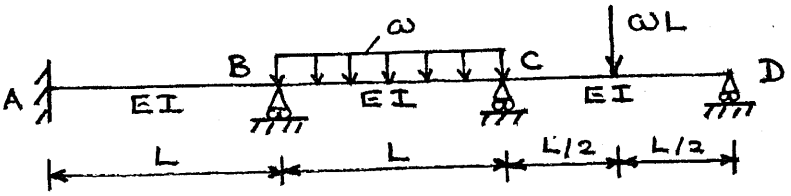


Fig.Q-2

3.(a) What are the three main properties of stress tensor? (3 marks)

(b) Write down the transformation law for the stress at a point. (3 marks)

(c) The stress tensor at a point with reference to axes x,y,z is given by

$$\sigma_{ij} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

Determine:

(a) Stress invariants
(b) Principal stresses
(c) Principal axes
(d) Directions of principal axes (14 marks)

4.(a) What is understood by a plane stress problem? (2 marks)

(b) Show that $\Phi = Ay^3 + Bxy + Cxy^3$ is a valid stress function. (2 marks)

(b) The above stress function is proposed to give the solution for a cantilever ($y = \pm\frac{d}{2}$; $0 \le x \le L$) ,carrying a concentrated end load of P.
Obtain values for constants A,B and C. (12 marks)

(d) Write expressions for stress field $\sigma_x, \sigma_y and \sigma_{xy}$. (4 marks)

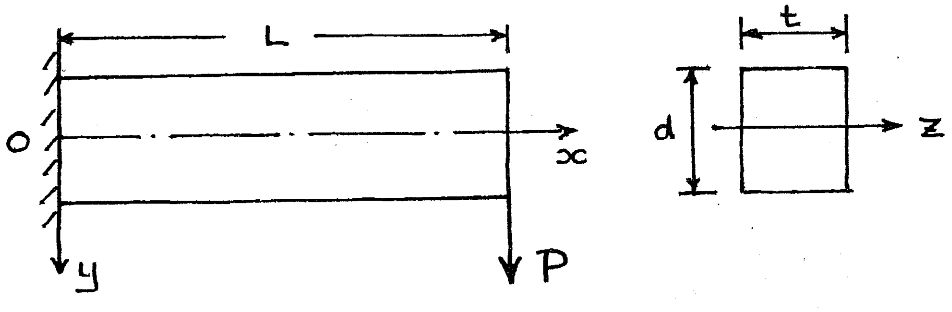


Fig. Q-4

5.(a) In Plastic Theory, **three conditions** apply when a structure is on the point of collapse. What are they? (3 marks)

(b) Compare the **safety factor** in elastic design with the **load factor** in plastic design. (3 marks)

(c) Determine the value of collapse load $\mathbf{q_c}$ for the continuous beam shown in Fig.Q-5, where the fully plastic moment is $\mathbf{M_p}$. (14 marks)

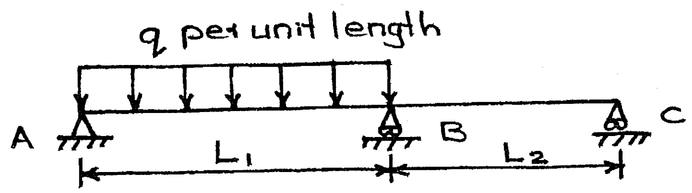


Fig. Q-5

6. Analyse the two-storey portal-type structure shown in Fig.Q-6 using Plastic analysis and determine the plastic moment $\mathbf{M_p}$. Use a load factor of **2**. (20 marks)

