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THE OPEN UNIVERSITY OF SRI LANKA
COMMONWEALTH EXECUTIVE MASTER OF BUSINESS/PUBLIC ADMINISTRATION
FINAL EXAMINATION – 2012
MCP 1607 – QUANTITATIVE TECHNIQUES FOR MANAGERS
DURATION : THREE (03) HOURS

DATE : 24th March 2012 **TIME : 9.30am – 12.30 pm**

INSTRUCTIONS TO CANDIDATES

a) Answer any five (05) Questions only.
b) Each question carries 20 marks.
c) Write your index number on every page.
d) Use of non-programmable calculator is allowed.
e) Graph paper will be provided.
f) Necessary statistical table and mathematical formulae annexed.

Q1. "Quantitative Techniques are used by managers at different levels in their day to day work. These techniques have their own limitations; however they have proved to be useful. What is important for managers is to know what the techniques are and what they are used for. They also should develop the skills of interpreting their problems to match the available techniques". Elaborate this statement with examples.

Q2. a) Find the differential coefficient of the following functions with respect to "x".

(i) $2x^4+3x^2+7x+4$ (ii) $(x^3+3)(2x^2+5x+3)$

(iii) $\dfrac{x^2+4}{x^3+3}$

b) If $y=x^3+3x^2+2x+1$ Find $\dfrac{d^2y}{dx^2}$

c) Management of a company is concerned about the money spent on sales promotion work. They have established the following relationship between monthly profit "P" and monthly money spent on promotion work "x".
(Both measured in Rs. "000")

$$P=500-4x^2+1600x$$

Find the optimal amount of money that should be spent on sales promotion in order to maximize profit.

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d) "A" and "B" are to matrices defined as follows.

$$A = \begin{bmatrix} 2 & 7 & 8 \\ 5 & 3 & 4 \\ 9 & 4 & 6 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 6 & 4 \\ 2 & 1 & 2 \\ 3 & 1 & 5 \end{bmatrix}$$

Evaluate the following

(i) A + B

(ii) A − B

(iii) A x B (Vector Multiplication)

e) "A", "B" and "C" are three cities. The distance from "A" to "B" is 20 km. The distance from "A" to "C" is 15 km and from "B" to "C" is 30 km.

(i) Develop a matrix that expresses the distance between any two cities.

(ii) Why is the above matrix developed called a symmetrical matrix.

(iii) If the fuel required to cover 1 km is 'k' and if the fuel required to travel from cities A, to the remaining two cities or from B or C to the remaining two cities are x, y *and* z respectively, develop a matrix equation connecting x, y *and* z to the original matrix developed.

Q3. a) A dice is thrown twice and the sum of scores calculated. Considering the sum of scores to be a random variable, write down the probability distribution function for the sum of scores.

b) Due to turnover and absenteeism at an assembly plant, 20% of the items are assembled by inexperienced employees. The management has determined that customers return 12% of the items assembled by inexperienced employees. Only 5% of items assembled by experience employees are returned.

(i) What is the probability that an item is returned?

(ii) Given that an item has been returned what is the probability that it was assembled by an inexperienced employee?

Q4. a) A motorist observed that the distance he could cover with a tank full or petrol is normally distributed with mean 350 km and standard deviation 50km.

(i) What is the probability that he could cover more than 400 km with a tank full of petrol?

(ii) What is the maximum distance he could cover with a tank full of petrol, so that he can be 95% sure (certain) that he will not run out of petrol?

b) Past experience indicates that 30% of individuals entering a certain store decide to make a purchase. What is the probability that at least two out of a group of five individuals who just entered would make a purchase?

c) A manager of a retail shop observes that on the average he receives two complaints a day. What is the probability that he will receive three complaints the next day?

Q5. The following table describes the daily output and the number of years of service of eight operators.

