THE OPEN UNIVERSITY OF SRI LANKA
Department Of Civil Engineering
Bachelor of Technology (Civil) – Level 5

CEX5233 – Structural Analysis

FINAL EXAMINATION – 2010/2011

Time Allowed: Three Hours

Date: 2011 - 03 – 11 (Friday) Time: 09.30 – 12.30 hrs

The Paper consists of 8 questions, Answer 5 questions only

1. The beam given in Figure Q1 has constant EI all over the length.

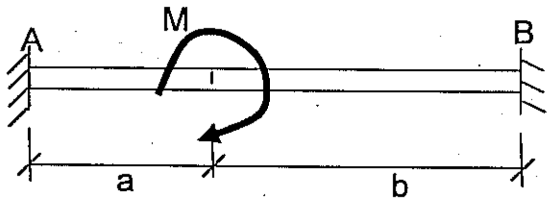


Figure Q1.

a. Determine the degree of static interterminacy [1-Mark]

b. Release the reaction and moment at B and formulate the flexibility matrix [8-Marks]

c. Draw the bending moment diagram for the released structure and determine the displacement and rotation at B. [5-Marks]

d. Hence using the flexibility method determine the moment and reaction at A and B. [6-Marks]

2. A frame structure shown in Figure Q2 has a uniformly distributed lateral load acting along CD (6 m) and a point load acting at the mid point of CB (4 m).

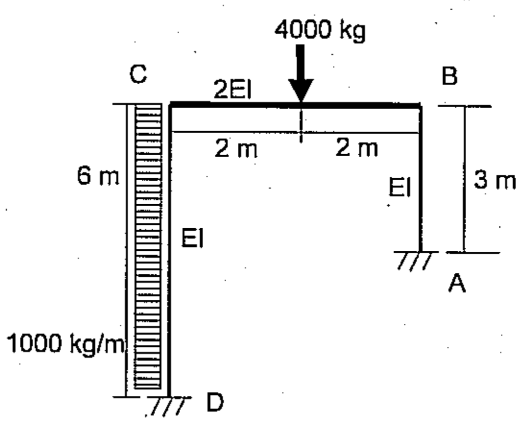


Figure Q2.

a) Determine the degree of kinematic indeterminacy [2-Marks]

b) Formulate the stiffness matrix (Use the charts provided with the exam paper) [8-Marks]

c) Determine the fixed end moments and support reactions at A and D [10-Marks]

3 a) Write three properties of a stress tensor. [3 Marks]

b) Stress tensor at a point in a continuum is given by(kN/m^2)

$$\sigma_{ij} = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix}$$

Determine

i) Stress invariants [6 Marks]

ii) Show that the principal stresses are -2 kN/m^2 , 1 kN/m^2 and 4kN/m^2 [5 Marks]

b) The state of stress tensor at a point can be written as

$$\sigma_{ij} = \begin{bmatrix} \sigma & a\sigma & b\sigma \\ a\sigma & \sigma & c\sigma \\ b\sigma & c\sigma & \sigma \end{bmatrix}$$

where a,b and c are constants and the σ is stress value. It was found that the stress vector vanishes on the octahedral plane $\underline{n} = \frac{1}{\sqrt{3}}e_1 + \frac{1}{\sqrt{3}}e_2 + \frac{1}{\sqrt{3}}e_3$

Determine a, b and c [6 Marks]

4 a) Determine the required plastic moment capacity of the beam(Mp) for the loading given in Figure Q4(a) [6 Marks]

b) Determine the required plastic moment capacity of the beam (Mp) for the loading given in Figure Q4(b) [6 Marks]

c) Determine the required plastic moment capacity of the beam (Mp) for the loading given in Figure Q4(c) [8 Marks]

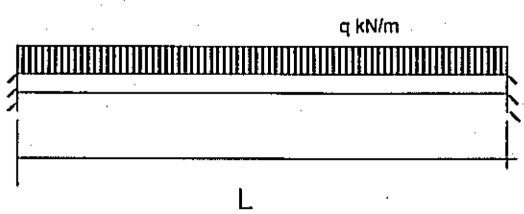


Figure Q4(a)

