00121

THE OPEN UNIVERSITY OF SRI LANKA

Bachelor of Technology (Civil) – Level 5

CEX 5233 - STRUCTURAL ANALYSIS

FINAL EXAMINATION - 2009/2010

Time Allowed : 3 hours

Date: 2010-03-12 (Friday) Time: 09.30 – 12.30 hrs.

The Paper consists of Eight (8) questions. Answer **Five (5)** questions

1.(a) Figure-Q1 shows sets of axes x_1', x_2' and x_1'', x_2'' which are obtained from x_1, x_2 by a clockwise rotation through θ and φ respectively.
Write down the array of direction cosines of the "dashed" axes referred to the "un-dashed" axes. (2 marks)

Write down the array of direction cosines of the "double-dashed" axes referred to the "dashed" axes.
(2 marks)

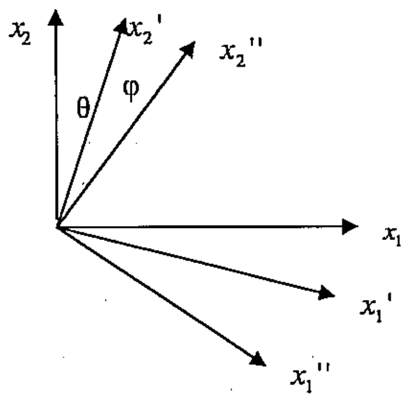


Figure-Q1

(b) The stress tensor at a point with reference to axes **x,y,z** is given by

$$\sigma_{ij} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 4 & \sqrt{3} \\ 0 & \sqrt{3} & 6 \end{bmatrix} \text{kN/m}^2$$

Determine:

(a) Stress invariants
(b) Principal stresses
(c) Principal axes
(d) Directions of principal axes (16 marks)

2.(a) What do you understand by a plane strain problem? (2 marks)

(b) Show that

$\Phi = Ay^2 + By^3 + \frac{q}{40d^3}y^5 - 3Cd^2xy + Cxy^3 - \frac{q}{4}x^2 + \frac{3q}{8d}x^2y - \frac{q}{8d^3}x^2y^3$ is a valid stress function. (2 marks)

(c) The above stress function is proposed to give the solution for a cantilever having a rectangular cross-section of unit width ($y = \pm d$; $0 \le x \le L$) ,carrying a concentrated end load of P and a uniformly distributed load q on the upper boundary.
Obtain values for constants **A,B** and **C.** (12 marks)

(d) Write expressions for stress field $\sigma_x, \sigma_y and \sigma_{xy}$. (4 marks)

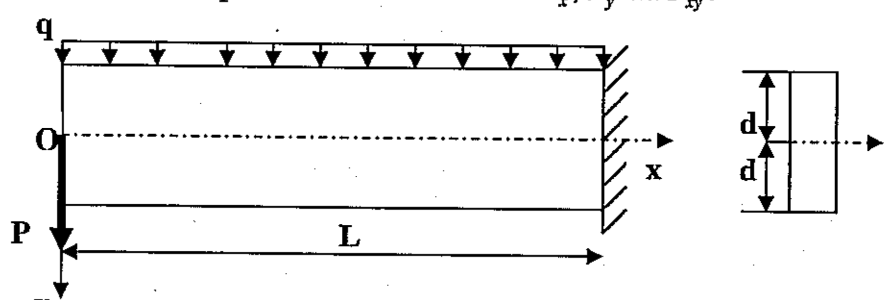


Figure-Q2

3. Analyse the continuous beam shown below, using flexibility method, for
(a) a uniformly distributed load of intensity **q** on spans **BC** and **CD** (12marks)

(b) a unit downward movement of support **C**
and draw the bending moment diagrams. (8marks)
The beam has a constant flexural rigidity **EI.**
(You may use Table-1 to determine flexibility coefficients)

