THE OPEN UNIVERSITY OF SRI LANKA

Bachelor of Technology (Civil) – Level 5

CEX5233 – STRUCTURAL ANALYSIS

FINAL EXAMINATION - 2012/2013

Time Allowed – 3 Hours

Date: 02nd August 2013 | Time: 09.30 – 12.30 Hrs

This paper consists of eight (8) questions. Answer **five (5)** questions.

QUESTION 1

(i) Write short notes on following experimental stress measurement techniques.

a) Electrical resistance strain gauge method (2 Marks)
b) Photoelasticity method (2 Marks)
c) Surface coating method (2 Marks)

(ii) An element of steel plate is subjected to in-plane stresses as follows.

$$\sigma_{xx} = 200N/mm^2,\ \sigma_{yy} = -100N/mm^2,\ \tau_{xy} = 40N/mm^2$$

Young's modulus of steel is 210 GPa, Poisson's ratio of steel is 0.3.

a) What magnitudes of strains would be recorded by strain gauges placed in the X, Y directions? (4 Marks)
b) Draw Mohr's circle of stresses, and determine the maximum and minimum principal stresses, and their orientation to the X direction. (4 Marks)
c) Determine the strains that would be measured by strain gauges placed to the X direction. (4 Marks)
d) Instead of determining the strains from the stresses, if you were asked to determine the state of stress by measuring direct strains, state the minimum number of strain gauges that you need. (2 Marks)

QUESTION 2

(i) Explain the difference between the yield moment and plastic moment. (4 Marks)
(ii) Consider following simply supported beam as shown in Figure 1(a). Flexural rigidity is EI.

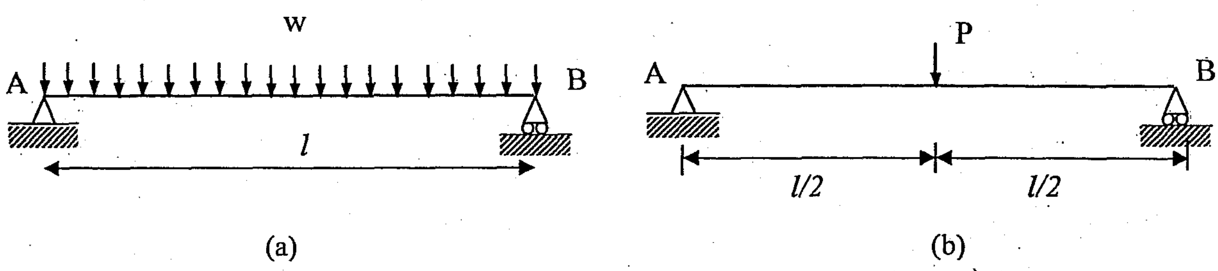

Figure 1: Simply supported beam with: (a) a uniformly distributed load; (b) central point load

a) The beam has a rectangular cross section with a breath "*b*" and depth "*d*". Determine the ratio of plastic moment (M_p) to yield moment (M_y). Hence, explain the practical significance of plastic analysis over elastic analysis. (6 Marks)
b) Determine the collapse load (w_{ult}) of the beam [Figure 1(a)] using the virtual work approach. You can consider that plastic moment as M_P. (4 Marks)
c) If the uniformly distributed load is removed and a point load (P) is applied as shown in Figure 1(b), determine the collapse load (P_{ult}) using the virtual work approach. (4 Marks)
d) Obtain the ratio of $\frac{w_{ult}l}{P_{ult}}$ and discuss the result. (2 Marks)

QUESTION 3

(i) General equilibrium equation is written as $\frac{\partial \sigma_{ij}}{\partial x_j} + F_i = 0$ in usual notation. Using this, write down equilibrium equations in three orthogonal directions. (3 Marks)
(ii) The stress tensor of a particular stress field is as

$$\sigma_{ij} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

a) Determine six independent stress components. (4 Marks)
b) Determine stress invariants. (4 Marks)
c) Determine the principal stresses. (4 Marks)
d) Determine the directions of principal axes. (5 Marks)

QUESTION 4

(i) Describe three characteristics of statical indeterminate structures. (3 Marks)

(ii) Consider the continuous beam shown in Figure 2. Flexural rigidity of two members, AB and BD is equal to EI. Uniformly distributed load (w) is acting on two members, AB and BD. Further, a concentrated load (wl) is acting on the member BD at C.