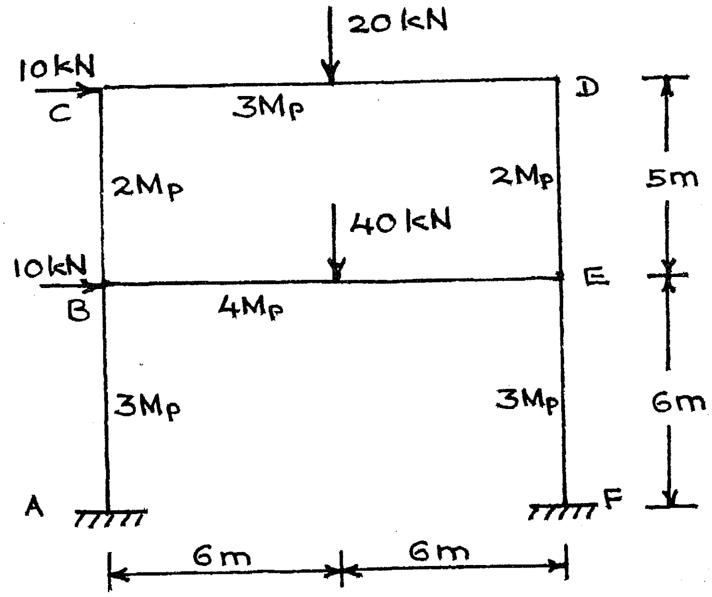


Fig. Q-6

7.(a) What are the general assumptions made in the membrane theory of thin shells? (3marks)

(b) Use the membrane theory of thin shells to find expressions for meridianal stress (σ_θ) and hoop stress (σ_Φ) for a spherical shell of constant thickness **h**, under its own weight **q** per unit area.
You may use the following governing equation for an axisymmetric shells under axisymmetric loading; (14 marks)

$$\frac{N_\theta}{r_2}+\frac{N_\Phi}{r_1}=-p_z$$

(c) Show that we can keep on reducing the thickness of the shell for material economy, as long as the serviceability requirements are met. (3 marks)

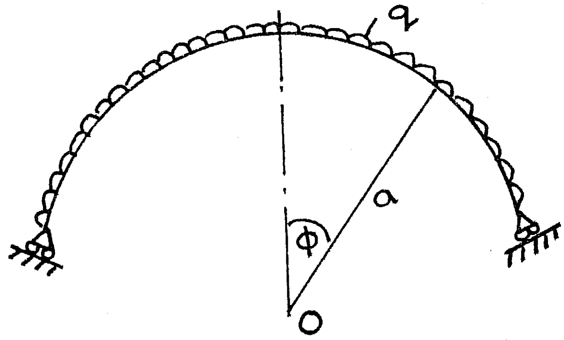


Fig. Q-7

8.(a) An isotropic annular plate of uniform thickness is simply supported at **r=a** and free at **r=b** as shown in the Fig.Q-8. It is subjected to a lateral concentric line total load of **P** along the inner periphery at radius **r=b**.
Derive an expression for the deflection of the plate, if the radial shear per unit length of the periphery at any distance **r** is given by

$$Q_r = D\frac{d}{dr}\left(\nabla^2\omega\right)= D\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{d\omega}{dr}\right)\right]$$ (15 marks)

(b) Hence find an expression for deflection of a circular plate simply supported at the edge and subjected to a concentrated load **P** at the centre. (5 marks)

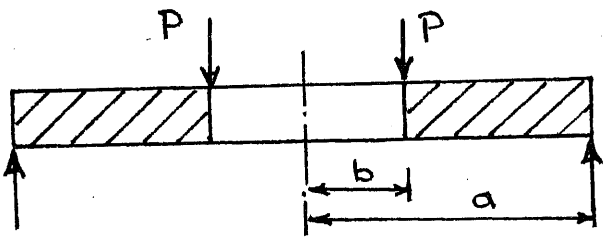
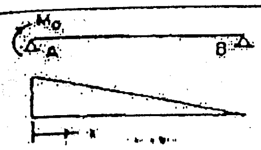
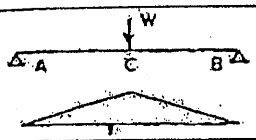
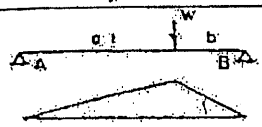
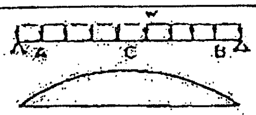
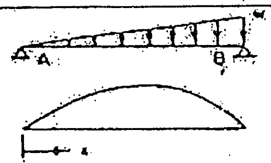
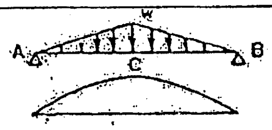
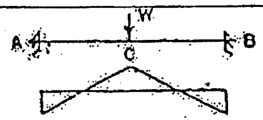
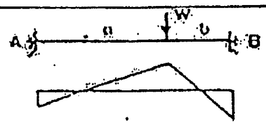
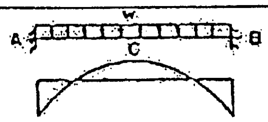
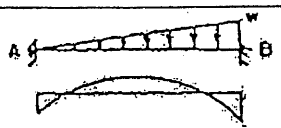
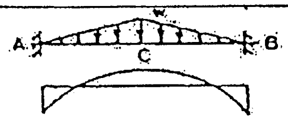


