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THE OPEN UNIVERSITY OF SRI LANKA

Department of Civil Engineering

Bachelor of Technology - Level 5

CEX5231 – MECHANICS OF FLUIDS

FINAL EXAMINATION 2012/2013

Time allowed : Three Hours

Date : 12th August, 2013 Time : 0930 – 1230 hrs.

ANSWER **ALL THREE** QUESTIONS IN **PART A** AND **ANY TWO** QUESTIONS IN **PART B.** ALL QUESTIONS CARRY EQUAL MARKS.

PART A

Answer **all three** questions

1) A horizontal open channel has a uniform triangular cross-section with a vertex angle of 90° as shown in Figure 1a. The channel is closed using a special gate that maintains an opening of height **h** with the same triangular cross-section as shown in Figure 1b . The channel carries a discharge of $1\ m^3/s$.

a) Define specific energy in an open channel flow.

b) What is the relationship between specific energy and the total head in an open channel flow? Explain your answer with a neat diagram.

c) Obtain a relationship between the specific energy of the flow and the flow depth for the given open channel.

d) Calculate the depth upstream of the gate when the height of the gate opening is 0.3 m (h = 0.3 m). State all your assumptions.

e) Calculate the force on the gate when the height of the opening is h = 0.3 m . State all your assumptions.

f) What effect will the contraction of the flow have on your answers to sections d) and e)?

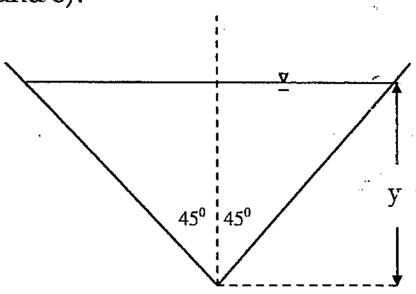

Figure 1a

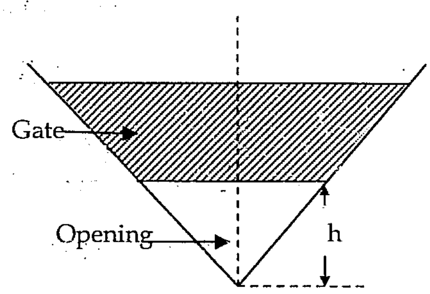

Figure 1b

2) A tank has a rectangular cross-section and is 2 m long and 0.5 m wide. The tank is filled with uniform fine sand of permeability of 2 m/day up to a depth of 1 m as shown in Figure 2. Steady uniform rainfall is simulated by allowing drops of water to fall on the top of the sand. The water soaks into the sand and forms a free surface within the sand layer, as shown in the Figure. For the first case the water level on the right hand side is maintained at a level 0.25 m above the bottom of the sand layer by allowing the water to flow out. Water is not allowed to flow out of the left hand side and it is observed that the height of the free surface at that end is 0.8 m above the bottom of the sand layer.

a) Derive, from first principles, a differential equation governing the height of the free surface above the bottom of the sand layer. State all your assumptions.

b) Using appropriate boundary conditions solve this differential equation and obtain an equation for the height of the free surface above the bottom of the sand layer. Explain your answer.

c) Calculate the rate of rainfall.

In the second case the rate of rainfall remains the same but now water is allowed to flow out of the left hand side as well as the right hand side. After conditions become steady it is found that 25% of the inflow from the rainfall flows out of the left hand side of the tank.

d) Sketch the height of the free surface in the sand layer for both cases and explain the differences.

e) Explain how you would calculate the height of the free surface on the left hand side for the second case. You DO NOT have to derive equations or perform calculations – you only have to state the method clearly.