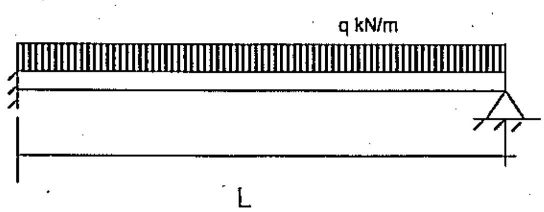


Figure Q4(b)

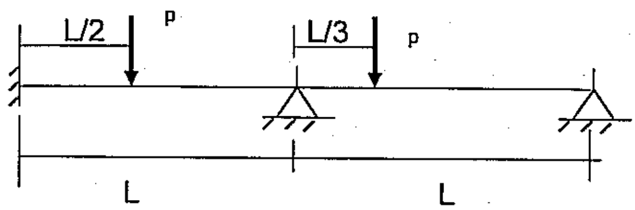


Figure Q4(c)

5 a) Three conditions are applied in plastic theory at the point of collapse. What are those? [3-Marks]

b) The structure given below is to be analyzed using plastic theory. The plastic moment of resistance of each member is Mp.

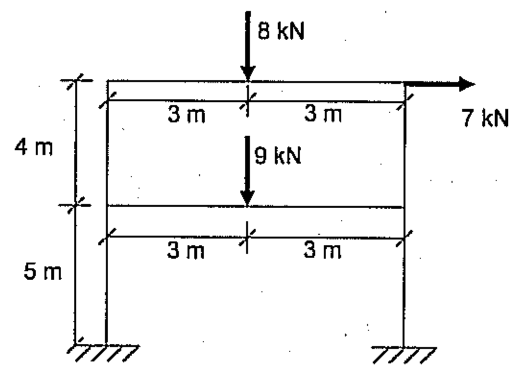


Figure Q5.

i) The study made by Baker, (1949) showed that several failure mechanisms can be identified such as two beam mechanisms, three sway mechanisms and three combined mechanisms for the frame shown in Figure Q5. Draw all the mechanisms indicating the location of plastic hinges. [4-Marks]

ii) Hence determine the plastic moment for

a) a beam mechanism [3-Marks]

b) a sway mechanism [3-Marks]

c) all combined mechanisms. [7-Marks]

6 a) Airy stress function is a popular technique that is used to solve elastic problems. Many two dimensional elastic problems require the use of polar coordinates to develop a solution. The biharmonic airy stress function in polar coordinates can be written as

$\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}\right)\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}\right)\phi=0$. The general solution to the above was proposed by Michell (1899) which contains many terms. However, for the axisymetric case the solution can be simplified and written as

$\phi = a_0 + a_1 \log r + a_2 r^2 + a_3 r^2 \log r$

The stresses can be derived using the following expressions

$$\sigma_r = \frac{1}{r}\frac{\partial \phi}{\partial r} + \frac{1}{r}\frac{\partial^2 \phi}{\partial \theta^2}, \quad \sigma_\theta = \frac{\partial^2 \phi}{\partial r^2} \text{ and } \sigma_{r\theta} = \frac{-\partial}{\partial r}\left(\frac{1}{r}\frac{\partial \phi}{\partial \theta}\right).$$

Show that $\sigma_r = 2a_3 \log r + \frac{a_1}{r^2} + a_3 + 2a_2$ and $\sigma_\theta = 2a_3 \log r - \frac{a_1}{r^2} + 3a_3 + 2a_2$

[10 Marks]

b) A thick walled cylinder under uniform pressure (Figure Q6) is to be analyzed using relationships derived in part (a). The stresses can be assumed as

$\sigma_r = \frac{A}{r^2} + B$ and $\sigma_\theta = -\frac{A}{r^2} + B$. Applying the suitable boundary conditions

determine A and B. Hence show that $\sigma_r = \frac{r_1^2 r_2^2 (p_2 - p_1)}{r_2^2 - r_1^2}\frac{1}{r^2} + \frac{r_1^2 p_1 - r_2^2 p_2}{r_2^2 - r_1^2}$