Table 1 Formulas for Beams

Structure	Shear	Moment	Slope	Deflection ↓
Simply supported Beam				
	$S_A = -\frac{M_o}{L}$	M_o	$\theta_A = \frac{M_o L}{3EI}$, $\theta_B = -\frac{M_o L}{6EI}$	$Y_{max} = 0.062\frac{M_o L^2}{EI}$ at $x = 0.422L$
	$S_A = \frac{W}{2}$	$M_o = \frac{WL}{4}$	$\theta_A = -\theta_B = \frac{WL^2}{16EI}$	$Y_c = \frac{WL^3}{48EI}$
	$S_A = \frac{Wb}{L}$, $S_B = \frac{Wa}{L}$	$M_o = \frac{Wab}{L}$	$\theta_A = \frac{Wab}{6EIL}(L+b)$, $\theta_B = -\frac{Wab}{6EIL}(L+a)$	$Y_o = \frac{Wa^2b^2}{3EIL}$
	$S_A = \frac{WL}{2}$	$M_c = \frac{WL^2}{8}$	$\theta_A = -\theta_B = \frac{W L^3}{24EI}$	$Y_c = \frac{5WL^4}{384EI}$
	$S_A = \frac{WL}{6}$, $S_B = \frac{WL}{3}$	$M_{max} = 0.064Wl^2$ at $x = 0.577L$	$\theta_A = \frac{7WL^3}{360EI}$, $\theta_B = -\frac{8WL^3}{360EI}$	$Y_{max} = 0.00652\frac{WL^4}{EI}$ at $x = 0.519L$
	$S_A = \frac{WL}{4}$	$M_c = \frac{WL^2}{12}$	$\theta_A = -\theta_B = \frac{5W L^3}{192EI}$	$Y_c = \frac{WL^4}{120EI}$
Fixed Beams				
	$S_A = \frac{W}{2}$	$M_c = \frac{WL}{8}$	$\theta_A = \theta_B = 0$	$Y_c = \frac{WL^3}{192EI}$
	$S_A = \frac{Wb^2}{L^3}(3a+b)$, $S_B = \frac{Wa^2}{L^3}(3b+a)$	$M_A = -\frac{Wab^2}{L^2}$, $M_B = -\frac{Wba^2}{L^2}$	$\theta_A = \theta_B = 0$	$Y_o = \frac{Wa^3b^3}{3EIL^3}$
	$S_A = \frac{WL}{2}$	$M_A = M_B = -\frac{WL^2}{12}$	$\theta_A = \theta_B = 0$	$Y_c = \frac{WL^4}{384EI}$
	$S_A = \frac{3WL}{20}$, $S_B = -\frac{7WL}{20}$	$M_A = -\frac{WL^2}{30}$, $M_B = -\frac{WL^2}{20}$	$\theta_A = \theta_B = 0$	$Y_{max} = 0.00131\frac{WL^4}{EI}$ at $x = 0.525L$
	$S_A = \frac{WL}{4}$	$M_A = M_B = -\frac{5WL^2}{96}$	$\theta_A = \theta_B = 0$	$Y_c = \frac{0.7WL^4}{384EI}$