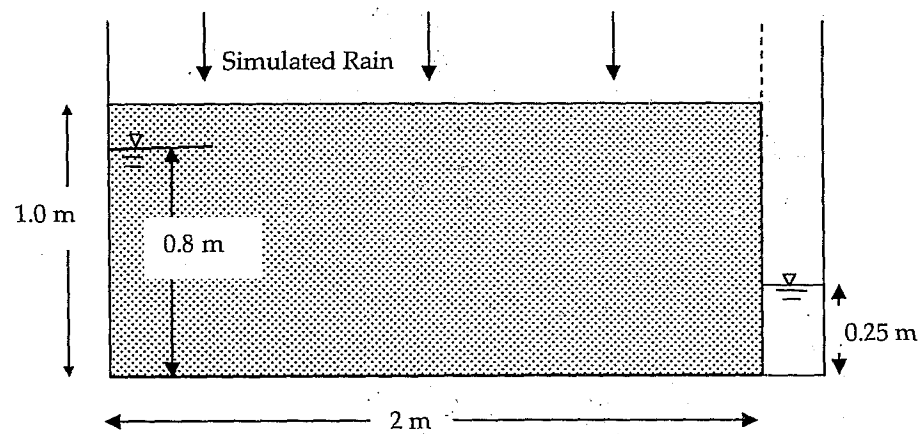

Figure 2

3) A long straight river has a rectangular cross-section of width **b**. The river has a uniform slope of S_o and has a plane bottom consisting of sand grains of a uniform diameter **d** and a density ρ_s. The river carries a discharge of **Q** of water of density ρ and viscosity μ with a uniform depth **h**. The acceleration due to gravity is **g**. The rate of sediment transport in the river as <u>submerged weight</u> of sediment is Q_s.

a) Are the variables b, S_o, d, ρ_s, Q, ρ, μ, h and g <u>independent</u> of each other? Explain your answer.

b) Obtain a non-dimensional relationship between the quantity Q_s and the <u>independent</u> variables that define the problem. Explain your answer.

c) Define the Reynolds number, the Froude number and the Shields parameter and explain their physical significance.

d) Discuss any relationship between the non-dimensional variables you have derived in section b) and the three non-dimensional variables defined in section c).

e) What comment would you make about a sediment transport formula that included ONLY the Shields' parameter?

PART B

Answer **any two** questions

4) An open channel has a rectangular cross-section with a width of 2 m and carries a steady discharge of 6.3 m^3/s. This channel has two sections, AB and BC, as shown in Figure 4. The section AB has a slope of 0.07 and a Manning's coefficient of 0.01 while the section BC has a slope of 0.0015 and a Manning's coefficient of 0.01. The flow far upstream and far downstream of B is observed to be uniform.

a) Show that a hydraulic jump will occur somewhere near B.

b) Sketch <u>two</u> possible free surface profile between A and B and classify the elements of the profile (from M1, M2, M3, S1, S2, S3, C1, C2, C3, etc.). Explain your answer.

c) Use the given information to determine which of the profiles sketched in section b) are correct in this case.

d) Explain how you would estimate the location of the hydraulic jump.

Note : The equations $\frac{dy}{dx} = \frac{S_0 - S_f}{1 - F_r^2}$ and $\frac{y_1}{y_2} = \frac{1}{2}\left(\sqrt{1 + 8F_{r2}^2} - 1\right)$ may be used.

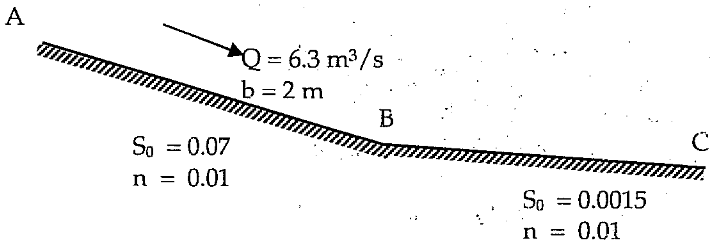

Figure 4

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5) a) Explain, using a neat figure, what is meant by a boundary layer.

b) When flying a kite it is sometimes necessary to run into the wind while trailing the kite behind until the kite reaches a certain elevation. Why is this necessary and why does it work?

c) Explain, using a neat figure, the difference between laminar and turbulent boundary layers for the same external velocity. Discuss the differences in the

i) Velocity profiles
ii) Thickness of the boundary layer
iii) Shear stress at the surface

d) Explain, using neat figures what is meant by "flow separation" and discuss its relationship to the boundary layer

e) Discuss the influence of the boundary layer on the drag force on objects.

6) A student wishes to analyze the ideal fluid flow resulting from a source of strength **m** in a corner, as shown in Figure 6a (z-plane). In order to simplify the analysis the flow is transformed to the ζ-plane as shown in Figure 6b.

a) Explain what is meant by an ideal fluid and a complex potential.

b) What is the transformation (relationship between z and ζ) that has been used?

c) What is the location of the source in the ζ-plane?

d) What is the complex potential in the ζ-plane? Explain your answer.

e) Locate any stagnation points in the z-plane.

f) Sketch the streamlines of the flow in the ζ-plane and the z-plane.

