THE OPEN UNIVERSITY OF SRI LANKA
Department Of Civil Engineering
Bachelor of Technology (Civil) - Level 5

CEX5233 - Structural Analysis

FINAL EXAMINATION – 2011/2012

Time Allowed: Three Hours

Date: 2012 - 03 – 02 (Friday) Time: 09.30 – 12.30 hrs

The Paper consists of 8 questions, Answer 5 questions only

1. The beam given in Figure Q1 has constant EI all over the length.

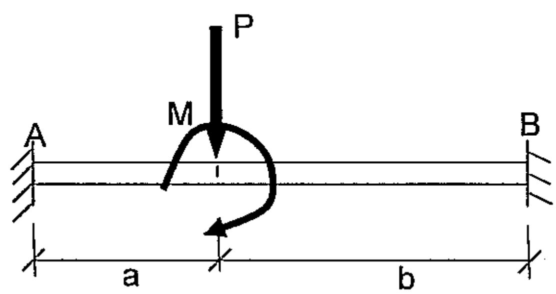

Figure Q1.

a. Determine the degree of static inderterminacy [1-Mark]
b. Release the reaction and moment at B and formulate the flexibility matrix [8-Marks]
c. Draw the bending moment diagram for the released structure and determine the displacement and rotation at B. [5-Marks]
d. Hence using the flexibility method determine the moment and reaction at A and B.[6-Marks]

2. A frame structure shown in Figure Q2 has a moment PL acting at B (4 m).

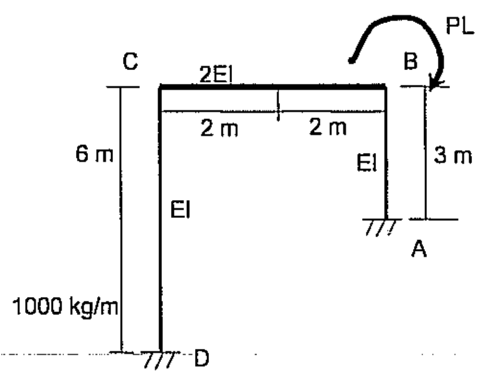

Figure Q2.

a) Determine the degree of kinematic indeterminacy [2-Marks]
b) Formulate the stiffness matrix (Use the charts provided with the exam paper) [8-Marks]

c) Determine the fixed end moments and support reactions at A and D [10-Marks]

3 a) The stress tensor at a point in the ground is given in the following matrix

$$\sigma_{ij} = \begin{bmatrix} 50 & 10 & 0 \\ 10 & 20 & 40 \\ 0 & 40 & 30 \end{bmatrix}$$

i) Determine Stress invariants [5- Marks]

ii) Determine the principal stresses [5- Marks]

b) A new coordinate system is obtained by rotating x and y 45^0 in counterclockwise about z axis Show that the rotational matrix is equal to,

$$A = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

[4-Marks]

c) Hence determine the stress state in the new coordinate system. [6 Marks]

4 a) Write three main characteristics of strain gauges [6 Marks]

b) Three strain components are measured by using a combination of gauges called strain rosette. Draw diagrams to show star rosette and delta rosette. [6 Marks]

c) Strain rosette readings are made at a critical point on the free surface in a structural steel member. The 60o rosette contains three wire gauges positioned at 0o, 60o and 120o. The readings were $\varepsilon_a = 100\mu$, $\varepsilon_b = 200\mu$ and $\varepsilon_c = -300\mu$. Determine principle strain and stresses and their directions. The material properties are E=200 GPa and v-0.3.[8 Marks].

5) a) Explain the terms safety factor and the load factor? [3-Marks]

b) The structure given below is to be analyzed using plastic theory. The moment capacity of each member is shown in the Figure Q5. Sketch all the possible collapse mechanism and determine the plastic moment capacity *Mp* [15-Marks]

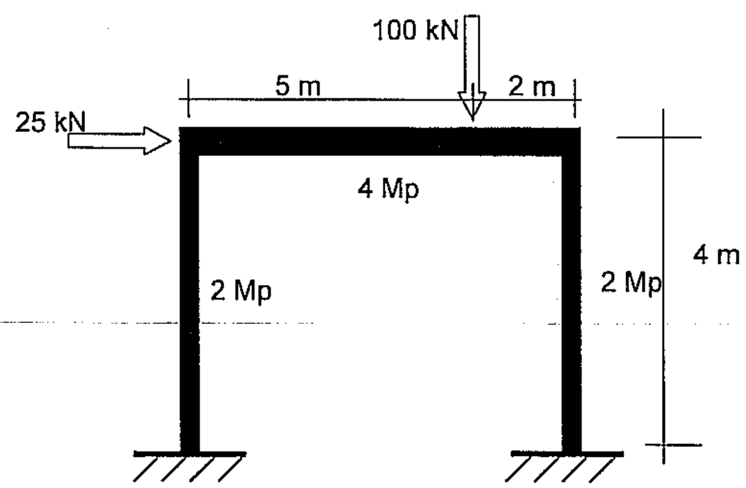

Figure Q5.