[10 Marks]

$$\sigma_\theta = \frac{-r_1^2 r_2^2 (p_2 - p_1)}{r_2^2 - r_1^2}\frac{1}{r^2} + \frac{r_1^2 p_1 - r_2^2 p_2}{r_2^2 - r_1^2}$$

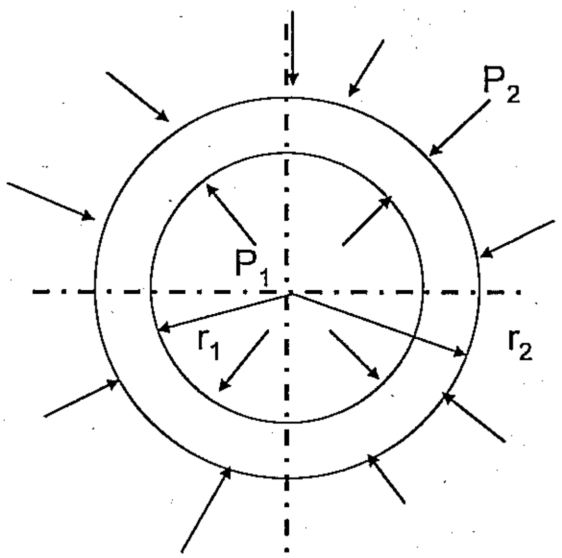


Figure Q6.

7. The Governing equation for plate bending is given by the following expressions

$$\frac{\partial^4 \omega}{\partial x^4} + \frac{2\partial^4 \omega}{\partial x^2 \partial y^2} + \frac{\partial^4 \omega}{\partial y^4} = \frac{q}{D}$$

All the terms have their normal meanings.

a) Write three main assumptions used in deriving the plate bending governing equation. [4 Marks]

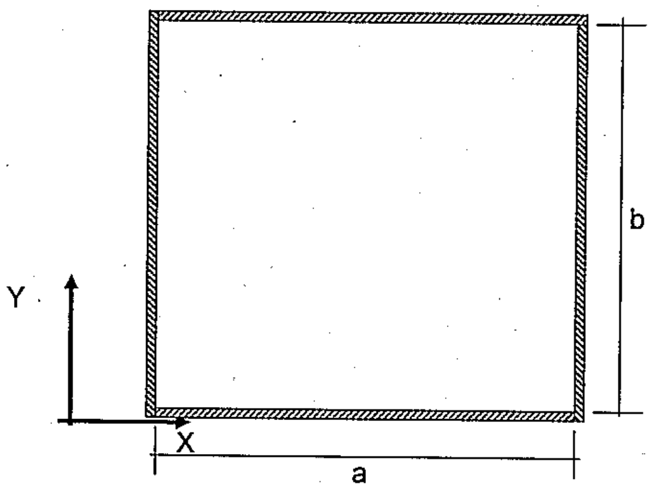
b) A simply supported rectangular plate (Figure Q7) is subjected to a sinusoidal load

$$q = q_0 \sin {\pi x}/{a} \sin {\pi y}/{b} .$$

If the deflection of the plate can be expressed as $\omega = C \sin {\pi x}/{a} \sin {\pi y}/{b}$

Show that ω is given by the following expression,

$$\omega = \frac{q_o}{D\pi^4\left(\frac{1}{a^2}+\frac{1}{b^2}\right)^2} \sin {\pi x}/{a} \sin {\pi y}/{b}$$ [8 Marks]



b) The first successful solution for the governing equation for plate bending was proposed by Naveier in 1820.

