

THE OPEN UNIVERSITY OF SRI LANKA  
 BACHELOR OF INDUSTRIAL STUDIES / BACHELOR OF TECHNOLOGY  
 TTZ5244 – QUANTITATIVE TECHNIQUES



FINAL EXAMINATION 2005/2006

DURATION – THREE HOURS

Date: 10<sup>th</sup> April 2006

TIME: 0930 -1230 hrs.

Answer 05 Questions including question 01 which is compulsory. All questions carry 20 marks amounting to a total of 100.

- (1) (i) Solve the following equations (02 marks)
- (a)  $3^x \times 27^x = 243$  (b)  $7^{2x+1} + 2 = 345$
- (ii) When interest is payable continuously at rate  $x$ , an investment of Rs.  $P$  accumulates to  $S = P \cdot e^{tx}$ , after  $t$  years. Express  $t$  in terms of other variables. How long one must invest Rs. 5000 at a rate of  $x = 0.05$  to get an accumulated amount of Rs. 6920? (02 marks)
- (iii) If  $f(x) = X^4 - 2X^3 - X^2 - 5$  what is the value of  $f(0)$ ,  $f(1)$  and  $f(-1)$  (02 marks)
- (iv) Calculate the gradient of the curve  $Y = 12X^3 - 4X + 5$  at the point (1,8) (02 marks)
- (v) Find the stationary points of the following function. (02 marks)
- $$y = x^3 - 3x^2$$
- (vi) Determine the second derivatives of the following function (02 marks)
- $$y = 12x^4 - 4x^3 + 3x^2 + 2x + 5$$
- (vii) Give an example for a unit matrix. (02 marks)

Consider following matrices for questions (viii) and (ix)

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 2 & 1 & 0 \\ -1 & 2 & 1 \\ 0 & -1 & 1 \end{pmatrix}$$

- (viii) Determine  $AB$  (02 marks)  
(ix) Prove that  $(AB)^T = B^T A^T$  (02 marks)  
(x) Find the determinant of the matrix A, if (02 marks)

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 5 \\ 1 & 5 & 12 \end{pmatrix}_{3 \times 3}$$

(02) (a) Define the important parameters for a straight line graph. (02 marks)

(b) Discuss the importance of straight line graph in business applications. (08 marks)

(c) Cost to publish "X" number of Text books amounts to Rs. 30,000.00. Additional expense of Rs. 30 per copy is incurred for various other activities. The publisher receives Rs. 100 per copy from sales, but must pay 10% of this to the author.

- (i) Write an expression to calculate the publisher's total profit in terms of the number of copies "X" printed. Assume that all copies are sold.  
(ii) Sketch a graph to show number of copies printed versus total profit.  
(iii) Determine how many copies must be sold to achieve break even point. i.e make a profit of exactly zero. (10 marks)

(03) (a) Differentiate the following functions with respect to X

(i)  $Y = 6X^4 - 3X^3 + 2X^2 + 6$

(ii)  $Y = 4 \log_e 4x$

(iii)  $Y = (3X + 5)^2$

(iv)  $e^{7x} - \log_e (3x)$  (16 Marks)

- (b) In economics price elasticity of demand is defined as  $-\frac{P}{Q} \times \left( \frac{dQ}{dP} \right)$   
 where 'Q' is the demand and 'P' is the price.  
 If the demand function Q is given by  $Q = 100 - P^2$   
 What is the elasticity of demand at  $P = 3$ ? (04 Marks)

(04) A small scale manufacturer of soft toys has found that when he charges Rs. 5 per item, he can sell 100 toys per week. If he lowers the price to Rs. 4, he can sell 120 toys a week.

(i) Assume that number of items sold (c), per week to be a linear function of price (p), (in the form of  $c = ap + b$ ), determine the constants a and b. (08 Marks)

(ii) If the revenue is given by  $r = c \times p$ , obtain an expression for revenue as a function of price. (04 Marks)

(iii) Calculate the price, in order to get the maximum revenue. (08 Marks)

(05) Write the following equation in a **matrix form** and solve the equations.

$$X + 2Y + 3Z = 14$$

$$3X + Y + 2Z = 11$$

$$2X + 3Y + Z = 11$$

(20 Marks)

(06) ABC Textile company produces three types of cloths A, B, and C. Three kinds of wool are required to produce the cloths. They are, red wool, green wool and blue wool. One Unit length of type A cloth needs 2m of red wool and 3m of blue wool; one unit length of type B cloth needs 3m of red wool, 2m of green wool and 2m of blue wool; and one unit length of type C cloth need 5m of green wool and 4m of blue wool. The company has a stock of only 80m of red wool, 100m of green wool and 150m of blue wool. It is assumed that the income obtained from one unit length of type A cloth is Rs. 3, type B cloth is Rs. 5 and that of type C cloth is Rs. 4. Formulate the problem as a linear programme. (20 Marks)

(07) A manufacturing company produces two products A & B with the profits earned per unit being Rs. 3 and Rs. 4 respectively. Each product is processed on two machines  $M_1$  and  $M_2$ . Product A requires 1 minute of processing time on  $M_1$  and 2 minutes on  $M_2$ . Product B requires 1 minute on  $M_1$  and 1 minute on  $M_2$ . Machine  $M_1$  is available for not more than 7 hrs and 30 mins, while machine  $M_2$  is available for 10 hrs during any working day.

(a) What are the variables in this problem? (02 Marks)

(b) What are the constraints of the problem? (04 Marks)

(c) Solve the formatted programme graphically to determine the number of units of products A and B to be manufactured to get maximum profit. (10 Marks)

(d) Calculate the maximum profit. (04 Marks)

(08) A certain garment factory wants to produce two products, shirts and blouses. The products have to be finished within one week, where the total number of productive labour hours available are 2000 hours. Time required to produce one shirt is 1 hour and the blouse is 2 hours. Fabric available with the company are 2300m, where a shirt needs 2m and a blouse 1m. A shirt and a blouse contribute Rs. 5 and Rs. 6 respectively to the profit of the company. The planning manager wishes to determine the number of shirts and blouses to be produced in order to maximize the total profit.

(i) Formulate the above problem as a linear programming model.

(ii) Determine the optimal product mix. (20 Marks)