6 a) Consider a rectangular beam, whose cross-section is defined by $\frac{-h}{2} \le x_2 \le \frac{h}{2}$ and $\frac{-b}{2} \le x_3 \le \frac{b}{2}$ and whose length, $0 \le x_1 \le l$ by, with the origin of the coordinates located at the center of the left cross-section.x1 = 0 (Figure Q6). The following Airy stress function is tested for this beam $\varphi = \alpha x_1 x_2^3 + \beta x_1 x_2$.

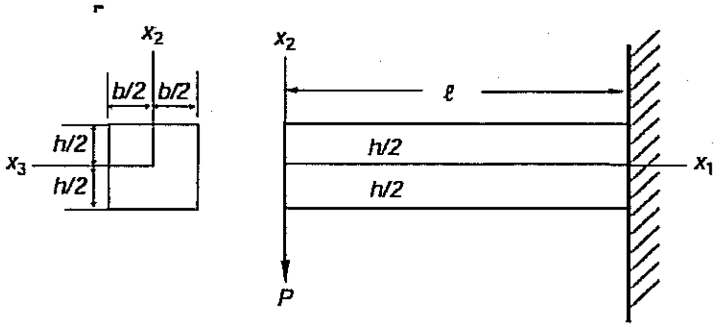

Figure Q6

a) Derive the stress components [10-Marks]
b) Write the boundary conditions when x_2=h/2, x_2=-h/2 and hence express β in terms of α [5-Marks]
c) Write the boundary conditions when x_1=0 plane and determine α and β [5–Marks]

7. The Governing equation for symmetrical bending of plates can be expressed as given below

$$\left[\frac{d^2}{dr^2}+\frac{1}{r}\frac{d}{dr}\right]\left[\frac{d^2}{dr^2}+\frac{1}{r}\frac{d}{dr}\right]w=\frac{q(r)}{D}$$

All the terms have their normal meanings.

a) Write two conditions that are needed to satisfy symmetrical bending [4 Marks]

b) Show that the above equation can be simplified to the following form

$$\frac{1}{r}\frac{d}{dr}\left\{r\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right)\right]\right\}=\frac{q(r)}{D}.$$ [4 Marks]

c) Show that for a solid plate the deflection is given by the following expression. $\omega = \frac{q_0 r^4}{64D} + C_2\frac{r^2}{4} + C_4$ where C_2 and C_4 are constants. [4 Marks]

d) Hence show that for a clamed plate deflection is given by

$$\omega = \frac{q_o}{64D}\left(a^2 - r^2\right)^2$$ [8 Marks]

8. Determine the membrane stress resultants N_x, N_θ, and $N_{\theta x}$ in a circular cylindrical section subtending an angle 2α at the axis under a udl of w per unit surface area. The length of the cylinder is L. The

sides of the cylinder are covered by diaphragm walls. The cylinder is supported on the edge beams. Assume that the origin of x coordinate is at the mid section of the cylinder.
The governing equations for a cylinder can be given as follows [20 Marks]

$$\frac{\partial N_x}{\partial x}+\frac{1}{a}\frac{\partial N_{\theta x}}{\partial \theta}+P_x=0$$

$$\frac{\partial N_{x\theta}}{\partial x}+\frac{1}{a}\frac{\partial N_\theta}{\partial \theta}+P_\theta=0$$

$$\frac{N_\theta}{a}+P_z=0$$

$$N_{\theta x}=N_{x\theta}$$

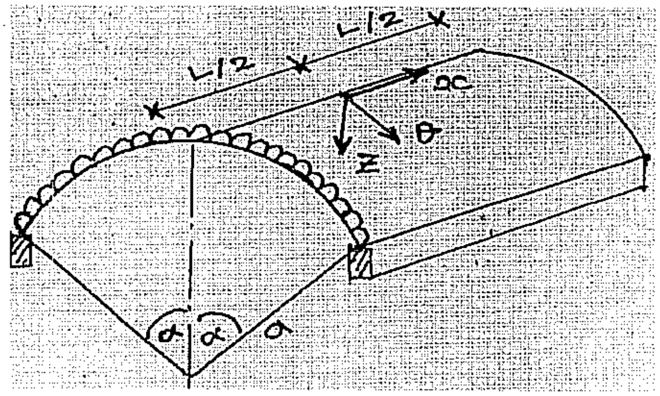

Figure Q8.