If $q = q_{(x,y)} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} a_{mn} \sin {m\pi x}/{a} \sin {n\pi y}/{b}$

Show that any particular co-efficient ($a_{m'n'}$) is given by the following expression

$$a_{m'n'} = \frac{4}{ab}\int_0^a\int_0^b q_{(x,y)} \sin(\frac{m\pi x}{a}) \sin(\frac{n\pi y}{b}) dxdy$$ [5 Marks]

You can assume that,

$\int_0^b \sin(\frac{n\pi y}{b})\sin(\frac{n'\pi y}{b})dy = b/2$ when $n=n'$ else $\int_0^b \sin(\frac{n\pi y}{b})\sin(\frac{n'\pi y}{b})dy = 0$

Hence show that for uniformly distributed load

$a_{m'n'} = \frac{4q}{\pi^2 mn}$ when m=1,3,5 and n=1,3,5

And for even number of m and n $a_{m'n'} = 0$ [3 Marks]

8. a) The governing equations for designing double curvature shells under membrane action are given by the following expressions

$$\frac{\partial^4 (rN_\phi)}{\partial\phi} - r_1 N_\theta \cos\phi = -P_\phi r r_1$$

$\frac{N_\theta}{r_2} + \frac{N_\phi}{r_1} = P_r$ All the terms have their usual meanings.

Write three main assumptions in the theory of shells [3 Marks]

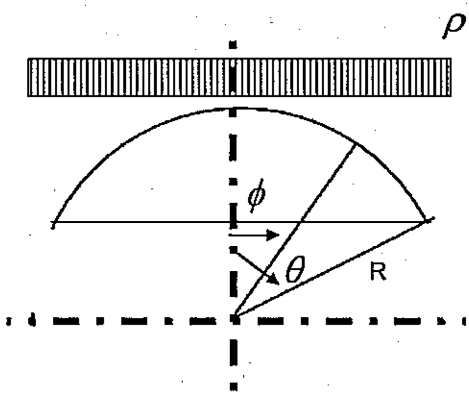


Figure Q8

b) A spherical dome shown in Figure Q8 has the radius R . The dome is subjected to a uniformly distributed load ρ over its plan area.

i) Show that $P_r = \rho\cos^2\phi$ and $P_\phi = \rho\cos\phi\sin\phi$ [5 Marks]

ii) Show that $N_\phi = {}^{-\rho R}\!/_{2}$ and $N_\theta = {}^{-\rho R Cos 2\phi}\!/_{2}$ [12 Marks]