OPERATOR NUMBER	NUMBER OF YEARS OF SERVICES	DAILY OUT PUT RS. "000"
1	7	1.0
2	12	2.0
3	25	4.0
4	5	0.6
5	10	1.8
6	30	4.2
7	20	3.6
8	15	3.0

(i) Calculate the coefficient of correlation between years of service and daily output.

(ii) Develop the regression equation of the form $y = a + bx$ where "y" is the output and "x" is years of service.

(iii) Estimate the daily output of an operator with 18 years of service.

(iv) Calculate the "Sum of Squares Error" Given by SSE.

(v) Calculate the error of the regression model given by "S_{yx}".

(vi) Calculate the coefficient of determination.

(vii) Calculate the standard error of the "b" coefficient given by "S_b"

(viii) Evaluate the 95% confidence interval for the "b" coefficient.

Q6. The following time series data explains the price of a kilogram of rice during 1996 to 2000. The centered moving average is all ready calculated.

YEAR	QUARTER	PRICE	CENTERED MOVING AVERAGE	YEAR	QUARTER	PRICE	CENTERED MOVING AVERAGE
1996	3	19.00	18.50	1998	3	23.50	22.90
	4	19.50	18.90		4	24.75	23.70
1997	1	20.50	19.12	1999	1	25.00	24.50
	2	17.00	19.56		2	23.00	25.40
	3	20.00	20.00		3	27.00	26.40
	4	22.00	20.50		4	28.50	27.40
1998	1	21.75	21.40	2000	1	29.00	28.25
	2	20.00	22.15		2	27.00	29.00

(i) Calculate the "Root Mean Square Error" (RMSE) for this forecasting model based on moving average.

(ii) Exponentially smooth the time series data with a smoothing constant of 0.2 (The initial average 18.50 may be taken as your first smoothed value)

(iii) Calculate the Root Mean Square Error (RMSE) for the forecasting model based on exponential smoothing.

(iv) Briefly discuss the two forecasting models.

Q7) Write short notes on the following.

a) Non-Linear Regression
b) Residual Analysis
c) Hetro – Scadasticity
d) Multi-Collenearity

MATHEMATICAL FORMULAE

1) Binomial Probability $^{n}C_{r}P^{r}q^{(n-r)}$

2) Poisson Probability $e^{-a}\frac{a^{x}}{x!}$

3) $$r=\frac{\sum xy-\frac{\left(\sum x\right)\left(\sum y\right)}{n}}{\sqrt{\left[\sum x^{2}-\frac{\left(\sum x\right)^{2}}{n}\right]\left[\sum y^{2}-\frac{\left(\sum y\right)^{2}}{n}\right]}}$$

4) $$b=\frac{n\sum xy-\sum x\sum y}{n\sum x^{2}-(\sum x)^{2}} \qquad a=\frac{\sum y}{n}-b\frac{\sum x}{n}$$