a) Determine the degree of statical indeterminacy of the beam. (2 Marks)
b) Draw a released structure. (2 Marks)
c) Determine the flexibility matrix for the drawn released structure. (5 Marks)
d) Determine moment at A and support reactions at B and D using the flexibility method. (8 Marks)

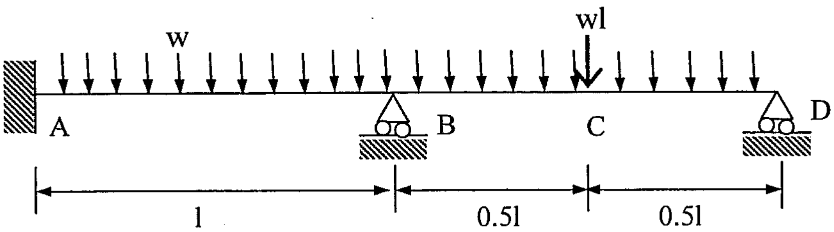

Figure 2: A continuous beam with a uniformly distributed load and point load

QUESTION 5

(i) What do you understand by kinematic indeterminacy of a structure. (3 Marks)

(ii) How does kinematic indeterminacy vary from statically indeterminacy of a structure? (2 Marks)

(iii) Consider the structure shown in Figure 3. Find the nodal displacements at B and C using the displacement method. You can neglect the axial deformation. (12 Marks)

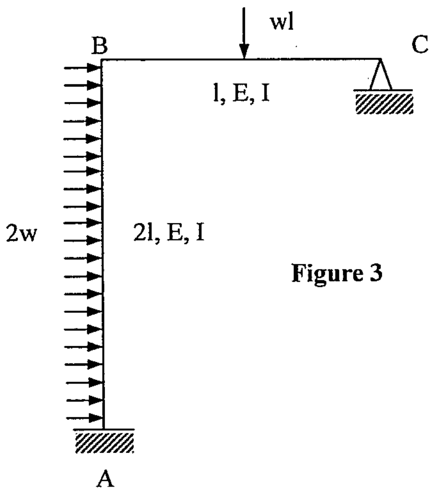

Figure 3

(iv) If the structure is going to fail due to the formation of plastic hinges, how many plastic hinges are required to form? (3 Marks)

QUESTION 6

(i) In the plastic theory, **certain conditions** apply when a structure is on the point collapse. Using these conditions how do you obtain,

(a) Lower bound theorem (2 Marks)
(b) Upper bound theorem (2 Marks)
(c) Uniqueness theorem (2 Marks)

(ii) A two-bay frame structure is shown in Figure 4. Dimensions and relative values of plastic moment (M_P) is given in the figure.

(a) Draw failure mechanisms. (3 Marks)
(b) Determine load factors for each failure mechanism. (7 Marks)
(c) Hence, determine the most probable failure mechanism. (2 Marks)
(d) Outline how you ensure that your solution is the unique solution. (2 Marks)

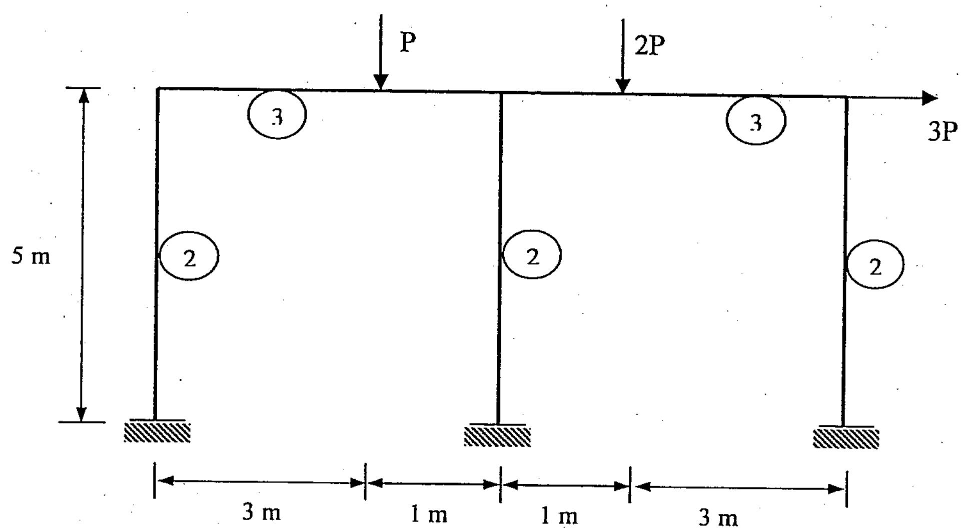

Figure 4: Two-bay frame structure

QUESTION 7

(i) What are the assumptions used in the theory of thin plates with small deflection. (4 Marks)