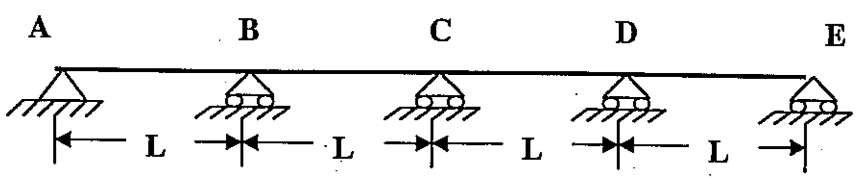


Figure-Q3

4. The plane frame shown in the figure-Q4, consists of rigidly connected members of constant flexural rigidity **EI**. Analyse the frame using displacement method and draw the Bending Moment diagram.
Neglect axial deformation. (20marks)
(You may use Table-1 to determine stiffness coefficients)

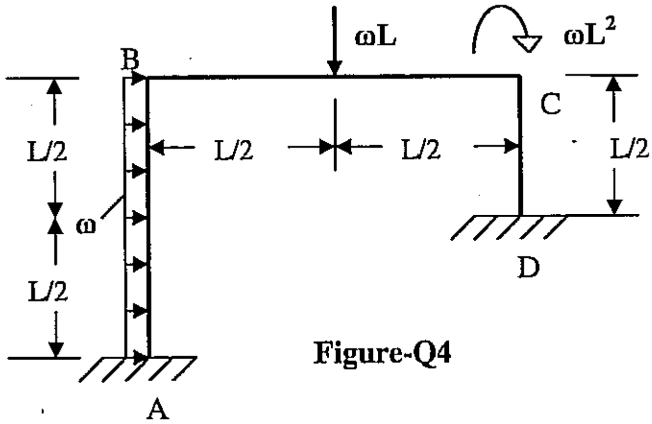


Figure-Q4

5.(a) What are the general assumptions made in the membrane theory of thin shells? (3marks)

A spherical dome of radius **a** is subjected to a uniformly distributed load **ω** on plan, as shown in the figure **Q-5**.

(b) Show that the expressions for Meridianal Stress (σ_Φ) and Hoop Stress (σ_θ) are given by

$$\sigma_\Phi = -\frac{a\omega}{2h} \quad ; \quad \sigma_\theta = -\frac{a\omega}{2h} Cos2\Phi$$

where h is the thickness of the shell. (12marks)

(c) Also show that the maximum value of φ that can be allowed is 45°, if the shell is made out of concrete. (5marks)

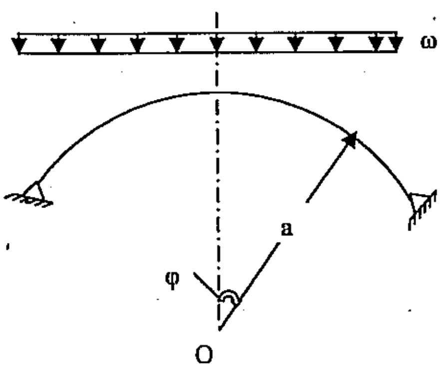


Figure Q-5

6. A circular plate of radius **a** is carrying a concentrated load **P** at the centre. Assuming that the edge of the plate is built-in, show that the deflection of the plate is given by;

$$\omega = \frac{Pr^2}{8\pi D} Log \frac{r}{a} + \frac{P}{16\pi D}\left(a^2 - r^2\right)$$ (13marks)

Hence determine the maximum deflection of the plate. (3marks)

Also find the moment M_r at built-in edge. (4marks)

7.(a) In Plastic Theory, **three conditions** apply when a structure is on the point of collapse. What are they? (3 marks)

(b) Analyse the rigid frame shown below using plastic method of analysis and determine the Plastic Moment M_p, for a load factor of **1.8**. (17 marks)