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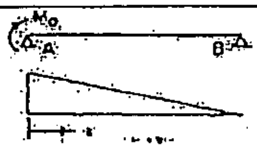
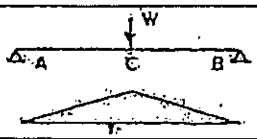
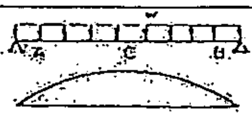
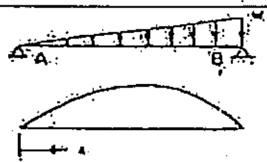
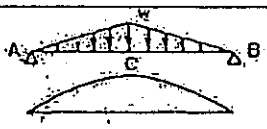
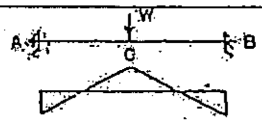
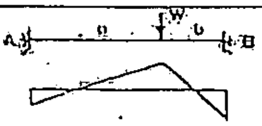
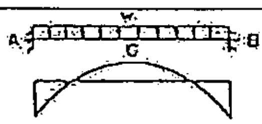
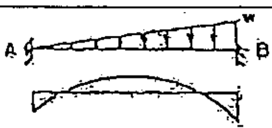
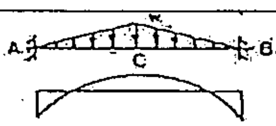
Table 1 Formulas for Beams

Structure	Shear	Moment	Slope	Deflection ↓
Simply supported Beam				
	$S_A=-\frac{M_o}{L}$	M_o	$\theta_A=\frac{M_oL}{3EI}$, $\theta_B=-\frac{M_oL}{6EI}$	$Y_{max}=0.062\frac{M_oL^2}{EI}$ at $x=0.422L$
	$S_A=\frac{W}{2}$	$M_o=\frac{WL}{4}$	$\theta_A=-\theta_B=\frac{WL^2}{16EI}$	$Y_c=\frac{WL^3}{48EI}$
	$S_A=\frac{Wb}{L}$ $S_B=\frac{Wa}{L}$	$M_o=\frac{Wab}{L}$	$\theta_A=\frac{Wab}{6EIL}(L+b)$ $\theta_B=-\frac{Wab}{6EIL}(L+a)$	$Y_o=\frac{Wa^2b^2}{3EIL}$
	$S_A=\frac{WL}{2}$	$M_c=\frac{WL^2}{8}$	$\theta_A=-\theta_B=\frac{W\ L^3}{24EI}$	$Y_c=\frac{5WL^4}{384EI}$
	$S_A=\frac{WL}{6}$ $S_B=\frac{WL}{3}$	$M_{max}=0.064Wl^2$ at $x=0.577L$	$\theta_A=\frac{7WL^3}{360EI}$ $\theta_B=-\frac{8WL^3}{360EI}$	$Y_{max}=0.00652\frac{WL^4}{EI}$ at $x=0.519L$
	$S_A=\frac{WL}{4}$	$M_c=\frac{WL^2}{12}$	$\theta_A=-\theta_B=\frac{5W\ L^3}{192EI}$	$Y_c=\frac{WL^4}{120EI}$
Fixed Beams				
	$S_A=\frac{W}{2}$	$M_c=\frac{WL}{8}$	$\theta_A=\theta_B=0$	$Y_c=\frac{WL^3}{192EI}$
	$S_A=\frac{Wb^2}{L^3}(3a+b)$ $S_B=\frac{Wa^2}{L^3}(3b+a)$	$M_A=-\frac{Wab^2}{L^2}$ $M_B=-\frac{Wba^2}{L^2}$	$\theta_A=\theta_B=0$	$Y_o=\frac{Wa^3b^3}{3EIL^3}$
	$S_A=\frac{WL}{2}$	$M_A=M_B=-\frac{WL^2}{12}$	$\theta_A=\theta_B=0$	$Y_c=\frac{WL^4}{384EI}$
	$S_A=\frac{3WL}{20}$ $S_B=-\frac{7WL}{20}$	$M_A=-\frac{WL^2}{30}$ $M_B=-\frac{WL^2}{20}$	$\theta_A=\theta_B=0$	$Y_{max}=0.00131\frac{WL^4}{EI}$ at $x=0.525L$
	$S_A=\frac{WL}{4}$	$M_A=M_B=-\frac{5WL^2}{96}$	$\theta_A=\theta_B=0$	$Y_c=\frac{0.7WL^4}{384EI}$