(ii) Governing equation for the uniformly loaded solid circular plate with symmetrical boundary condition is given in polar coordinates as,

$$\nabla^4 w = \frac{1}{r}\frac{d}{dr}\left\{r\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right)\right]\right\} = \frac{q_0}{D}$$

where q_0 is a constant, w is the deflection at a general point.

a) Obtain the general expression for w as

$$w=\frac{q_0r^4}{64D}+\frac{C_1r^2}{4}(\log r-1)+C_2\frac{r^2}{4}+C_3\log r+C_4$$ (6 Marks)

b) This plate is fixed at edge ($r = a$), hence obtain the simplified expression for w. (5 Marks)

c) Obtain the maximum deflection of the plate. (5 Marks)

QUESTION 8

(i) What are the assumptions used in the theory of thin shells. (5 Marks)

Governing equations for axisymmetric shells are given as (15 Marks)

$$\frac{\partial}{\partial S}(rN_S)+\frac{\partial N_{\theta s}}{\partial\theta}-\frac{\partial r}{\partial S}N_\theta+rP_S=0$$

$$\frac{\partial}{\partial S}(rN_{s\theta})+\frac{\partial N_\theta}{\partial\theta}+\frac{\partial r}{\partial S}N_{\theta s}+rP_\theta=0$$

$$r_1N_\theta+r_2N_S+r_1r_2P_Z=0$$

where coordinate system (r, s, ϕ) are as shown in Figure 5(a).

Using above relationships, find the membrane stress distribution (N_θ and N_S) in a conical water tank supported at the top as shown in Figure 5(b). You can consider the specific weight of water as γ_w.

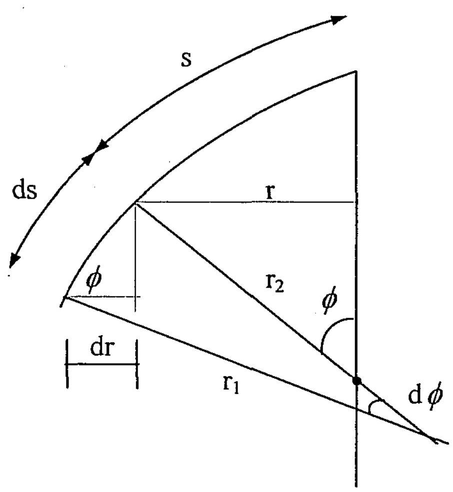

Figure 5(a)

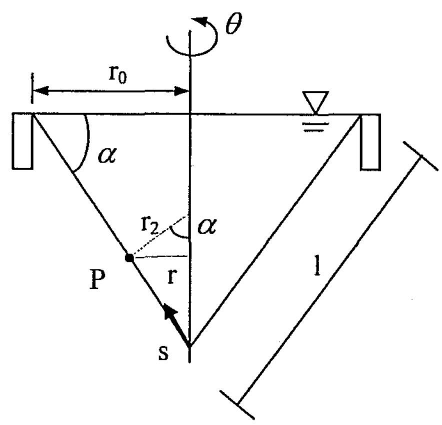

Figure 5(b)

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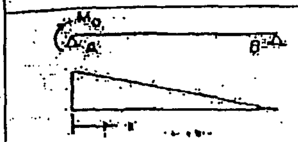
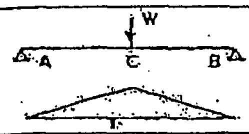
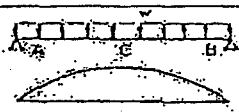
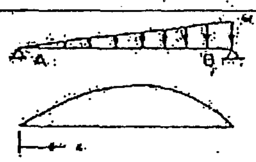
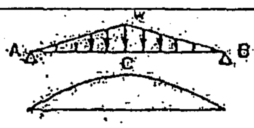
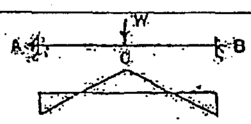
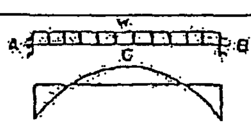
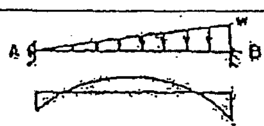
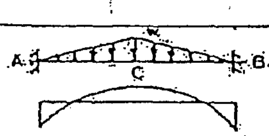
Table 1 Formulas for Beams