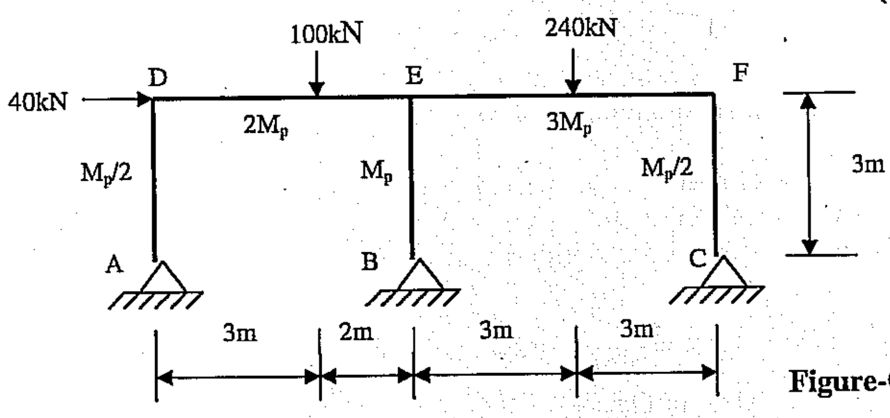


Figure-Q7

8.(a) Determine the collapse load for the fixed beam shown in the figure below, if the Plastic Moment Capacity of the beam is **MP**. (10 marks)

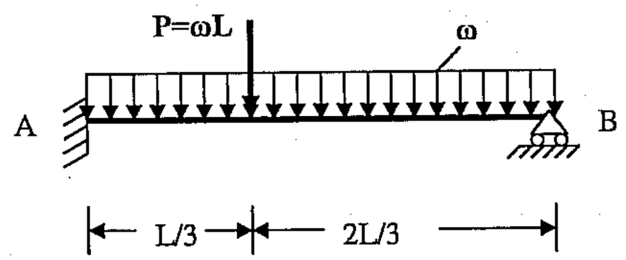


Figure-Q8(a)

(b) Determine the ultimate load for the propped cantilever shown in the figure below, if the Plastic Moment Capacity of the beam is **MP**. (10 marks)

Figure-Q8(b)

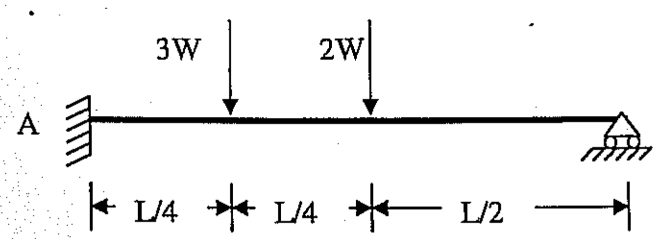
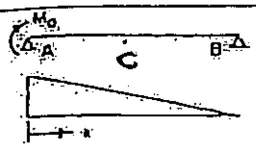
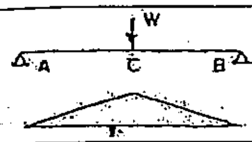
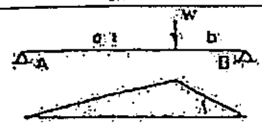
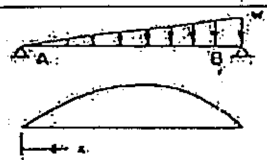
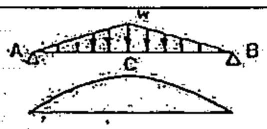
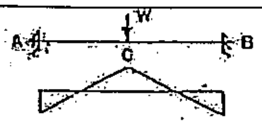
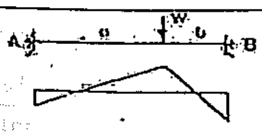
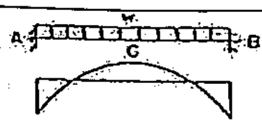
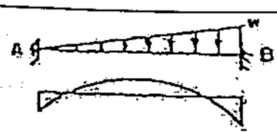
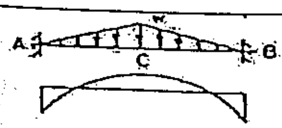