Table 1 Formulas for Beams

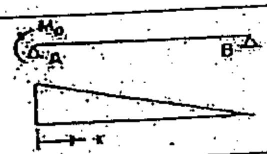
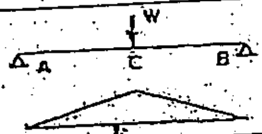
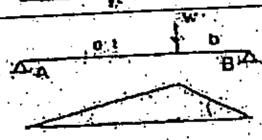
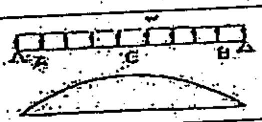
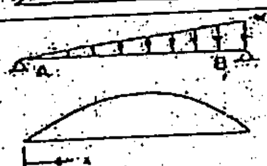
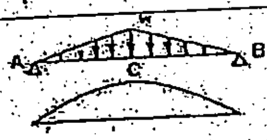
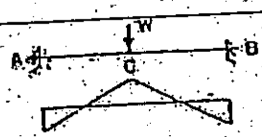
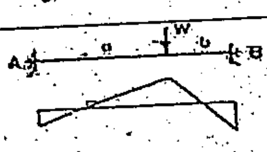
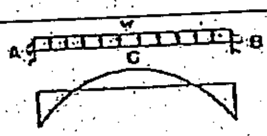
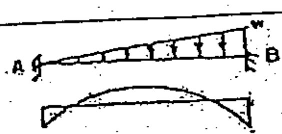
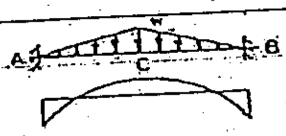
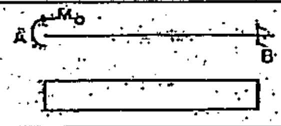
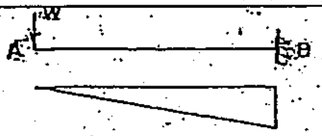
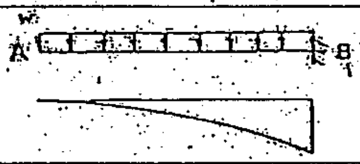
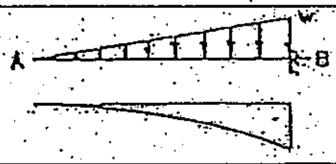
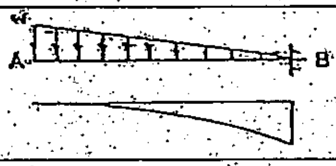
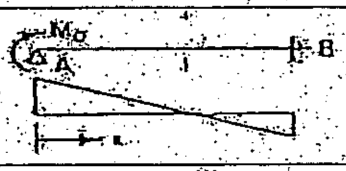
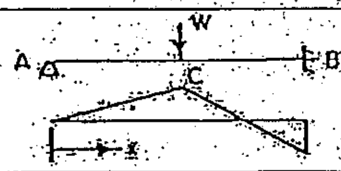
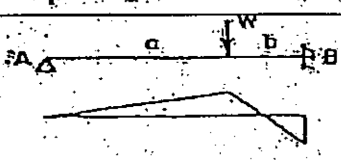
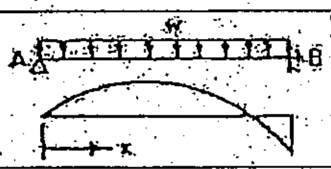
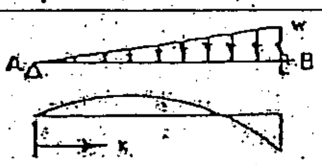
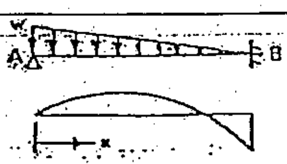
Structure	Shear	Moment	Slope	Deflection ↓
Simply supported Beam				
	$S_A=-\frac{M_o}{L}$	M_o	$\theta_A=\frac{M_oL}{3EI}$ $\theta_B=-\frac{M_oL}{6EI}$	$Y_{max}=0.062\frac{M_oL^2}{EI}$ $at\ x=0.422L$
	$S_A=\frac{W}{2}$	$M_o=\frac{WL}{4}$	$\theta_A=-\theta_B=\frac{WL^2}{16EI}$	$Y_c=\frac{WL^3}{48EI}$
	$S_A=\frac{Wb}{L}$ $S_B=\frac{Wa}{L}$	$M_o=\frac{Wab}{L}$	$\theta_A=\frac{Wab}{6EIL}(L+b)$ $\theta_B=-\frac{Wab}{6EIL}(L+a)$	$Y_o=\frac{Wa^2b^2}{3EIL}$
	$S_A=\frac{WL}{2}$	$M_c=\frac{WL^2}{8}$	$\theta_A=-\theta_B=\frac{WL^3}{24EI}$	$Y_c=\frac{5WL^4}{384EI}$
	$S_A=\frac{WL}{6}$ $S_B=\frac{WL}{3}$	$M_{max}=0.064Wl^2$ $at\ x=0.577L$	$\theta_A=\frac{7WL^3}{360EI}$ $\theta_B=-\frac{8WL^3}{360EI}$	$Y_{max}=0.00652\frac{WL^4}{EI}$ $at\ x=0.519L$
	$S_A=\frac{WL}{4}$	$M_c=\frac{WL^2}{12}$	$\theta_A=-\theta_B=\frac{5WL^3}{192EI}$	$Y_c=\frac{WL^4}{120EI}$
Fixed Beams				
	$S_A=\frac{W}{2}$	$M_c=\frac{WL}{8}$	$\theta_A=\theta_B=0$	$Y_c=\frac{WL^3}{192EI}$
	$S_A=\frac{Wb^2}{L^3}(3a+b)$ $S_B=\frac{Wa^2}{L^3}(3b+a)$	$M_A=-\frac{Wab^2}{L^2}$ $M_B=-\frac{Wba^2}{L^2}$	$\theta_A=\theta_B=0$	$Y_o=\frac{Wa^3b^3}{3EIL^3}$
	$S_A=\frac{WL}{2}$	$M_A=M_B=-\frac{WL^2}{12}$	$\theta_A=\theta_B=0$	$Y_c=\frac{WL^4}{384EI}$
	$S_A=\frac{3WL}{20}$ $S_B=-\frac{7WL}{20}$	$M_A=-\frac{WL^2}{30}$ $M_B=-\frac{WL^2}{20}$	$\theta_A=\theta_B=0$	$Y_{max}=0.00131\frac{WL^4}{EI}$ $at\ x=0.525L$
	$S_A=\frac{WL}{4}$	$M_A=M_B=-\frac{5WL^2}{96}$	$\theta_A=\theta_B=0$	$Y_c=\frac{0.7WL^4}{384EI}$