Structure	Shear	Moment	Slope	Deflection
Simply supported Beam				
	$S_A=-\frac{M_o}{L}$	M_o	$\theta_A=\frac{M_oL}{3EI}$, $\theta_B=-\frac{M_oL}{6EI}$	$Y_{max}=0.062\frac{M_oL^2}{EI}$ at $x=0.422L$
	$S_A=\frac{W}{2}$	$M_o=\frac{WL}{4}$	$\theta_A=-\theta_B=\frac{WL^2}{16EI}$	$Y_c=\frac{WL^3}{48EI}$
	$S_A=\frac{Wb}{L}$ $S_B=\frac{Wa}{L}$	$M_o=\frac{Wab}{L}$	$\theta_A=\frac{Wab}{6EIL}(L+b)$ $\theta_B=-\frac{Wab}{6EIL}(L+a)$	$Y_o=\frac{Wa^2b^2}{3EIL}$
	$S_A=\frac{WL}{2}$	$M_c=\frac{WL^2}{8}$	$\theta_A=-\theta_B=\frac{W\ L^3}{24EI}$	$Y_c=\frac{5WL^4}{384EI}$
	$S_A=\frac{WL}{6}$ $S_B=\frac{WL}{3}$	$M_{max}=0.064Wl^2$ at $x=0.577L$	$\theta_A=\frac{7WL^3}{360EI}$ $\theta_B=-\frac{8WL^3}{360EI}$	$Y_{max}=0.00652\frac{WL^4}{EI}$ at $x=0.519L$
	$S_A=\frac{WL}{4}$	$M_c=\frac{WL^2}{12}$	$\theta_A=-\theta_B=\frac{5W\ L^3}{192EI}$	$Y_c=\frac{WL^4}{120EI}$
Fixed Beams				
	$S_A=\frac{W}{2}$	$M_c=\frac{WL}{8}$	$\theta_A=\theta_B=0$	$Y_c=\frac{WL^3}{192EI}$
	$S_A=\frac{Wb^2}{L^3}(3a+b)$ $S_B=\frac{Wa^2}{L^3}(3b+a)$	$M_A=-\frac{Wab^2}{L^2}$ $M_B=-\frac{Wba^2}{L^2}$	$\theta_A=\theta_B=0$	$Y_o=\frac{Wa^3b^3}{3EIL^3}$
	$S_A=\frac{WL}{2}$	$M_A=M_B=-\frac{WL^2}{12}$	$\theta_A=\theta_B=0$	$Y_c=\frac{WL^4}{384EI}$
	$S_A=\frac{3WL}{20}$ $S_B=-\frac{7WL}{20}$	$M_A=-\frac{WL^2}{30}$ $M_B=-\frac{WL^2}{20}$	$\theta_A=\theta_B=0$	$Y_{max}=0.00131\frac{WL^4}{EI}$ at $x=0.525L$
	$S_A=\frac{WL}{4}$	$M_A=M_B=-\frac{5WL^2}{96}$	$\theta_A=\theta_B=0$	$Y_c=\frac{0.7WL^4}{384EI}$