Table 1 Formulas for Beams

Structure	Shear	Moment	Slope	Deflection ↓
Simply supported Beam				
	$S_A = -\frac{M_o}{L}$	M_o	$\theta_A = \frac{M_o L}{3EI}$ $\theta_B = -\frac{M_o L}{6EI}$	$Y_{max} = 0.062\frac{M_o L^2}{EI}$ $at\ x = 0.422L$ $Y_c = M_o L^2/16EI$
	$S_A = \frac{W}{2}$	$M_o = \frac{WL}{4}$	$\theta_A = -\theta_B = \frac{WL^2}{16EI}$	$Y_c = \frac{WL^3}{48EI}$
	$S_A = \frac{Wb}{L}$ $S_B = \frac{Wa}{L}$	$M_o = \frac{Wab}{L}$	$\theta_A = \frac{Wab}{6EIL}(L+b)$ $\theta_B = -\frac{Wab}{6EIL}(L+a)$	$Y_o = \frac{Wa^2b^2}{3EIL}$
	$S_A = \frac{WL}{2}$	$M_c = \frac{WL^2}{8}$	$\theta_A = -\theta_B = \frac{W\ L^3}{24EI}$	$Y_c = \frac{5WL^4}{384EI}$
	$S_A = \frac{WL}{6}$ $S_B = \frac{WL}{3}$	$M_{max} = 0.064Wl^2$ $at\ x = 0.577L$	$\theta_A = \frac{7WL^3}{360EI}$ $\theta_B = -\frac{8WL^3}{360EI}$	$Y_{max} = 0.00652\frac{WL^4}{EI}$ $at\ x = 0.519L$
	$S_A = \frac{WL}{4}$	$M_c = \frac{WL^2}{12}$	$\theta_A = -\theta_B = \frac{5W\ L^3}{192EI}$	$Y_c = \frac{WL^4}{120EI}$
Fixed Beams				
	$S_A = \frac{W}{2}$	$M_c = \frac{WL}{8}$	$\theta_A = \theta_B = 0$	$Y_c = \frac{WL^3}{192EI}$
	$S_A = \frac{Wb^2}{L^3}(3a+b)$ $S_B = \frac{Wa^2}{L^3}(3b+a)$	$M_A = -\frac{Wab^2}{L^2}$ $M_B = -\frac{Wba^2}{L^2}$	$\theta_A = \theta_B = 0$	$Y_o = \frac{Wa^3b^3}{3EIL^3}$
	$S_A = \frac{WL}{2}$	$M_A = M_B = -\frac{WL^2}{12}$	$\theta_A = \theta_B = 0$	$Y_c = \frac{WL^4}{384EI}$
	$S_A = \frac{3WL}{20}$ $S_B = -\frac{7WL}{20}$	$M_A = -\frac{WL^2}{30}$ $M_B = -\frac{WL^2}{20}$	$\theta_A = \theta_B = 0$	$Y_{max} = 0.00131\frac{WL^4}{EI}$ $at\ x = 0.525L$
	$S_A = \frac{WL}{4}$	$M_A = M_B = -\frac{5WL^2}{96}$	$\theta_A = \theta_B = 0$	$Y_c = \frac{0.7WL^4}{384EI}$