Table 1 Formulas for Beams

Structure	Shear	Moment	Slope	Deflection ↓
Cantilever Beam				
	0	M_o	$\theta_A = \frac{M_o L}{EI}$	$Y_A = \frac{M_o L^2}{2EI}$
	W	$M_B = -WL$	$\theta_A = -\frac{WL^2}{2EI}$	$Y_A = \frac{WL^3}{3EI}$
	$S_B = -WL$	$M_B = -\frac{WL^2}{2}$	$\theta_A = -\frac{WL^3}{6EI}$	$Y_A = \frac{WL^4}{8EI}$
	$S_B = -\frac{WL}{2}$	$M_B = -\frac{WL^2}{6}$	$\theta_A = -\frac{WL^3}{24EI}$	$Y_A = \frac{WL^4}{8EI}$
	$S_B = -\frac{WL}{2}$	$M_B = -\frac{WL^2}{2}$	$\theta_A = -\frac{WL^3}{8EI}$	$Y_A = \frac{11WL^4}{120EI}$
Propped Cantilever				
	$S_A = -\frac{3M_o}{2L}$	$M_B = -\frac{M_o}{2}$	$\theta_A = -\frac{M_o L}{4EI}$	$Y_{max} = \frac{M_o L^2}{27EI}$ at $x = \frac{L}{3}$
	$S_A = -\frac{3M_o}{2L}$	$M_B = -\frac{3WL}{16}$ $M_c = \frac{5WL}{32}$	$\theta_A = \frac{WL^2}{32EI}$	$Y_{max} = 0.00962\frac{WL^3}{ELI}$ at $x = 0.447$
	$S_A = \frac{Wb^2}{2L^3}(a+2L)$ $S_B = -\frac{Wa}{2L^3}(3L^2 - a^2)$	$M_B = -\frac{Wab}{L^2}(a+\frac{b}{2})$	$\theta_A = \frac{Wab^2}{4EIL}$	$Yo = \frac{Wa^2b^2}{12EIL^3}(3L+a)$
	$S_A = +\frac{3WL}{8}$	$M_B = -\frac{WL^2}{8}$	$\theta_A = \frac{WL^3}{48EI}$	$Y_{max} = 0.0054\frac{WL^4}{ELI}$ at $x = 0.422L$
	$S_A = +\frac{WL}{10}$	$M_{max} = 0.03WL^2$ at $x = 0.447L$ $M_B = -\frac{WL^2}{15}$	$\theta_A = \frac{WL^3}{120EI}$	$Y_{max} = 0.00239\frac{WL^4}{EI}$ at $x = 0.447L$
	$S_A = +\frac{11WL}{40}$	$M_{max} = 0.0423WL^2$ at $x = 0.329L$ $M_B = -\frac{7WL^2}{120}$	$\theta_A = \frac{WL^3}{80EI}$	$Y_{max} = 0.00305\frac{WL^4}{EI}$ at $x = 0.402L$