5) $$SST=\sum y^{2}-n\bar{y}^{2}$$
$$\left(\bar{y}=\frac{\sum y}{n}\right)$$

6) $$SSE=\sum y^{2}-a\sum y-b\sum xy$$

7) Coefficient of Determination $=1-\frac{SSE}{SST}$

8) $$S_{yx}=\sqrt{\frac{SSE}{(n-2)}}$$

9) $SSX=\sum x^{2}-n\bar{x}^{2}$ Where $\bar{x}=\frac{\sum x}{n}$

10) $$S_{b}=\frac{S_{yx}}{\sqrt{SSX}}$$

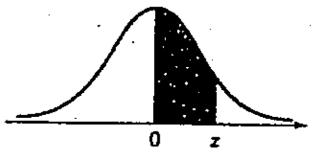
11) Smoothing Model

$$F_{t}=\alpha D_{t}+(1-\alpha)F_{t-1}$$

12) $$RMSE=\sqrt{\frac{\sum(ERROR)^{2}}{n}}$$

	Level of Significance for One-Tailed Test					
	.10	*.05*	*.025*	*.01*	*.005*	*.0005*
	Level of Significance for Two-Tailed Test					
d.f.	*.20*	*.10*	*.05*	*.02*	*.01*	*.001*
1	3.078	6.314	12.706	31.821	63.657	636.619
2	1.886	2.920	4.303	6.965	9.925	31.598
3	1.638	2.353	3.182	4.541	5.841	12.941
4	1.533	2.132	2.776	3.747	4.604	8.610
5	1.476	2.015	2.571	3.365	4.032	6.859
6	1.440	1.943	2.447	3.143	3.707	5.959
7	1.415	1.895	2.365	2.998	3.499	5.405
8	1.397	1.860	2.306	2.896	3.355	5.041
9	1.383	1.833	2.262	2.821	3.250	4.781
10	1.372	1.812	2.228	2.764	3.169	4.587
11	1.363	1.796	2.201	2.718	3.106	4.437
12	1.356	1.782	2.179	2.681	3.055	4.318
13	1.350	1.771	2.160	2.650	3.012	4.221
14	1.345	1.761	2.145	2.624	2.977	4.140
15	1.341	1.753	2.131	2.602	2.947	4.073
16	1.337	1.746	2.120	2.583	2.921	4.015
17	1.333	1.740	2.110	2.567	2.898	3.965
18	1.330	1.734	2.101	2.552	2.878	3.922
19	1.328	1.729	2.093	2.539	2.861	3.883
20	1.325	1.725	2.086	2.528	2.845	3.850
21	1.323	1.721	2.080	2.518	2.831	3.819
22	1.321	1.717	2.074	2.508	2.819	3.792
23	1.319	1.714	2.069	2.500	2.807	3.767
24	1.318	1.711	2.064	2.492	2.797	3.745
25	1.316	1.708	2.060	2.485	2.787	3.725
26	1.315	1.706	2.056	2.479	2.779	3.707
27	1.314	1.703	2.052	2.473	2.771	3.690
28	1.313	1.701	2.048	2.467	2.763	3.674
29	1.311	1.699	2.045	2.462	2.756	3.659
30	1.310	1.697	2.042	2.457	2.750	3.646
40	1.303	1.684	2.021	2.423	2.704	3.551
60	1.296	1.671	2.000	2.390	2.660	3.460
120	1.289	1.658	1.980	2.358	2.617	3.373
∞	1.282	1.645	1.960	2.326	2.576	3.291

Normal Curve Areas

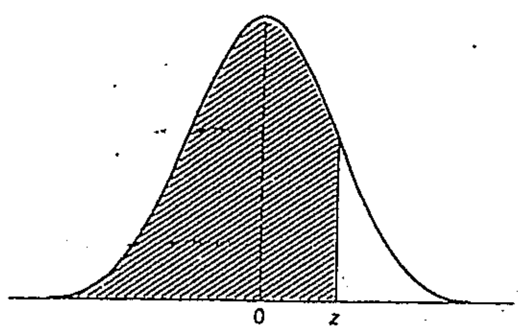


z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990

Source: Abridged from Table I of A. Hald, *Statistical Tables and Formulas* (New York: John Wiley & Sons, Inc.), 1952. Reproduced by permission of A. Hald and the publisher.

Table A2. Values of z, the standard normal variable, from 0·0 by steps of 0·01 to 3·9, showing the cumulative probability up to z.
(Probability correct to 4 decimal places).