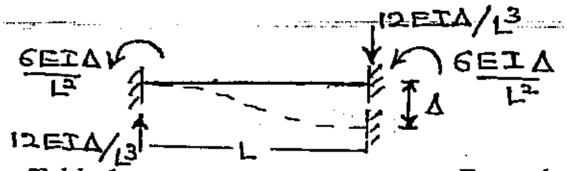
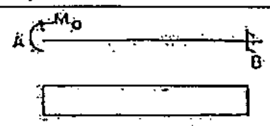
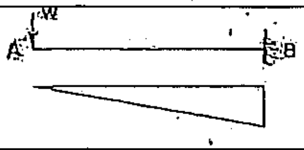
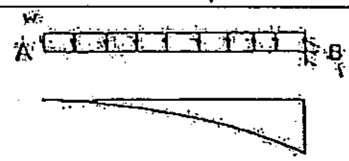
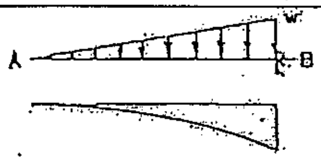
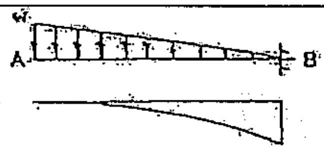
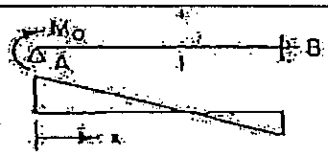
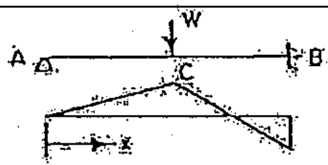
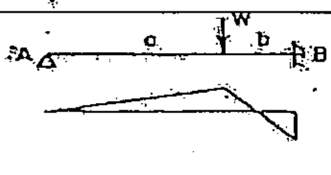
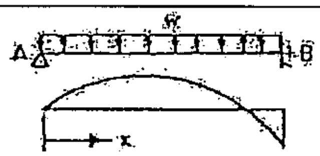
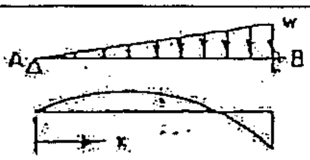
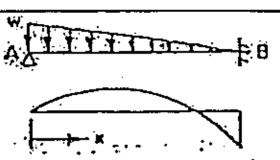


Table 1

Formulas for Beams

Structure	Shear	Moment	Slope	Deflection
Cantilever Beam				
	0	M_o	$\theta_A = \frac{M_o L}{EI}$	$Y_A = \frac{M_o}{2}$…
	W	$M_B = -WL$	$\theta_A = -\frac{WL^2}{2EI}$	$Y_A = \frac{W}{3}$…
	$S_B = -WL$	$M_B = -\frac{WL^2}{2}$	$\theta_A = -\frac{WL^3}{6EI}$	$Y_A = \frac{W}{8E}$…
	$S_B = -\frac{WL}{2}$	$M_B = -\frac{WL^2}{6}$	$\theta_A = -\frac{WL^3}{24EI}$	$Y_A = \frac{W}{8E}$…
	$S_B = -\frac{WL}{2}$	$M_B = -\frac{WL^2}{2}$	$\theta_A = -\frac{WL^3}{8EI}$	$Y_A = \frac{11W}{120E}$…
Propped Cantilever				
	$S_A = -\frac{3M_o}{2L}$	$M_B = -\frac{M_o}{2}$	$\theta_A = -\frac{M_o L}{4EI}$	$Y_{max} = \frac{M_o}{27}$… at $x=$…
	$S_A = -\frac{3M_o}{2L}$	$M_B = -\frac{3WL}{16}$ $M_c = \frac{5WL}{32}$	$\theta_A = \frac{WL^2}{32EI}$	$Y_{max} = 0.00962$… at $x = 0.4$…
	$S_A = \frac{Wb^2}{2L^3}(a+2L)$ $S_B = -\frac{Wa}{2L^3}(3L^2 - a^2)$	$M_B = -\frac{Wab}{L^2}(a + \frac{b}{2})$	$\theta_A = \frac{Wab^2}{4EIL}$	$Yo = \frac{Wa^2 b^2}{12EIL^3}$(…
	$S_A = +\frac{3WL}{8}$	$M_B = -\frac{WL^2}{8}$	$\theta_A = \frac{WL^3}{48EI}$	$Y_{max} = 0.0054$… at $x = 0.$…
	$S_A = +\frac{WL}{10}$	$M_{max} = 0.03WL^2$ at $x = 0.447L$ $M_B = -\frac{WL^2}{15}$	$\theta_A = \frac{WL^3}{120EI}$	$Y_{max} = 0.00239$… at $x = 0.$…
	$S_A = +\frac{11WL}{40}$	$M_{max} = 0.0423WL^2$ at $x = 0.329L$ $M_B = -\frac{7WL^2}{120}$	$\theta_A = \frac{WL^3}{80EI}$	$Y_{max} = 0.00305$… at $x = 0.$…