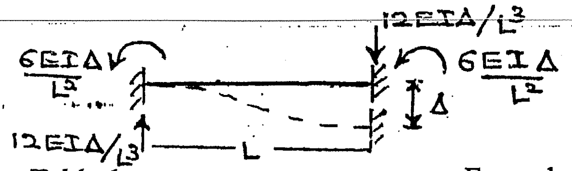
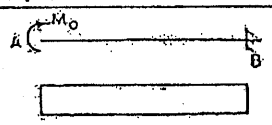
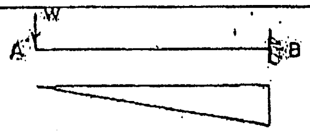
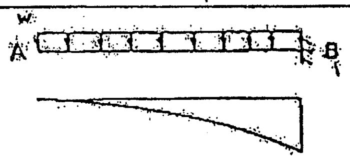
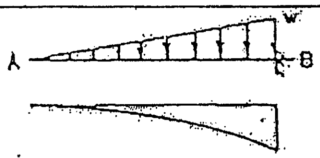
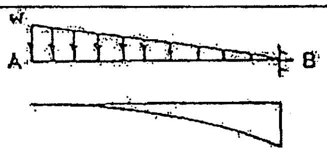
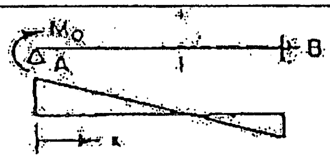
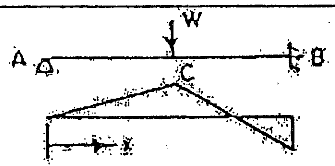
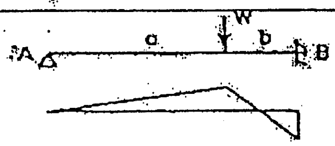
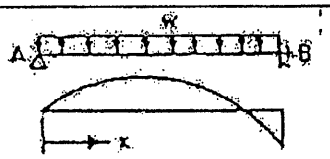
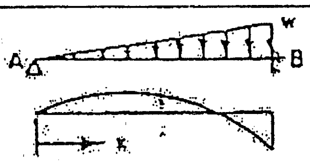
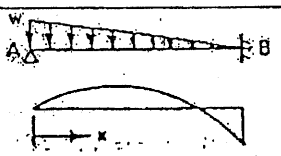


Table 1 Formulas for Beams

Structure	Shear	Moment	Slope	Deflection ↓
Cantilever Beam				
	0	M_o	$\theta_A = \frac{M_o L}{EI}$	$Y_A = \frac{M_o L^2}{2EI}$
	W	$M_B = -WL$	$\theta_A = -\frac{WL^2}{2EI}$	$Y_A = \frac{WL^3}{3EI}$
	$S_B = -WL$	$M_B = -\frac{WL^2}{2}$	$\theta_A = -\frac{WL^3}{6EI}$	$Y_A = \frac{WL^4}{8EI}$
	$S_B = -\frac{WL}{2}$	$M_B = -\frac{WL^2}{6}$	$\theta_A = -\frac{WL^3}{24EI}$	$Y_A = \frac{WL^4}{8EI}$
	$S_B = -\frac{WL}{2}$	$M_B = -\frac{WL^2}{2}$	$\theta_A = -\frac{WL^3}{8EI}$	$Y_A = \frac{11WL^4}{120EI}$
Propped Cantilever				
	$S_A = -\frac{3M_o}{2L}$	$M_B = -\frac{M_o}{2}$	$\theta_A = -\frac{M_o L}{4EI}$	$Y_{max} = \frac{M_o L^2}{27EI}$ at $x = \frac{L}{3}$
	$S_A = -\frac{3M_o}{2L}$	$M_B = -\frac{3WL}{16}$ $M_c = \frac{5WL}{32}$	$\theta_A = \frac{WL^2}{32EI}$	$Y_{max} = 0.00962\frac{WL^3}{ELI}$ at $x = 0.4471$
	$S_A = \frac{Wb^2}{2L^3}(a+2L)$ $S_B = -\frac{Wa}{2L^3}(3L^2 - a^2)$	$M_B = -\frac{Wab}{L^2}(a+\frac{b}{2})$	$\theta_A = \frac{Wab^2}{4EIL}$	$Yo = \frac{Wa^2b^2}{12EIL^3}(3L+$
	$S_A = +\frac{3WL}{8}$	$M_B = -\frac{WL^2}{8}$	$\theta_A = \frac{WL^3}{48EI}$	$Y_{max} = 0.0054\frac{WL^4}{ELI}$ at $x = 0.42$
	$S_A = +\frac{WL}{10}$	$M_{max} = 0.03WL^2$ at $x = 0.447L$ $M_B = -\frac{WL^2}{15}$	$\theta_A = \frac{WL^3}{120EI}$	$Y_{max} = 0.00239\frac{WL}{EI}$ at $x = 0.447$
	$S_A = +\frac{11WL}{40}$	$M_{max} = 0.0423WL^2$ at $x = 0.329L$ $M_B = -\frac{7WL^2}{120}$	$\theta_A = \frac{WL^3}{80EI}$	$Y_{max} = 0.00305\frac{WL}{EI}$ at $x = 0.402$