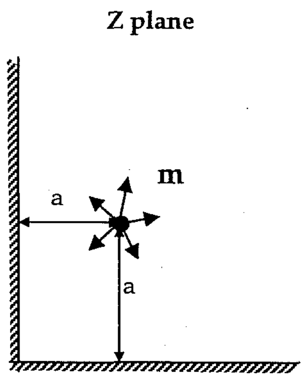

Figure 6a

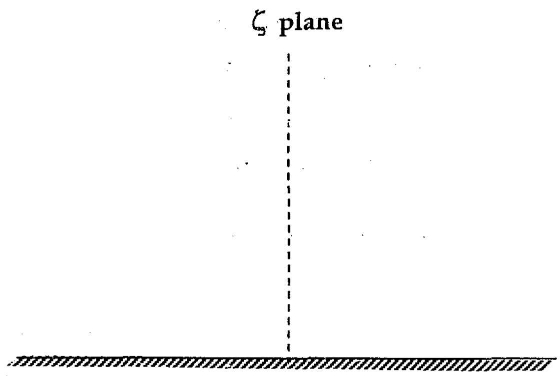

Figure 6b

7) The Navier-Stokes equations for a two-dimensional flow can be written in the form

$$\frac{\partial \underline{q}}{\partial t} + (\underline{q}.\underline{\nabla})\underline{q} = -\frac{1}{\rho}\underline{\nabla}(p) + \frac{\mu}{\rho}\nabla^2 \underline{q} \quad (1) \qquad \text{and} \qquad \nabla.\underline{q} = 0 \quad (2)$$

where $\underline{q} = u\underline{i} + v\underline{j}$ is the velocity vector and p is the piezometric pressure.

a) Derive Equation (2). Explain your answer using a neat diagram.

b) Explain, using a neat diagram, how Equation (1) has been derived. You DO NOT have to derive the equation – you have only to state the fundamental principle and list the steps followed.

A fluid density ρ and viscosity μ flows steadily and uniformly between two stationary horizontal parallel plates that are a distance **h** apart under the influence of a pressure gradient.

c) Use the Navier-Stokes equations to obtain a differential equation for the velocity distribution. Explain your answer.

d) Use the differential equation obtained in section c) to obtain an expression for the discharge per unit width between the plates.

8) Three Tanks – X, Y and Z – are connected by 3 pipes – AD, BD and CD – that all meet at a junction D as shown in Figure 8. All three pipes have a length of 100 m, a diameter of 100 mm and a friction factor of 0.01 . The differences between the water levels of the three tanks are shown in the figure.

Water is pumped out of Tank X by a Pump, P, as shown in the figure. The characteristics of the pump are given in Table 8.

a) Calculate the discharge in the three pipes when the water level differences are as shown in the figure. Explain your method.

Discharge (litres/second)	0	10	20	30	40
Pump Head (metres)	15	12	9	6	3

Table 8

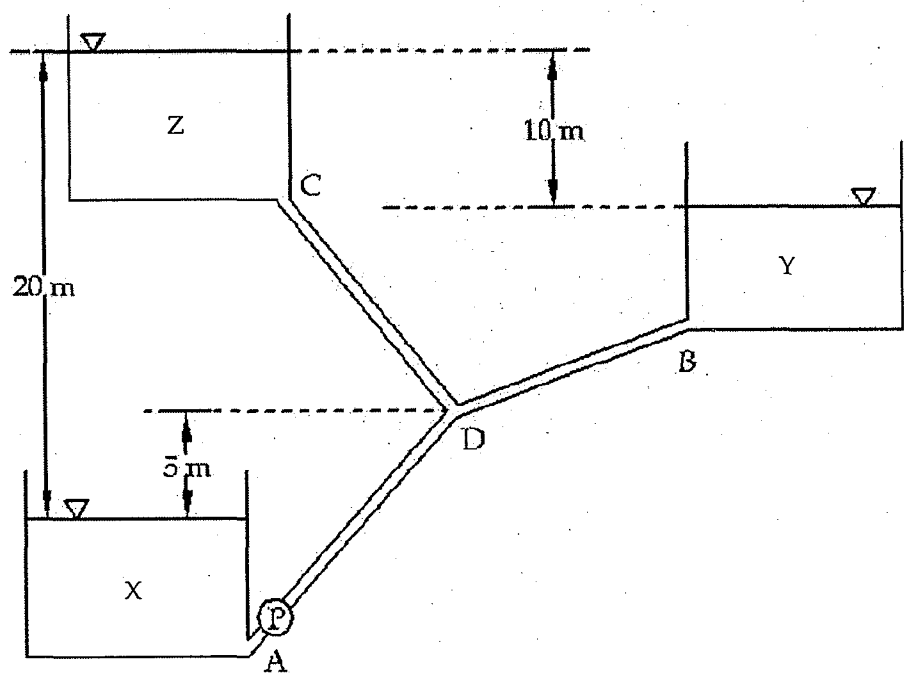

Figure 8