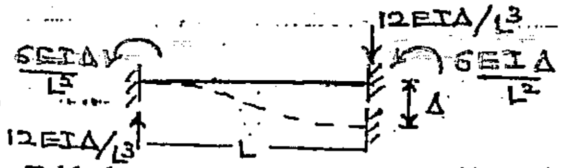
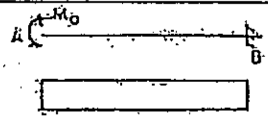
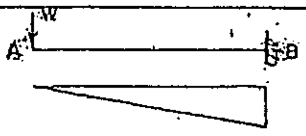
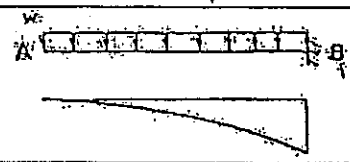
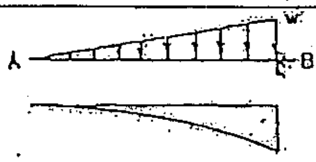
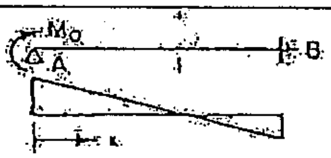
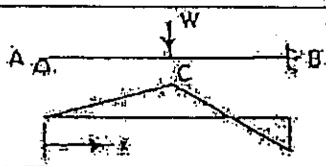
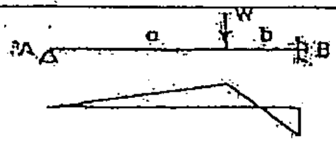
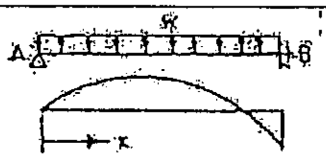
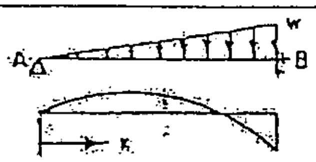
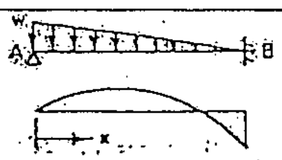


Table 1

Formulas for Beams

Structure	Shear	Moment	Slope	Deflection ↓
Cantilever Beam				
	0	M_o	$\theta_A = \frac{M_o L}{EI}$	$Y_A = \frac{M_o L^2}{2EI}$
	W	$M_B = -WL$	$\theta_A = -\frac{WL^2}{2EI}$	$Y_A = \frac{WL^3}{3EI}$
	$S_B = -WL$	$M_B = -\frac{WL^2}{2}$	$\theta_A = -\frac{WL^3}{6EI}$	$Y_A = \frac{WL^4}{8EI}$
	$S_B = -\frac{WL}{2}$	$M_B = -\frac{WL^2}{6}$	$\theta_A = -\frac{WL^3}{24EI}$	$Y_A = \frac{WL^4}{8EI}$
	$S_B = -\frac{WL}{2}$	$M_B = -\frac{WL^2}{2}$	$\theta_A = -\frac{WL^3}{8EI}$	$Y_A = \frac{11WL^4}{120EI}$
Propped Cantilever				
	$S_A = -\frac{3M_o}{2L}$	$M_B = -\frac{M_o}{2}$	$\theta_A = -\frac{M_o L}{4EI}$	$Y_{max} = \frac{M_o L^2}{27EI}$ $at\ x = \frac{L}{3}$
	$S_A = -\frac{3M_o}{2L}$	$M_B = -\frac{3WL}{16}$ $M_c = \frac{5WL}{32}$	$\theta_A = \frac{WL^2}{32EI}$	$Y_{max} = 0.00962\frac{WL^3}{ELI}$ $at\ x = 0.4471$
	$S_A = \frac{Wb^2}{2L^3}(a+2L)$ $S_B = -\frac{Wa}{2L^3}(3L^2 - a^2)$	$M_B = -\frac{Wab}{L^2}(a+\frac{b}{2})$	$\theta_A = \frac{Wab^2}{4EIL}$	$Yo = \frac{Wa^2b^2}{12EIL^3}(3L+a)$
	$S_A = +\frac{3WL}{8}$	$M_B = -\frac{WL^2}{8}$	$\theta_A = \frac{WL^3}{48EI}$	$Y_{max} = 0.0054\frac{WL^4}{ELI}$ $at\ x = 0.422L$
	$S_A = +\frac{WL}{10}$	$M_{max} = 0.03WL^2$ $at\ x = 0.447L$ $M_B = -\frac{WL^2}{15}$	$\theta_A = \frac{WL^3}{120EI}$	$Y_{max} = 0.00239\frac{WL^4}{EI}$ $at\ x = 0.447L$
	$S_A = +\frac{11WL}{40}$	$M_{max} = 0.0423WL^2$ $at\ x = 0.329L$ $M_B = -\frac{7WL^2}{120}$	$\theta_A = \frac{WL^3}{80EI}$	$Y_{max} = 0.00305\frac{WL^4}{EI}$ $at\ x = 0.402L$