z	0·00	0·01	0·02	0·03	0·04	0·05	0·06	0·07	0·08	0·09
0·0	·5000	·5040	·5080	·5120	·5160	·5199	·5239	·5279	·5319	·5359
·1	·5398	·5438	·5478	·5517	·5557	·5596	·5636	·5675	·5714	·5753
·2	·5793	·5832	·5871	·5910	·5948	·5987	·6026	·6064	·6103	·6141
·3	·6179	·6217	·6255	·6293	·6331	·6368	·6406	·6443	·6480	·6517
·4	·6554	·6591	·6628	·6664	·6700	·6736	·6772	·6808	·6844	·6879
·5	·6915	·6950	·6985	·7019	·7054	·7088	·7123	·7157	·7190	·7224
·6	·7257	·7291	·7324	·7357	·7389	·7422	·7454	·7486	·7517	·7549
·7	·7580	·7611	·7642	·7673	·7704	·7734	·7764	·7794	·7823	·7852
·8	·7881	·7910	·7939	·7967	·7995	·8023	·8051	·8078	·8106	·8133
·9	·8159	·8186	·8212	·8238	·8264	·8289	·8315	·8340	·8365	·8389
1·0	·8413	·8438	·8461	·8485	·8508	·8531	·8554	·8577	·8599	·8621
·1	·8643	·8665	·8686	·8708	·8729	·8749	·8770	·8790	·8810	·8830
·2	·8849	·8869	·8888	·8907	·8925	·8944	·8962	·8980	·8997	·9015
·3	·9032	·9049	·9066	·9082	·9099	·9115	·9131	·9147	·9162	·9177
·4	·9192	·9207	·9222	·9236	·9251	·9265	·9279	·9292	·9306	·9319
·5	·9332	·9345	·9357	·9370	·9382	·9394	·9406	·9418	·9429	·9441
·6	·9452	·9463	·9474	·9484	·9495	·9505	·9515	·9525	·9535	·9545
·7	·9554	·9564	·9573	·9582	·9591	·9599	·9608	·9616	·9625	·9633
·8	·9641	·9649	·9656	·9664	·9671	·9678	·9686	·9693	·9699	·9706
·9	·9713	·9719	·9726	·9732	·9738	·9744	·9750	·9756	·9761	·9767
2·0	·9772	·9778	·9783	·9788	·9793	·9798	·9803	·9808	·9812	·9817
·1	·9821	·9826	·9830	·9834	·9838	·9842	·9846	·9850	·9854	·9857
·2	·9861	·9864	·9868	·9871	·9875	·9878	·9881	·9884	·9887	·9890
·3	·9893	·9896	·9898	·9901	·9904	·9906	·9909	·9911	·9913	·9916
·4	·9918	·9920	·9922	·9925	·9927	·9929	·9931	·9932	·9934	·9936
·5	·9938	·9940	·9941	·9943	·9945	·9946	·9948	·9949	·9951	·9952
·6	·9953	·9955	·9956	·9957	·9959	·9960	·9961	·9962	·9963	·9964
·7	·9965	·9966	·9967	·9968	·9969	·9970	·9971	·9972	·9973	·9974
·8	·9974	·9975	·9976	·9977	·9977	·9978	·9979	·9979	·9980	·9981
·9	·9981	·9982	·9982	·9983	·9984	·9984	·9985	·9985	·9986	·9986
3·0	·9987	·9987	·9987	·9988	·9988	·9989	·9989	·9989	·9990	·9990
·1	·9990	·9991	·9991	·9991	·9992	·9992	·9992	·9992	·9993	·9993
·2	·9993	·9993	·9994	·9994	·9994	·9994	·9994	·9995	·9995	·9995
·3	·9995	·9995	·9995	·9996	·9996	·9996	·9996	·9996	·9996	·9997
·4	·9997	·9997	·9997	·9997	·9997	·9997	·9997	·9997	·9997	·9998
·5	·9998	·9998	·9998	·9998	·9998	·9998	·9998	·9998	·9998	·9998
·6	·9998	·9998	·9999	·9999	·9999	·9999	·9999	·9999	·9999	·9999
·7	·9999	·9999	·9999	·9999	·9999	·9999	·9999	·9999	·9999	·9999
·8	·9999	·9999	·9999	·9999	·9999	·9999	·9999	·9999	·9999	·9999
·9	1·0000									



The curve is $N(0, 1)$, the standard normal variable. The table entry is the shaded area $\Phi(z) = \Pr(Z < z)$. For example, when $z = 1{\cdot}96$ the shaded area is 0·9750. Critical values of the standard normal distribution will be found in the bottom row of Table A3.