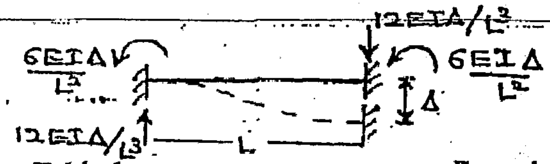
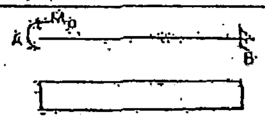
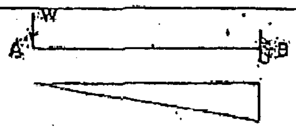
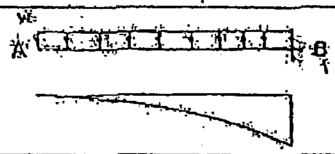
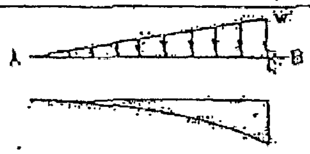
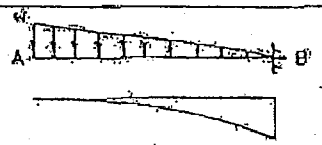
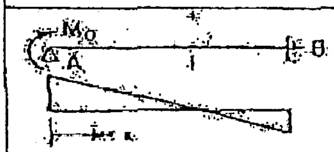
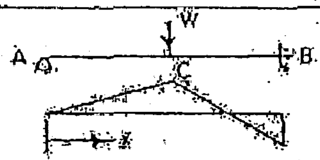
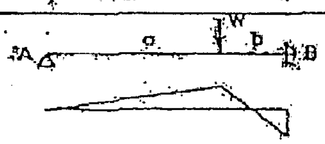
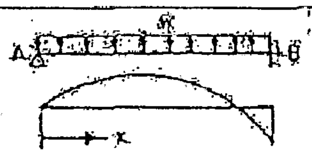
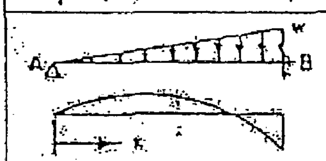
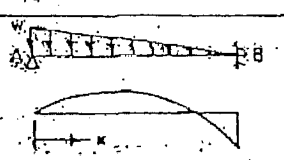


Table 1

Formulas for Beams

Structure	Shear	Moment	Slope	Deflection ↓
Cantilever Beam				
	0	M_o	$\theta_A = \frac{M_o L}{EI}$	$Y_A = \frac{M_o L^2}{2EI}$
	W	$M_B = -WL$	$\theta_A = -\frac{WL^2}{2EI}$	$Y_A = \frac{WL^3}{3EI}$
	$S_B = -WL$	$M_B = -\frac{WL^2}{2}$	$\theta_A = -\frac{WL^3}{6EI}$	$Y_A = \frac{WL^4}{8EI}$
	$S_B = -\frac{WL}{2}$	$M_B = -\frac{WL^2}{6}$	$\theta_A = -\frac{WL^3}{24EI}$	$Y_A = \frac{WL^4}{8EI}$
	$S_B = -\frac{WL}{2}$	$M_B = -\frac{WL^2}{2}$	$\theta_A = -\frac{WL^3}{8EI}$	$Y_A = \frac{11WL^4}{120EI}$
Propped Cantilever				
	$S_A = -\frac{3M_o}{2L}$	$M_B = -\frac{M_o}{2}$	$\theta_A = -\frac{M_o L}{4EI}$	$Y_{max} = \frac{M_o L^2}{27EI}$ at $x = \frac{L}{3}$
	$S_A = -\frac{3M_o}{2L}$	$M_B = -\frac{3WL}{16}$ $M_c = \frac{5WL}{32}$	$\theta_A = \frac{WL^2}{32EI}$	$Y_{max} = 0.00962\frac{WL…}{EI}$ at $x = 0.447…$
	$S_A = \frac{Wb^2}{2L^3}(a+2L)$ $S_B = -\frac{Wa}{2L^3}(3L^2 - a^2)$	$M_B = -\frac{Wab}{L^2}(a + \frac{b}{2})$	$\theta_A = \frac{Wab^2}{4EIL}$	$Yo = \frac{Wa^2 b^2}{12EIL^3}(3L…$
	$S_A = +\frac{3WL}{8}$	$M_B = -\frac{WL^2}{8}$	$\theta_A = \frac{WL^3}{48EI}$	$Y_{max} = 0.0054\frac{WL…}{EI}$ at $x = 0.42…$
	$S_A = +\frac{WL}{10}$	$M_{max} = 0.03WL^2$ at $x = 0.447L$ $M_B = -\frac{WL^2}{15}$	$\theta_A = \frac{WL^3}{120EI}$	$Y_{max} = 0.00239\frac{WL…}{EI}$ at $x = 0.44…$
	$S_A = +\frac{11WL}{40}$	$M_{max} = 0.0423WL^2$ at $x = 0.329L$ $M_B = -\frac{7WL^2}{120}$	$\theta_A = \frac{WL^3}{80EI}$	$Y_{max} = 0.00305\frac{WL…}{E…}$ at $x = 0.40…$